Stellar Astronomy and Astrophysics (SS12)

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Exercise 5 for May 29, 2012

1. Convection:

In the lecture the Schwartzschild criterion for convection was derived. Convection occurs, if the logarithmic temperature gradient $\nabla = \left(\frac{d\ln T}{d\ln P}\right)$ for radition is larger than that for the adiabatic one:

$$\nabla_{\rm rad} > \nabla_{\rm ad} = 1 - \frac{1}{\gamma} \tag{1}$$

with $\gamma = \frac{c_P}{c_V}$ being the ratio between the specific heats. This equation was derived under the assumption that $\frac{d\rho_{\rm blob}}{dr} < \frac{d\rho_{\rm sur}}{dr}$ is equivalent to $\frac{dT_{\rm blob}}{dr} > \frac{dT_{\rm sur}}{dr}$. For this argument we used the equation of state, which in the case of an ideal gas is $P = \frac{\rho R}{\mu} T$. However, the argument is only valid if the mean molecular weight μ is constant.

Assume now that due to nuclear burning there is a chemical gradient in the star (μ is a function of the radial coordinate r). How would the convection criterion change if we assume that such a chemical gradient exists?

Ansatz: $\frac{d\rho}{\rho} = \lim_{dr\to 0} \frac{\rho(r+dr)-\rho(r)}{\rho(r)}$. Convection occurs, if the density $\rho_{\rm blob}(r+dr) = \rho_{\rm ad}(r+dr)$ is smaller than $\rho_{\rm sur}(r+dr)$. Further assume that the chemical compositon in the blob remains constant $\mu_{\rm blob}(r+dr) = \mu_{\rm blob}(r)$, while it is changing in the surrounding $\mu_{\rm sur}(r) \neq \mu_{\rm sur}(r+dr)$.

Is convection easier or suppressed if $\mu_{\rm sur}(r+dr) < \mu_{\rm sur}(r)$? Do you see any problem using the modified convection criterion?

2. Hydrostatic Equation:

Show that the equation of hydrostatic equilibrium may be written as

$$\frac{dP}{d\tau} = \frac{g}{\kappa} \tag{2}$$

with P being the pressure, g being the local gravitational acceleration and τ being the optical depth defind for the mass absorption coefficient κ.

3. Pressure in a white dwarf:

a) Estimate the density of a white dwarf that has a solar mass packed into a sphere with about 0.01 solar radii.
b) Estimate the degeneracy pressure in the interiour of this white dwarf and compare this pressure with the thermal pressure of a gas that has a temperature of 10^7 K.