Homework Assignment #2 is due Wednesday, Oct. 28, 2015

Theoretical Astrophysics (MKTP2)

Winter Semester 2015/2016

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1. **Boltzmann equation with external potential** 25 pt

   Consider a gas at constant temperature T in an external gravitational potential $\Phi(\vec{x})$. Assume that the distribution function can be separated in the form $f = g(\vec{x})\, f_0(\vec{w})$, where

   $$f_0(\vec{w}) = \left(\frac{m}{2\pi\, k\, T}\right)^{3/2} \exp\left(-\frac{m\, \vec{w}^2}{2k\, T}\right) \tag{1}$$

   is the Maxwell distribution function. Determine $g(\vec{x})$ from the Boltzmann equation.

2. **Validity of the hydrodynamic approach** 25 pt

   Giant molecular clouds (GMCs) in the Milky Way have densities of $\sim 100\,\mathrm{H_2}$ molecules per cubic centimeter and sizes of several tens of parsec. The typical temperature is $10\,\mathrm{K}$ which translates into a sound speed of $c_\mathrm{s} \approx 0.2\,\mathrm{km\,s^{-1}}$. Consider a volume element in the 6-dimensional phase space $d^3x d^3w$, where the spatial part d^3x is the size of the Earth and the velocity part $d^3w \sim 10^{-6}\langle c_\mathrm{s}^2\rangle^{3/2}$ centered around the bulk (center of mass) velocity of the cloud. Note that GMCs are partially ionized due cosmic rays and the interstellar radiation field with ionization degrees around 10^{-7} in the dense and well-shielded regions.

   (a) What is the approximate number of gas particles in $d^3x d^3w$?

   (b) What is the average distance $\mathrm{H_2}$ particles travel between two collisions?

   (c) The evolution of GMCs is often modeled with hydrodynamic simulations. Consider the volume element defined above and discuss whether this theoretical approach is justified or not.

   (d) The interstellar medium is threaded by magnetic fields. Do your arguments change if magnetic fields are taken into account.

   Now consider the air you are currently breathing. The density is about $1.2\times10^{-3}\,\mathrm{g\,cm^{-3}}$.

   (e) What is the number of gas particles in a phase space volume element with $d^3x \sim 10^{-6}\,\mathrm{cm^3}$ and $d^3w \sim 10^{-6}\langle c_\mathrm{s}^2\rangle^{3/2}$? And how does $(d^3x)^{1/3}$ compare to the typical mean free path of the molecules in air?

   It is sufficient to provide *very rough* order of magnitude estimates. For (c) and (d) consider the fundamental assumptions of hydrodynamics and discuss their validity.