Stellar Astronomy and Astrophysics (SS11)

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Exercise 4 (for Tuesday , June 7, 2010, 14.00 st.)

1. Convection:

In the lecture the Schwartzschild criterion for convection was derived. Convection occurs, if the logarithmic temperature gradient $\nabla = \left(\frac{d\ln T}{d\ln P}\right)$ for radition is larger than that for the adiabatic one:

$$\nabla_{\rm rad} > \nabla_{\rm ad} = 1 - \frac{1}{\gamma} \tag{1}$$

with $\gamma = \frac{c_P}{c_V}$ being the ratio between the specific heats. This equation was derived under the assumption that $\frac{d\rho_{\rm blob}}{dr} < \frac{d\rho_{\rm sur}}{dr}$ is equivalent to $\frac{dT_{\rm blob}}{dr} > \frac{dT_{\rm sur}}{dr}$. For this argument we used the equation of state, which in the case of an ideal gas is $P = \frac{\rho R}{\mu}T$. However, the argument is only valid if the mean molecular weight μ is constant.

Assume now that due to nuclear burning there is a chemical gradient in the star (μ is a function of the radial coordinate r). How would the convection criterion change if we assume that such a chemical gradient exists?

Ansatz: $\frac{d\rho}{\rho} = \frac{\rho(r+dr)-\rho(r)}{\rho(r)}$. Convection occurs, if the density $\rho_{\rm blob}(r+dr) = \rho_{\rm ad}(r+dr)$ is smaller than $\rho_{\rm sur}(r+dr)$. Further assume that the chemical compositon in the blob remains constant $\mu_{\rm blob}(r+dr) = \mu_{\rm blob}(r)$, while it is changing in the surrounding $\mu_{\rm sur}(r) \neq \mu_{\rm sur}(r+dr)$.

Is convection easier or suppressed if $\mu_{\rm sur}(r+dr) < \mu_{\rm sur}(r)$? Do you see any problem using the modified convection criterion?

2. Did it ever happen?

In the lecture we have calculated that the thermal energy of protons is by far not high enough to overcome the Coulomb barrier. Our assumptions were that the protons must come closer to each other than 10^{-15} m= 10^{-13} m. We equate the mean thermal energy $\frac{3}{2}kT$ with the Coulomb energy at that distance:

$$\frac{3}{2}kT = \frac{Z_1 Z_2 e^2}{r} \tag{2}$$

with $Z_1 = Z_2 = 1$ (charge of the protons), $r = 10^{-13}$ cm, $k = 1.38 \cdot 10^{-16}$ erg K^{-1} (Boltzmann constant), $e = 4.80 \cdot 10^{-10}$ esu.

Then it follows that

$$T = \frac{2 Z_1 Z_2 e^2}{3kr} = 1.1 \cdot 10^{10}\,\mathrm{K} \tag{3}$$

The temperature in the sun is, however, only about 14 million K. If we had the temperature of 10^{10} K, the fusion would be explosive (because a large fraction of the protons could fuse). Since the energies are distributed according to the Maxwell-Boltzmann distribution which has a tail (albeit exponentially suppressed) some particles have energies much higher than the average. The fraction of particles with the required energy is therefore

$$\text{fraction} = e^{-\frac{1.1\cdot 10^10}{1.4\cdot 10^7}} \approx 10^{-341} \tag{4}$$

If we assume that we have 10^{80} atoms in the visible universe, we can assume that it is almost impossible that any particle will ever have the right energy. However, our argument is only "instantaneous". Let us assume that all matter in the visible universe is made up of hydrogen and has a temperature of 14 million K and a density of the solar center ($160\,\mathrm{g\ cm^{-3}}$). Did it ever happen in a Hubble time (10^{10} yrs) that a particle classically had the right energy to fuse? Assume that large-angle collisions between protons happen when the kinetic energy equals the Coulomb energy. A proton mass is $1.67 \cdot 10^{-24}\,\mathrm{g}$.

3. Relative Abundances for CNO in equilibrium:

Assume that the CNO cycle is in equilibrium and the temperature is about $T = 2 \cdot 10^7\,\mathrm{K}$. In this case the lifetimes against proton capture is $\tau(^{15}\mathrm{N}) = 30$ years, $\tau(^{13}\mathrm{C}) = 1600$ years, $\tau(^{12}\mathrm{C}) = 6600$ years, $\tau(^{14}\mathrm{N}) = 6 \cdot 10^5$ years. Oxygen decays in $\tau(^{15}\mathrm{O}) = 1$ minute. What are the abundances of these CNO isotopes in equilibrium?