Homework Assignment #5 is due Wednesday, Nov. 18, 2015

Theoretical Astrophysics (MKTP2)

Winter Semester 2015/2016

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1. **Parker wind solution** *25 pt*

   Consider a steady, radial flow of an ideal gas in the gravitational field of a star. Assume a polytropic equation of state,

   $$P = K\,\rho^{\gamma}\,, \tag{1}$$

   where K is constant along the streamlines and $\gamma < 5/3$.

   (a) Show that the continuity equation can be written as

   $$4\pi\, r^2\, \rho\, v = \dot{M}\,, \tag{2}$$

   where v is the radial velocity and $\dot{M}$ is the constant rate of change of mass. Derive the relevant Euler equation for this spherically symmetric system.

   (b) Show that a smooth solution containing both sub- and supersonic regions of the flow exists only if

   $$v^2 = c_s^2 \quad \text{at} \quad r = \frac{G\,M}{2c_s^2}\,, \tag{3}$$

   where $c_s = \sqrt{\gamma P/\rho}$ is the speed of sound (which depends on r!).

   (c) Imposing this condition and the boundary conditions

   $$\rho = \rho_* \quad \text{and} \quad c_s = c_* \tag{4}$$

   at the surface $r = r_*$ of the star, find the mass loss rate in the wind in the limit of low surface velocity $v_* \ll c_*$, given that the surface temperature is large compared to the "virial" temperature, i.e. $c_*^2 \gg G\,M/r_*$.

   (d) Find the location of the sonic point and discuss the behavior of the solutions as $\gamma \to 5/3$.

2. **Evolution of a supernova remnant** *25 pt*

Assume first the supernova remnant in its "adiabatic" phase: all the mass of the remnant is concentrated in a thin shell located at the position of the shock at radius, $r = r_s(t)$, where $M \approx 4\pi r_s^3 \rho_1/3$ with ρ_1 being the density of the interstellar medium. Furthermore, the pressure $P(t)$ interior to the shock can be considered uniform and the equations of motion for the thin shell is then

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(M\dot{r}_s\right) = 4\pi\, r_s^2\, P \tag{5}$$

where $\dot{r}_s \equiv \mathrm{d}r_s/\mathrm{d}t$.

(a) Use the jump conditions for a strong shock of an ideal gas with an adiabatic index $\gamma = 5/3$ to estimate the thickness Δr of the shell in terms of r_s (assume the shell density equals the post-shock density).

(b) Given that in the Sedov phase the total internal energy of the gas in the remnant equals 80% of the explosion energy E_{SN}, show that the equations of motion have a solution of the form

$$r_s = A\, t^{\alpha}\,. \tag{6}$$

Find the constants α and A.

Assume now that the shell cools rapidly. Because the cooling rate in the gas is proportional to the density squared there is a phase in the evolution when the thermal energy of the freshly shocked gas can no longer be shared evenly throughout the remnant, as it was in the adiabatic phase discussed above. Most of the energy is radiated away and a cool dense shell forms around the still hot interior. To a good approximation the interior can be described as a hot adiabatic gas bubble of constant mass (again equation of state $P \propto \rho^{\gamma}$ with $\gamma = 5/3$). The evolution of the blast wave is now driven by the adiabatic expansion of this bubble.

(c) Show that this pressure-driven snow plow phase admits again a solution of the form

$$r_s \propto t^{\beta}\,, \tag{7}$$

and find the index β.