Stellar Astronomy and Astrophysics (SS07):

Exercise 1 (for May 5, 2007)

1. The Stellar IMF:

The distribution of stellar masses at birth in the solar neighbourhood can be described by the following functional form:

$$\xi(m) = \begin{cases} 0.26m^{-0.5} & \text{for} \quad 0.01 < m \leq 0.08 \\ 0.035m^{-1.3} & \text{for} \quad 0.08 < m \leq 0.5 \\ 0.019m^{-2.3} & \text{for} \quad 0.5 < m < 100. \end{cases}$$

The quantity $\xi(m)dm$ indicates the number of objects per cubic parsec, pc^3, in the mass interval m to $m + dm$, with mass m given in units of solar mass $M_\odot$. Objects with $m < 0.08$ are brown dwarfs. Their mass is too small for hydrogen burning in the center. Stars with $m > 100$ are not stable.

a: The quantity $\xi(m)$ is the differential number density in the (linear) mass interval m to $m + dm$. Often, however, it is better to consider the differential number density ξ_L in the (logarithmic) mass bin $\log_{10} m$ to $\log_{10} m + d\log_{10} m$. Calculate ξ_L and plot $\log_{10} \xi_L$ versus $\log_{10} m$.

b: What is the average stellar mass?

c: Sirius is the brightest star on the sky. Actually, it is a binary system, with Sirius A being 2.1 times heavier than the Sun (with a spectral type A1V) and Sirius B being a white dwarf. The system is at at distance of ≈ 2.6 pc. How many stars with the mass of Sirius A and heavier do you expect within a distance of 10 pc from the Sun?

d: An alternative description of the IMF is

$$\xi^*(m) = \begin{cases} 0.158 \exp[-0.5(\log_{10} m - \log_{10} m_c)^2/\sigma^2] & \text{for} \quad m \leq 1.0 \\ 0.044m^{-1.3} & \text{for} \quad m \; 1.0 \end{cases}$$

with a characteristic mass $m_c = 0.08$, and a width of the distribution $\sigma = 0.69$ at small masses. For $m < 1.0$ the distribution is a log normal, above one solar mass it follows a power law with the classical Salpeter slope.

To compare both distributions, compute the number of stars in the mass range $0.01 < m < 1.0$ for ξ and ξ^*.

2. Intensity, total flux:

Suppose that the intensity of a light bulb varies with direction as

$$I(\theta) = \frac{1}{2} I(0)(1 + \cos\theta)$$

Draw this intensity distribution vs. θ and vs. $\mu = \cos\theta$.

What do you obtain for J, F, and K in terms of the forward intensity $I(0)$?

3. Blackbody:

1. By what factor should the temperature of a black body be increased so that

a: The integrated flux (over all frequencies) is doubled?

b: The frequency at which the intensity is greatest is doubled?

2. What is the total number of photons inside an oven set at 200 Celsius with a volume of 1 m^3.

3. The tungsen filament of a light bulb has a temperature of 2700 K. If we assume blackbody radiation:

a: What amount of the energy is radiated in the visible range of the electromagnetic spectrum (4000-7000 Å).

b: What is the energy per blackbody photon at the center of the sun ($T = 1.58 \cdot 10^7$ K and in the solar photosphere ($T = 5770$ K) expressed in electron Volts (1eV=$1.6 \cdot 10^{-12}$ erg).