Homework Assignment #3 is due Wednesday, Nov. 4, 2015

Theoretical Astrophysics (MKTP2)

Winter Semester 2015/2016

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1. **Equation of state of a degenerate electron gas** *25 pt*

   Consider a degenerate electron gas at zero temperature. That means that all quantum levels up to the Fermi momentum p_F are filled and all others are empty. The phase-space probability density is then given by

   $$f(\vec{q},\vec{p}) = \begin{cases} 2/h^3 & \text{for} \quad p \leq p_F \ , \\ 0 & \text{for} \quad p > p_F \ . \end{cases} \tag{1}$$

   The factor 2 takes into account the two spin orientations of the electrons.

   Use this distribution function to compute the density and pressure and show that this gas has a polytropic equation of state,

   $$P = K\rho^\gamma \ , \tag{2}$$

   Find the index γ and the constant K for the non-relativistic case. Assume that each electron is accompanied by one proton with negligible kinetic energy.

2. **Estimate of viscosity** *20 pt*

   (a) Consider a gas flowing with a mean velocity u_i in the i-direction. What is the equilibrium velocity distribution f_0 in the absence of external forces?

   (b) Now suppose there is a mean velocity gradient in the j-direction such that $\partial u_\mathrm{i}/\partial j \neq 0$. Solve for the velocity distribution function f, assuming that this velocity gradient is a small perturbation.

   (c) Show that the ij-component of the stress tensor, S_{ij}, can be written as

   $$S_{ij} = -\eta \frac{\partial u_i}{\partial j}, \tag{3}$$

   and give an expression for η.

3. **Equation of hydrostatic balance** *(bonus points 5 pt)*

   Using the equation of hydrostatic balance, obtain a crude estimate of the central temperature of the Sun. Hint: Approximate the differential operator by the finite difference between the solar surface and center.