Stellar Astronomy and Astrophysics (SS09):

Exercise 1 (for April 20, 2008)

1. Pressure-free collapse of homogeneous isothermal spheres

Take an isothermal sphere with constant density. Neglecting the effects of pressure, one can analytically derive the free-fall time as well as the time evolution of the density. Start doing this by looking at the equations of hydrodynamics and consider them in one dimension.

$$\begin{aligned}
\rho \frac{d\vec{v}}{dt} &= -\vec{\nabla} P - \rho \vec{\nabla} \Phi && \text{equation of motion} \\
\frac{d\rho}{dt} + \rho \vec{\nabla} \cdot \vec{v} &= 0 && \text{continuity equation} \\
\vec{\nabla}^2 \Phi &= 4\pi G \rho && \text{Poisson equation} \\
\left(\frac{dP}{d\rho} \right) &= c_s^2 && \text{equation of state}
\end{aligned}$$

Here ρ, $\vec{v}$, P, and Φ are density, velocity, pressure and gravitational potential, respectively. And G and c_s are gravitational constant and isothermal sound speed.

Combine these equations to find $\vec{v}(t)$ as function of radius $\vec{r}(t)$. Solve to find the collapse time. Consider again the continuity equation to find $\rho(t)$.

Note, that this exercise is optional and should be regarded as a help for a more troughoutly understanding of the lecture on star formation. It does not need to be solved at home but will be discussed during the tutorial.

2. Intensity, total flux:

Suppose that the intensity of a light bulb varies with direction as

$$I(\theta) = \frac{1}{2} I(0)(1 + \cos\theta)$$

Draw this intensity distribution vs. θ and vs. $\mu = \cos\theta$.

What do you obtain for J, F, and K in terms of the forward intensity $I(0)$?

3. Blackbody:

1. By what factor should the temperature of a black body be increased so that

a: The integrated flux (over all frequencies) is doubled?

b: The frequency at which the intensity is greatest is doubled?

2. What is the total number of photons inside an oven set at 200 Celsius with a volume of 1 m^3.

3. The tungsen filament of a light bulb has a temperature of 2700 K. If we assume blackbody radiation:

a: What amount of the energy is radiated in the visible range of the electromagnetic spectrum (4000-7000 Å).

b: What is the energy per blackbody photon at the center of the sun ($T = 1.58 \cdot 10^7$ K and in the solar photosphere ($T = 5770$ K) expressed in electron Volts (1eV=$1.6 \cdot 10^{-12}$ erg).

4. Energy density:

Calculate and plot the energy density u_ν and u_λ of the cosmic background radiation ($T = 2.725\,\mathrm{K}$) and the integrated energy density.