Stellar Astronomy and Astrophysics (SS09):

Exercise 5 (for June 15, 2009)

1. Period-Luminosity relation for Cepheids:

In the lecture we derived

1. the period-density relation $P \propto \rho^{-1/2}$,
2. A Mass-Luminosity relation $L \propto M^{\alpha}$,
3. and have shown that the instability strip for pulsation is restricted to a narrow strip of almost constant $T_{\rm eff}$ in the HRD.

a) If we assume that for Cepheids $\alpha = 4$ (a somewhat higher value than for main-sequence stars), show that a period-luminosity relation ($\log L$ vs. $\log P$ exists. Hint: Use Stefan-Boltzmann's law.

b) Assume that the instability strip is not at $T_{\rm eff} = $ const. but more accurately given by $\log L = \beta \log T_{\rm eff} + \delta$. How does the period-luminosity function look like in this case?

c) Is this relation sufficient to determine the distance of a Cepheid from measuring its period and an apparent magnitude?

2. Convection:

In the lecture the Schwartzschild criterion for convection was derived. Convection occurs, if the logarithmic temperature gradient $\nabla = \left(\frac{d\ln T}{d\ln P}\right)$ for radition is larger than that for the adiabatic one:

$$\nabla_{\rm rad} > \nabla_{\rm ad} = 1 - \frac{1}{\gamma} \tag{1}$$

with $\gamma = \frac{c_P}{c_V}$ being the ratio between the specific heats. This equation was derived under the assumption that $\frac{d\rho_{\rm blob}}{dr} < \frac{d\rho_{\rm sur}}{dr}$ is equivalent to $\frac{dT_{\rm blob}}{dr} > \frac{dT_{\rm sur}}{dr}$. For this argument we used the equation of state, which in the case of an ideal gas is $P = \frac{\rho R}{\mu} T$. However, the argument is only valid if the mean molecular weight μ is constant.

Assume now that due to nuclear burning there is a chemical gradient in the star (μ is a function of the radial coordinate r). How would the convection criterion change if we assume that such a chemical gradient exists?

Ansatz: $\frac{d\rho}{\rho} = \frac{\rho(r+dr)-\rho(r)}{\rho(r)}$. Convection occurs, if the density $\rho_{\rm blob}(r + dr) = \rho_{\rm ad}(r + dr)$ is smaller than $\rho_{\rm sur}(r + dr)$. Further assume that the chemical compositon in the blob remains constant $\mu_{\rm blob}(r + dr) = \mu_{\rm blob}(r)$, while it is changing in the surrounding $\mu_{\rm sur}(r) \neq \mu_{\rm sur}(r + dr)$.

Is convection easier or suppressed if $\mu_{\rm sur}(r + dr) < \mu_{\rm sur}(r)$? Do you see any problem using the modified convection criterion?