Stellar Astronomy and Astrophysics (SS07):

Exercise 4 (for July 5, 2007)

1. Bolzmann formula:

a At what temperature will equal numbers of atoms have electrons in the ground state and in the second excited state ($n = 3$)?

b As the temperature $T \to \infty$, how will the electrons in the hydrogen atoms be distributed, according to the Boltzmann equation? That is, what will be the relative numbers of electrons in the $n = 1, 3, 3, \ldots$ orbitals? Will thins in fact be the distribution that actually occurs?

2. Saha formula:

The Saha formula

$$\frac{n^{j+1} n_e}{n^j} = 2 \frac{Z^{j+1}}{Z^j} \left(\frac{2\pi m_e k T}{h^2} \right)^{3/2} \mathrm{e}^{\chi_i / kT}$$

was derived in the lecture.

Consider a box of electrically neutral hydrogen that is maintained in a constant unit volume ($n_e = n_{\mathrm{II}}$). The total density of hydrogen atoms is $n_t = n_{\mathrm{I}} + n_{\mathrm{II}} = \rho/(m_p + n_e) \approx \rho/n_p$, with $n_{\mathrm{I}}, n_{\mathrm{II}}, n_{\mathrm{e}}$ being the density of neutral hydrogen atoms, ionized hydrogen atoms, electron, respectively, and ρ, m_p, m_e being the mass density, and the masses of protons and electrons.

a Derive the quadratic equation $\left(\frac{N_{\mathrm{II}}}{N_t}\right)^2 + \left(\frac{N_{\mathrm{II}}}{N_t}\right) f(T) + g(T) = 0$.

b What are the approximate values for Z_{I} and Z_{II}?

c Assume $\rho = 10^{-9}$ g cm-3. Solve the quadratic equation for temperatures between T=5000 K and T=25000 K and plot the result.

3. Critical mass for collapse:

Consider the stability of a non-rotating, spherical, isothermal gas cloud. Compare the critical mass for gravitational collapse in the Jeans case (self-gravitating, constant-density sphere embedded in infinite homogeneous background medium) with the Bonnor-Ebert case (selfgravitating, isothermal sphere in pressure equilibrium with external medium).

4. Rotating collapse:

Consider the collapse of an initially spherical, isothermal, rotating gas cloud.

a Show that conservation of angular momentum will stop further contraction in the plane perpendicular to the rotational axis when the radius reaches

$$r_f = \frac{\omega_0^2 r_0^4}{2GM_r} \,,$$

where M_r is the mass interior to r, and ω_0 and r_0 are the original angular velocity and radius of the cloud's surface, respectively. Assume that the initial radial velocity of the cloud is zero and that $r_f \ll r_0$. Assume furthermore that the cloud remains in rigid-body rotation during the collapse. This is a wrong assumption, but helps to simplify the calculation.

b If the cloud has a mass of $1\,\mathrm{M}_\odot$ and an initial radius of 0.5 pc, and if collapse is halted at a radius of 100 AU, what is the initial angular velocity of the cloud? And what are the circular velocities at the equator, initially as well as at the final stage? Assume that the initial moment of inertia is approximately that of a uniform sphere ($I = 2/5Mr^2$), while the final moment of inertia is that of a uniform disk $I = 1/2Mr^2$).

c Compute the rotational period of the disk that forms. Compare that with the orbital time of a planet around a solar mass star at 100 AU distance .