Stellar Astronomy and Astrophysics (SS12)

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Exercise 8 for June 19, 2012

Some aspects of the stellar main sequence

8.1 Energy production in the solar interior: the pp cycle

Calculate the reaction rate $R_{\rm pp}$ and the mean lifetime of a proton in the first step of the pp chain at the solar center. Recall that

$$R_{\rm pp} = \frac{6.48 \times 10^{-15}}{\mu Z_{\rm p} Z_{\rm p}} \frac{1}{2} \frac{n_{\rm p} n_{\rm p}}{[{\rm cm}^6]} \frac{S(E_0)}{[{\rm keV\,barn}]} \left(\frac{E_{\rm G}}{4kT}\right)^{2/3} \exp\left[-3\left(\frac{E_{\rm G}}{4kT}\right)\right] {\rm cm}^{-3}\,{\rm s}^{-1}\,,$$

with the Gamov energy $E_{\rm G}$ as defined in the lecture. Note that the factor $1/2$ accounts for the fact that we deal with the reaction between identical particles. The parameters μ, n, Z, k, and T are reduced mass, number density, atomic charge, Boltzmann constant, and temperature, respectively.
The density of the solar core is $\rho_{\rm c} = 148\,{\rm g\,cm}^{-3}$ and the temperature is $T_{\rm c} = 15.6 \times 10^6\,{\rm K}$. Assume for simplicity that the mass fraction of hydrogen is 50%, and use the value $S_{\rm pp}(E_0) = 3.8 \times 10^{-22}\,{\rm keV\,barn}$ for the nuclear fusion factor.

8.2 Simple estimate of the energy production in massive stars

Consider a star with a mass of $10\,{\rm M}_\odot$ that is solely composed of hydrogen initially. Its central core contains 20% of the total mass. Assume that during the star's lifetime the core material is completely converted into ^{56}Fe. How much nuclear energy is released during this process? What fraction of this energy is provided by the first step in this reaction, that is the fusion of $^1_1$H into $^4_2$He? How long does this main sequence evolution last? Recall that the relation between luminosity and mass is $L \propto M^{3.5}$.

8.3 Solar wind

The typical density of the solar wind at the distance of the Earth (1 AU) is 10 particles per cm^3. They have a velocity of $\sim 500\,{\rm km\,s}^{-1}$. If the solar wind is composed mostly of protons, what is the mass loss rate of the Sun? Assume the wind is isotropic.

8.4 Solar x-rays

Assume that the Sun is a perfect blackbody with a surface temperature of 5.800 K.

- Estimate the intensity of the Sun's soft x-ray radiation at a frequency of $\nu = 2 \times 10^{16}\,{\rm Hz}$.
- How many photons in the frequency interval $2 \times 10^{16} \pm 10^{14}\,{\rm Hz}$ are emitted from the photosphere during the Sun's lifetime of $\sim 10^{10}\,{\rm yr}$?
- Where do the Sun's x-rays come from?