Stellar Astronomy and Astrophysics (SS09):

Exercise 4 (for June 2, 2009, Tuesday, 2pm)

1. Did it ever happen?

In the lecture we have calculated that the thermal energy of protons is by far not high enough to overcome the Coulomb barrier. Our assumptions were that the protons must come closer to each other than 10^{-15} m= 10^{-13} m. We equate the mean thermal energy $\frac{3}{2}kT$ with the Coulomb energy at that distance:

$$\frac{3}{2}kT = \frac{Z_1 Z_2 e^2}{r} \tag{1}$$

with $Z_1 = Z_2 = 1$ (charge of the protons), $r = 10^{-13}$ cm, $k = 1.38 \cdot 10^{-16}$ erg K^{-1} (Boltzmann constant), $e = 4.80 \cdot 10^{-10}$ esu.

Then it follows that

$$T = \frac{2Z_1 Z_2 e^2}{3kr} = 1.1 \cdot 10^{10}\,\mathrm{K} \tag{2}$$

The temperature in the sun is, however, only about 14 million K. If we had the temperature of 10^{10} K, the fusion would be explosive (because a large fraction of the protons could fuse). Since the energies are distributed according to the Maxwell-Boltzmann distribution which has a tail (albeit exponentially suppressed) some particles have energies much higher than the average. The fraction of particles with the required energy is therefore

$$\text{fraction} = e^{-\frac{1.1 \cdot 10^{1}0}{1.4 \cdot 10^{7}}} \approx 10^{-341} \tag{3}$$

If we assume that we have 10^{80} atoms in the visible universe, we can assume that it is almost impossible that any particle will ever have the right energy. However, our argument is only "instantaneous". Let us assume that all matter in the visible universe is made up of hydrogen and has a temperature of 14 million K and a density of the solar center (160 g cm^{-3}). Did it ever happen in a Hubble time (10^{10} yrs) that a particle classically had the right energy to fuse? Assume that large-angle collisions between protons happen when the kinetic energy equals the Coulomb energy. A proton mass is $1.67 \cdot 10^{-24}$ g.

2. Relative Abundances for CNO in equilibrium:

Assume that the CNO cycle is in equilibrium and the temperature is about $T = 2 \cdot 10^7$ K. In this case the lifetimes against proton capture is $\tau(^{15}\mathrm{N}) = 30$ years, $\tau(^{13}\mathrm{C}) = 1600$ years, $\tau(^{12}\mathrm{C}) = 6600$ years, $\tau(^{14}\mathrm{N}) = 6 \cdot 10^5$ years. Oxygen decays in $\tau(^{15}\mathrm{O}) = 1$ minute. What are the abundances of these CNO isotopes in equilibrium?