Exercises belonging to lecture
Observational Astronomy MKEP5 (SS 2011)

Sheet 10

1. **Some special properties of Fourier transforms**
   The Fourier transform of a function $f(x)$ is defined by

$$g(u) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi i u x} dx \tag{1}$$

You can use the appendix on Fourier transforms to answer the following questions:

(a) Suppose $f(x) = e^{ikx}$. Give the Fourier transform $g(u)$. Show that a shift in x by an amount l leads to a complex rotation (i.e. rotation in the complex plane) of $g(u)$. Give the amount of rotation. How does it depend on the wavenumber k?

(b) Suppose $f(x) = \delta(x - x_0)$. Give the Fourier transform $g(u)$. The case of $x_0 = 0$ is a special case: the Fourier transform is real everywhere. What happens if $x_0 \neq 0$? And how does this behave for ever increasing x_0?

With this knowledge you will be able to understand that the phase of the complex visibility $\mathcal{V}$ tells something about the position of things on the sky.

(c) Suppose $f(x) = e^{-x^2/\theta^2}$ for some $\theta > 0$. Give the Fourier transform $g(u)$. How does it change if θ changes?

With this knowledge you will be able to understand the relation between the visibility amplitude function $V(b)$ and the size of an object on the sky (large object = small visibility, small object = large visibility).

2. **VLT-MIDI mid-infrared interferometry of an extended source**
   A convenient way of formulating the Van Cittert-Zernike Theorem is that the complex visibility $\mathcal{V}$ as a function of u and v (the coordinates of the uv-plane) measured with an interferometric telescope array is the Fourier transform of the image on the sky, normalized by the total flux:

$$\mathcal{V}(u, v) = \frac{1}{F} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I(n_x, n_y) e^{-2\pi i (u n_x + v n_y)} dn_x dn_y \tag{2}$$

where n_x and n_y are the transverse components of the direction vector $\vec{n}$ toward the sky, with the z-coordinate pointing toward the pointing direction of the interferometer. Note: The integration domain is not *really* infinity - it is at most the size of the primary beam (the PSF of a single telescope).

(a) Given that, in our definition, the visibility $\mathcal{V}$ has an amplitude between 0 and 1, and assuming that the source is smaller than the primary beam, explain why we use $1/F$ as normalization factor. Hint: for a special point in the uv-plane we *know* by definition what $\mathcal{V}$ must be - which point is that and what is $\mathcal{V}$ there?

With the Very Large Telescope (VLT) on Paranal in Chile you can do mid-infrared interferometry. The instrument for that is called MIDI. Unfortunately the VLT has "only" 4 large telescopes, and often you only get observing time on two of them to do your interferometric measurements. This means that you measure just a few visibility points in the uv-plane. Moreover, you only measure the visibility *amplitude* $V = |\mathcal{V}|$, not the complex visibility $\mathcal{V}$. This means that you cannot do image reconstruction. The only thing you can do is to make a model of what you think the image looks like on the sky, then compute the visibility V for a given baseline, and then compare to the measured visibility and see if it fits. The simplest model of a source is perhaps a disk on the sky with uniform intensity I and angular radius θ_r (so that the solid angle of this disk is $\pi\theta_r^2$ and thus the flux is $F = \pi\theta_r^2 I$).

(b) Give, for this model, the expression of the visibility amplitude V as a function of projected baseline b of a pair of telescopes (b is, in CGS units, expressed in cm), for given wavelength λ (which is, in the case of MIDI roughly 10 μm, but you can keep the expression general for arbitrary λ).

(c) If the disk has a radius of $\theta_r = 0.05$ arcseconds on the sky, at which projected baseline b will the visibility amplitude $V(b)$ reach its first null (i.e. at which b is $V(b) = 0$)?

(d) How large is the disk on the sky if we measure the first null at twice that value for b?

3. **Radio interferometry map of an extended source**
With ALMA you will be able to get millimeter-wave images of objects with unprecedented sensitivity and spatial resolution. As with all long baseline interferometers you have to make sure that you have enough uv-coverage. Suppose you want to make a millimeter-wave image of some extended object on the sky which has structure on both small and large angular scales.

   (a) If you have a compact telescope array configuration (the dishes are close together), which structures of your source will you *not* be able to see?

   (b) If you have an extended telescope array configuration (the dishes are far apart), which structures of your source will you *not* be able to see?

4. **The "near field" of an interferometer**
Suppose we have an interferometric radio telescope array consisting of three telescopes arranged along a line, with spacing L. If a perfectly plane electromagnetic wave with wavelength λ hits the telescope, then the phase difference ϕ_{12} between telescopes 1 and 2 must be identical to the phase shift ϕ_{23} between telescopes 2 and 3. However, if the object to be observed is not at infinite distance, but at some finite distance d, then the wave crests are not perfect plane waves, but instead they are slightly curved (i.e. spherical with the center of the sphere being the source at distance d). The "near field" of an interferometer is defined as the distance d out to which this curvature effect can cause changes in the phase of order 1.

   (b) Show that this near field is defined as

$$d_{\text{near}} \simeq \frac{L^2}{\lambda} \tag{3}$$

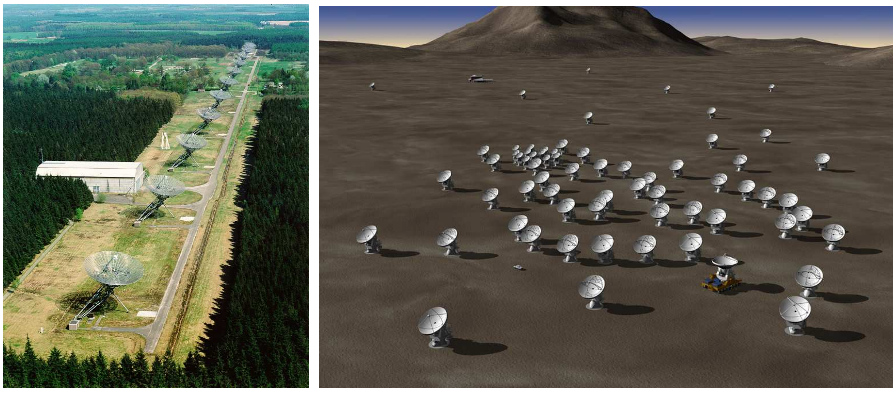

Figure 1: Left: The Westerbork radio array in the Netherlands. Right: Artist impression of future status of the Atacama Large Millimeter Array (ALMA) in Chile.

(Tip: Do not bother about factors of a few; the functional form is important).

(c) Argue that you can use the above 3-telescope interferometer to measure the distance to the source, as long as the source is in the near field.

(d) Show that if we make a $\lambda = 21$ cm space radio interferometer with one radio telescope on Earth, and two space-based radio telescopes at 1 AU (= distance Earth-Sun = 1.5×10^{13} cm) on each side of the Earth, we can measure the distance to the Andromeda Galaxy (in other words: that the Andromeda galaxy, which is at roughly 800 kiloparsec distance, becomes near-field!).

5. **A few conceptual questions...**

(a) In Fig. 1-left you see the Westerbork radio telescope array in the Netherlands. The dishes are aligned along a line. It seems that you have uv-coverage (and therefore spatial resolution) only in a single direction. Can one nevertheless acquire good uv-coverage with this linear telescope array, and if yes, how?

(b) Would this trick still work if this array would have been located at the Earth's equator and arranged north-south?

(c) In addition to being on a line, the dishes of Westerbork also have regular spacing. Modern radio interferometers (such as ALMA, see Fig. 1-right) have the dishes located at seemingly "random" positions. What is the advantage of that compared to the regular spacing of the Westerbork telescope?

(d) As seen in Fig. 1-right, a number of ALMA dishes are arranged close to each other, while others are spaced increasingly farther away. Explain why this is done so.