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A new outcome of binary–binary scattering

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ABSTRACT

A recently discovered solution of the three-body problem represents a new possible byproduct of binary–binary scattering. This is demonstrated by an explicit example. A way of detecting the new orbit automatically is discussed, and we give estimates related to the probability of its formation.

Key words: gravitation – celestial mechanics, stellar dynamics – binaries: general.

1 INTRODUCTION

Binary–binary scattering concerns the dynamical behaviour of two binaries that approach from a large distance, interact for a while, and produce new or altered configurations: two different binaries receding from each other, a binary and two ejected single stars, a hierarchical triple and an escaping single star, etc. (Mikkola 1984). The importance of binary–binary scattering lies in applications to the dynamics of star clusters, which are significantly endowed with a primordial population of binaries (Hut et al. 1992). Many of these congregate in the core of the cluster, where their mutual interactions release enough energy to halt core collapse, and indeed to power their entire evolution (Goodman & Hut 1989).

Our quantitative information on binary–binary scattering comes from statistical analysis of large numbers of Monte Carlo experiments. In particular, what are needed are cross-sections for the release of energy and for destruction of binaries. For this reason it is necessary to analyse the outcome of each experiment automatically, i.e. without visual inspection. This is done in terms of known approximate solutions, e.g. the ejection of one single star on a nearly hyperbolic orbit relative to a hierarchical triple system; in turn this consists of an outlying third body on a nearly elliptic orbit about an inner binary.

What has changed this picture recently is the discovery of a new type of solution in the three-body problem. Chenciner & Montgomery (2000) have proved the existence of a periodic solution in which three stars of equal mass maintain comparable distances (Fig. 1). Furthermore, it appears to be stable and to persist for modest changes in the masses as well as in the initial conditions. Therefore it is a new candidate for a possible outcome of binary–binary scattering. [It cannot be created by the interaction of three stars alone, as in binary–single scattering, by a well-known theorem; see Littlewood (1986) for an informal account.]

In this short note we show how one may automatically recognize orbits that resemble the periodic orbit discovered by Chenciner & Montgomery, in the case of planar motion; such orbits we refer to as '8-like'. We also give an example of the formation of an 8-like orbit in binary–binary scattering, and present evidence relating to the probability of this outcome.

2 CREATION OF AN 8-LIKE TRIPLE SYSTEM

It proved not at all difficult to find a scattering event in which a fourth star, approaching the periodic orbit from a substantial but finite distance, produces two binaries (Fig. 2). Time reversal then yields an example of an 8-like orbit created by the interaction of two binaries. (The asymptotic orbit of the triple system is not exactly the periodic orbit of Chenciner & Montgomery, because of perturbations by the fourth body as it recedes to infinity.)

Modest changes in the impact parameter of the fourth body (by about 0.1 in the units of Fig. 2) produced qualitatively the same outcome. All other scattering events studied resulted in a binary and two single escapers. These results suggest that the cross-section for formation of two binaries is of the order of a few per cent of the cross-section for all close encounters (defined here as those in which the minimum distance of the fourth body would bring it within the orbit of its target).

In order to estimate the cross-section of the inverse process (production of an 8-like orbit) in binary–binary scattering experiments, it is necessary to be able to recognize such an outcome automatically. It is not difficult to detect an outcome in which a fourth particle is escaping from an interacting triple system, and so it is necessary only to be able to recognize an 8-like orbit automatically. We propose that this is done by examining the sequence of collinear configurations of the three bodies.

In the orbit of Chenciner & Montgomery (2000), collinear configurations occur in the sequence …123123123…, or its reverse, where the digit denotes the body in the middle. We propose that 8-like orbits are *defined* by this property. The only other kind of permanently bound triple system that is considered in this context (a hierarchical triple) would exhibit a sequence of only two alternating digits.

In order to test this procedure we have selected equal-mass planar triple systems in Monte Carlo fashion by the following

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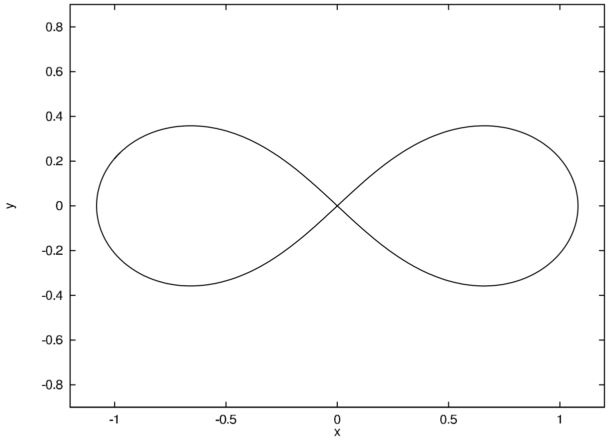

Figure 1. The periodic orbit of Chenciner & Montgomery (2000). The three bodies follow the same track at intervals of one-third of the period. In each period there are six collinear configurations. Units are such that $Gm = 1$, where m is the individual particle mass. Initial conditions (Simó 2000) are $\boldsymbol{r}_1 = -\boldsymbol{r}_2 = (-0.970\,004\,36, 0.243\,087\,53)$, $\boldsymbol{r}_3 = \mathbf{0}$, $2\dot{\boldsymbol{r}}_1 = 2\dot{\boldsymbol{r}}_2 = -\dot{\boldsymbol{r}}_3 = -(0.932\,407\,37, 0.864\,731\,46)$.

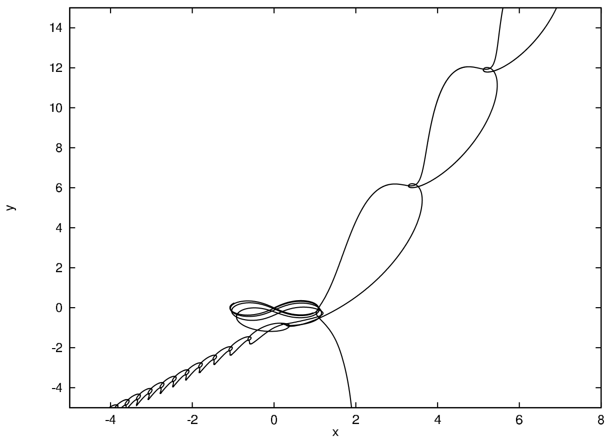

Figure 2. Formation of an 8-like orbit in a binary–binary encounter. The 'initial' conditions were obtained by adding to the initial conditions of Fig. 1 a fourth body with $m = 1$, $\boldsymbol{r} = (2, -10)$, $\dot{\boldsymbol{r}} = (0, 1)$.

process. The coordinates and velocity components were chosen randomly in (0, 1) and then converted to the barycentric frame. By adding the same suitably chosen angular velocity to all three particles the angular momentum was then set to zero. Finally velocities, coordinates and masses were scaled to virial equilibrium in standard units (Heggie & Mathieu 1986). Systems created in this way were then integrated numerically until the sequence of collinear configurations differed from that of an 8-like orbit, or until a maximum time was reached. The configurations were tested at intervals of 0.01 time units. If an eccentric tight binary forms it can be difficult to detect the changing collinear configurations, and so an orbit was also deemed to be not 8-like if the number of collinear configurations detected in time t was less than $t/10$ for $t > 10$. (For the orbit of Chenciner & Montgomery there would be about 12 collinear configurations to $t = 10$ in these units.)

The result of computations with nearly 10^4 sets of initial conditions revealed that, while about 6 per cent remained 8-like until $t = 10$, this fraction had reduced to about 0.2 per cent by $t = 100$ and very roughly 0.03 per cent by $t = 1000$. Fig. 3 shows the orbit of one of the particles in one of these long-lived 8-like configurations. The implication that 8-like configurations are rare is consistent with informal experiments in which the initial conditions of the periodic orbit of Chenciner & Montgomery are altered. For example, if the coordinates of the initial central particle are changed by as much as about 0.13 (in standard units), it is found that 8-like behaviour does not persist after about $t = 100$. Since there are four initial conditions to vary (if we restrict to zero angular momentum, fixed energy and virial equilibrium, and allow for time-translation invariance), it is not surprising that the proportion of 8-like orbits is so low. This conclusion is reinforced when it is realized that our choice of initial conditions (planar motions, zero angular momentum) favours orbits in the vicinity of the orbit of Chenciner & Montgomery. On the other hand, our definition of an 8-like orbit is not immediately applicable to non-planar motions.

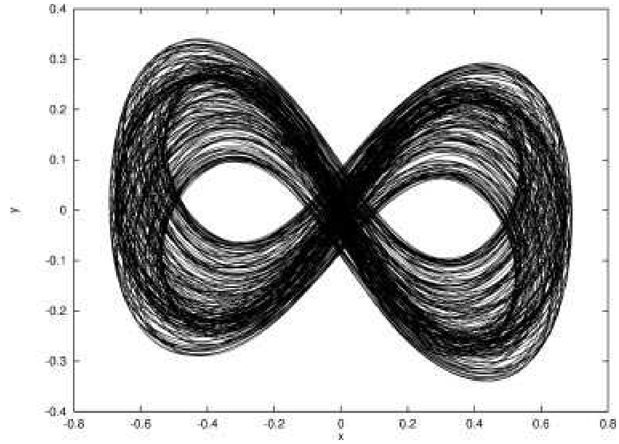

Figure 3. A long-lived 8-like orbit. The units are described in the text, as well as the Monte Carlo procedure used for the selection of the initial conditions. The orbit of only one of the particles is shown.

3 CONCLUSION

We have shown that a newly discovered periodic orbit of the three-body problem (Chenciner & Montgomery 2000) represents a new possible outcome for binary–binary scattering. We have defined (at least for planar motions) a class of orbits, which we call '8-like', the behaviour of which resembles that of the new periodic orbit. We have shown that, in a reasonable ensemble of initial conditions, the proportion leading to 8-like behaviour is small. Finally we have argued that the probability of its occurrence in a binary–binary scattering event is a very small fraction of 1 per cent.

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