Stellar Astronomy and Astrophysics (SS12)

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Exercise 7 for June 12, 2012

Some aspects of energy production in stars

7.1 Did it ever happen?

In the lecture we have calculated that the thermal energy of protons is by far not high enough to overcome the Coulomb barrier. Our assumptions were that the protons must come closer to each other than $10^{-15}\,\text{m} = 10^{-13}\,\text{cm}$. We equate the mean thermal energy $\frac{3}{2}kT$ with the Coulomb energy at that distance:

$$\frac{3}{2}kT = \frac{Z_1 Z_2 e^2}{r} \tag{1}$$

with $Z_1 = Z_2 = 1$ (charge of the protons), $r = 10^{-13}\,\text{cm}$, $k = 1.38 \cdot 10^{-16}\,\text{erg K}^{-1}$ (Boltzmann constant), $e = 4.80 \cdot 10^{-10}\,\text{esu}$. Then it follows that

$$T = \frac{2Z_1 Z_2 e^2}{3kr} = 1.1 \cdot 10^{10}\,\text{K} \tag{2}$$

The temperature in the Sun is, however, only about 14 million K. If we had the temperature of 10^{10} K, the fusion would be explosive (because a large fraction of the protons could fuse). Since the energies are distributed according to the Maxwell-Boltzmann distribution which has a tail (albeit exponentially suppressed) some particles have energies much higher than the average. The fraction of particles with the required energy is therefore

$$\text{fraction} = e^{-\frac{1.1 \cdot 10^{10}}{1.4 \cdot 10^{7}}} \approx 10^{-341} \tag{3}$$

If we assume that we have 10^{80} atoms in the visible universe, we can assume that it is almost impossible that any particle will ever have the right energy. However, our argument is only “instantaneous”. Let us assume that all matter in the visible universe is made up of hydrogen and has a temperature of 14 million K and a density of the solar center ($160\,\text{g cm}^{-3}$). Did it ever happen in a Hubble time (for simplicity assume 10^{10} yr) that a particle classically had the right energy to fuse? Assume that large-angle collisions between protons happen when the kinetic energy equals the Coulomb energy. A proton mass is $1.67 \cdot 10^{-24}$ g.

7.2 Relative abundances for CNO in equilibrium:

Assume that the CNO cycle is in equilibrium and the temperature is about $T = 2 \cdot 10^7$ K. In this case the lifetimes against proton capture are $\tau(^{15}\text{N}) = 30$ years, $\tau(^{13}\text{C}) = 1600$ years, $\tau(^{12}\text{C}) = 6600$ years, $\tau(^{14}\text{N}) = 6 \cdot 10^5$ years. Oxygen decays in $\tau(^{15}\text{O}) = 1$ minute. What are the abundances of these CNO isotopes in equilibrium?

7.3 Detecting neutrinos:

Neutrinos have an extremely small cross section σ_ν for interaction with other matter. Typically $\sigma_\nu \approx 10^{-48}\,\mathrm{m}^2$.

a: Calculate the mean free path of a neutrino at densities that are typical for the interior of the Sun.

b: Starting from the amount of energy produced in the Sun to keep up its luminosity (see the lecture), and assuming that all energy comes from hydrogen burning via the pp chain, estimate how many neutrinos we should detect per second in a cubic water tank of 100 m side length. (Try to come up with reasonable estimates rather than accurate numbers, and do not worry about neutrino oscillations.)