Stellar Astronomy and Astrophysics (SS12)

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Exercise 2 for May 8, 2012

2.1 Pressure-Free Collapse of Homogeneous Isothermal Spheres as Simple Model for Protostellar Collapse

Take an isothermal sphere with constant density. Neglecting the effects of pressure, one can analytically derive the free-fall time as well as the time evolution of the density. Start doing this by looking at the equations of hydrodynamics and consider them in one dimension.

$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} P - \rho \vec{\nabla} \Phi \qquad \text{equation of motion}$$

$$\frac{d\rho}{dt} + \rho \vec{\nabla} \cdot \vec{v} = 0 \qquad \text{continuity equation}$$

$$\vec{\nabla}^2 \Phi = 4\pi G \rho \qquad \text{Poisson equation}$$

$$\left(\frac{dP}{d\rho} \right) = c_s^2 \qquad \text{equation of state}$$

Here ρ, $\vec{v}$, P, and Φ are density, velocity, pressure and gravitational potential, respectively. And G and c_s are gravitational constant and isothermal sound speed. Combine these equations to find $\vec{v}(t)$ as function of radius $\vec{r}(t)$. Solve to find the collapse time. Consider again the continuity equation to find $\rho(t)$.