Stellar Astronomy and Astrophysics (SS11)

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Exercise 2

1. Pressure-free collapse of homogeneous isothermal spheres

Take an isothermal sphere with constant density. Neglecting the effects of pressure, one can analytically derive the free-fall time as well as the time evolution of the density. Start doing this by looking at the equations of hydrodynamics and consider them in one dimension.

$$
\begin{aligned}
\rho\frac{d\vec{v}}{dt} &= -\vec{\nabla}P - \rho\vec{\nabla}\Phi && \text{equation of motion} \\
\frac{d\rho}{dt} + \rho\vec{\nabla}\cdot\vec{v} &= 0 && \text{continuity equation} \\
\vec{\nabla}^2\Phi &= 4\pi G\rho && \text{Poisson equation} \\
\left(\frac{dP}{d\rho}\right) &= c_s^2 && \text{equation of state}
\end{aligned}
$$

Here ρ, $\vec{v}$, P, and Φ are density, velocity, pressure and gravitational potential, respectively. And G and c_s are gravitational constant and isothermal sound speed.

Combine these equations to find $\vec{v}(t)$ as function of radius $\vec{r}(t)$. Solve to find the collapse time. Consider again the continuity equation to find $\rho(t)$.

2. Pre-main-sequence contraction:

Consider for simplicity fully convective protostars and use the virial theorem to obtain order of magnitude estimates.

a Describe qualitatively the behavior of the central temperature T_c if the object contracts? What is the reason for this behavior?

b For a one solar-mass object, estimate how long the pre-main-sequence contraction phase will last. Assume the object radiates with constant luminosity equal to the present-day Sun.

c Why does the contraction eventually stop?

3. Central temperature of the Sun:

Using the equation of hydrostatic balance, obtain a crude estimate of the central temperature of the present-day Sun.