Assignment #5: due Thursday, Nov. 13, 2008

Theoretical Astrophysics

Winter 2008/2009
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1. **Rankine-Hugoniot conditions** 35 pt

   (a) Show that if gas flows into a planar, stationary shock front along the shock normal, the density, velocity and pressure on either side of the shock are related by the following three jump conditions:

$$\rho_1 v_1 = \rho_2 v_2, \tag{1}$$

$$p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2 \tag{2}$$

$$\frac{v_1^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_1}{\rho_1} = \frac{v_2^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_2}{\rho_2} \tag{3}$$

   where γ is the adiabatic index of the gas, and where the subscripts 1 and 2 refer to the upstream and downstream quantities, respectively. [*Hint: use the hydrodynamic equations in conservative form to identify the fluxes of particles, momentum and energy entering and leaving the shock front.*]

   (b) Show that the downstream pressure p_2 can be written as

$$p_2 = p_1 + \rho_1 v_1 (v_1 - v_2) \,. \tag{4}$$

   (c) Using Eq. 4 and the shock jump conditions, find an expression for the compression ratio $r \equiv \rho_2/\rho_1$ in terms of the upstream Mach number $\mathcal{M} = v_1/c_1$, where $c_1 = \sqrt{\gamma p_1/\rho_1}$ is the upstream sound speed. [*Hint: begin by eliminating* p_2]

   (d) A supernova remnant shock moves at 5 000 km/sec into the interstellar medium (temperature $\approx 10^4$ K). Assuming the downstream medium consists of ionised hydrogen in full thermodynamic equilibrium, estimate its temperature.

   (e) What would be the approximate temperature behind a shock front moving at 50 000 km/ sec, if full thermodynamic equilibrium were established? Why does this approximation fail?

2. **Evolution of a supernova remnant** 30 pt

Assume first the supernova remnant in its "adiabatic" phase: all the mass of the remnant is concentrated in a thin shell located at the position of the shock at radius, $r = r_s(t)$, where $M \approx 4\pi r_s^3 \rho_1/3$ with ρ_1 being the density of the interstellar medium. Furthermore, the pressure $P(t)$ interior to the shock can be considered uniform and the equations of motion for the thin shell is then

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(M\dot{r}_s\right) = 4\pi\, r_s^2\, P \tag{5}$$

where $\dot{r}_s \equiv \mathrm{d}r_s/\mathrm{d}t$.

(a) Use the jump conditions for a strong shock of an ideal gas with an adiabatic index $\gamma = 5/3$ to estimate the thickness Δr of the shell in terms of r_s (assume the shell density equals the post-shock density).

(b) Given that in the Sedov phase the total internal energy of the gas in the remnant equals 80% of the explosion energy E_{SN}, show that the equations of motion have a solution of the form

$$r_s = A\, t^{\alpha}\,. \tag{6}$$

Find the constants α and A.

Assume now that the shell cools rapidly. Because the cooling rate in the gas is proportional to the density squared there is a phase in the evolution when the thermal energy of the freshly shocked gas can no longer be shared evenly throughout the remnant, as it was in the adiabatic phase discussed above. Most of the energy is radiated away and a cool dense shell forms around the still hot interior. To a good approximation the interior can be described as a hot adiabatic gas bubble of constant mass (again equation of state $P \propto \rho^{\gamma}$ with $\gamma = 5/3$). The evolution of the blast wave is now driven by the adiabatic expansion of this bubble.

(c) Show that this pressure-driven snow plow phase admits again a solution of the form

$$r_s \propto t^{\beta}\,, \tag{7}$$

and find the index β.