Stellar Astronomy and Astrophysics (SS08):

Exercise 1 (for April 24, 2008)

1. The Stellar IMF:

The distribution of stellar masses at birth in the solar neighbourhood can be described by the following functional form:

$$\xi(m) = \begin{cases} 0.26m^{-0.5} & \text{for} \quad 0.01 < m \leq 0.08 \\ 0.035m^{-1.3} & \text{for} \quad 0.08 < m \leq 0.5 \\ 0.019m^{-2.3} & \text{for} \quad 0.5 < m < 100. \end{cases}$$

The quantity $\xi(m)dm$ indicates the number of objects per cubic parsec, pc^3, in the mass interval m to $m+dm$, with mass m given in units of solar mass $M_\odot$. Objects with $m < 0.08$ are brown dwarfs. Their mass is too small for hydrogen burning in the center. Stars with $m > 100$ are not stable.

a: The quantity $\xi(m)$ is the differential number density in the (linear) mass interval m to $m + dm$. Often, however, it is better to consider the differential number density ξ_L in the (logarithmic) mass bin $\log_{10} m$ to $\log_{10} m + d\log_{10} m$. Calculate ξ_L and plot $\log_{10} \xi_L$ versus $\log_{10} m$.

b: What is the average stellar mass?

c: Sirius is the brightest star on the sky. Actually, it is a binary system, with Sirius A being 2.1 times heavier than the Sun (with a spectral type A1V) and Sirius B being a white dwarf. The system is at at distance of ≈ 2.6 pc. How many stars with the mass of Sirius A and heavier do you expect within a distance of 10 pc from the Sun?

2. Protostellar collapse:

Consider the idealized case of the collapse of a pressure-free isothermal sphere with constant initial density.

a How does the collapse timescale depend on density?

b For an initial density of $n(\mathrm{H_2}) = 10^5\ \mathrm{cm}^{-3}$, estimate the collapse timescale.

c Speculate on how *real* protostellar collapse may deviate from the above idealized picture. List three characteristic differences.

3. Pre-main-sequence contraction:

Consider for simplicity fully convective protostars and use the virial theorem to obtain order of magnitude estimates.

a Describe qualitatively the behavior of the central temperature T_c if the object contracts? What is the reason for this behavior?

b For a one solar-mass object, estimate how long the pre-main-sequence contraction phase will last. Assume the object radiates with constant luminosity equal to the present-day Sun.

c Why does the contraction eventually stop?