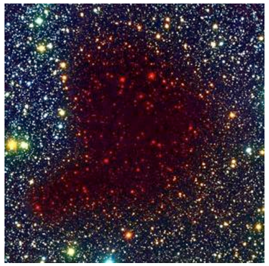
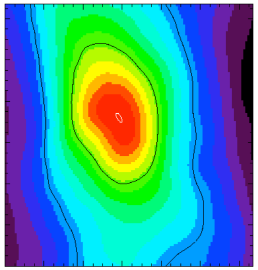
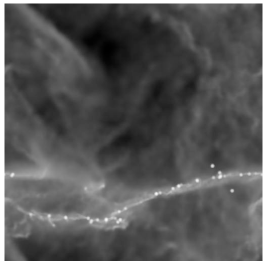
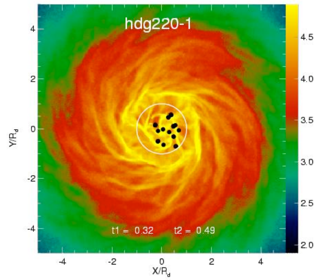
# Modeling Star Cluster Formation






**Ralf Klessen**

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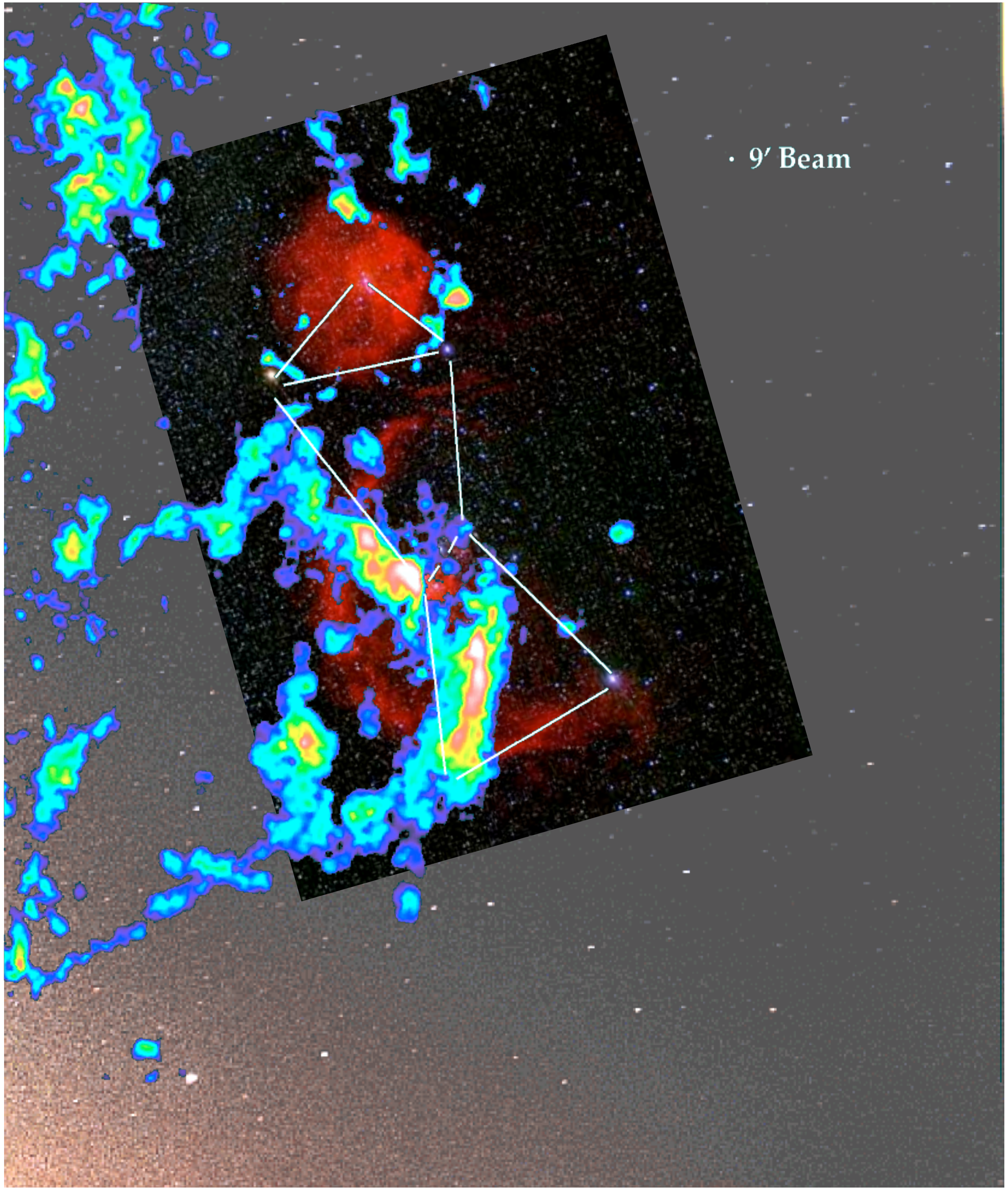
Institut für Theoretische Astrophysik

# Agenda

- phenomenology
  - Orion
  - Taurus
- interplay between gravity and turbulence
- examples and predictions
  - star cluster formation: dynamics
  - star cluster formation: thermodynamics
    --> stellar initial mass function

# phenomenology

Ralf Klessen: Zürich, Sept. 17, 2007

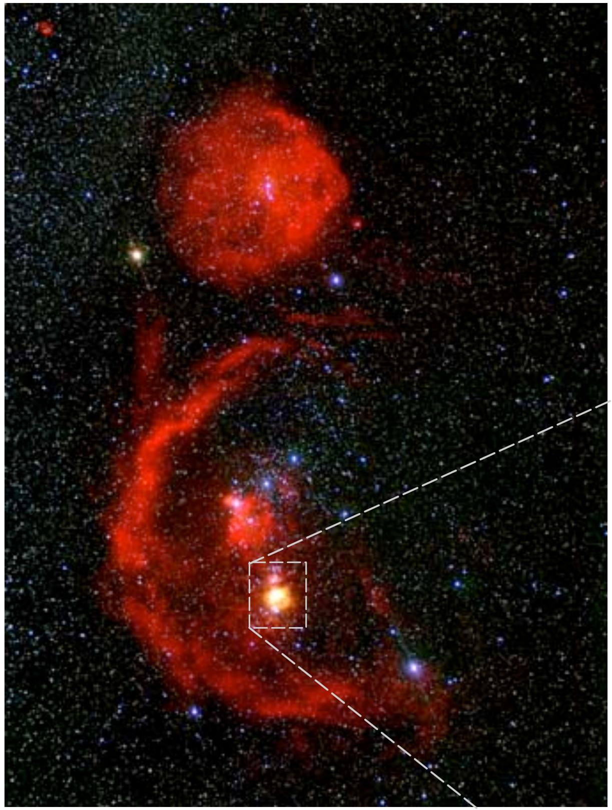



# Star formation in Orion

We see

- *Stars* (in visible light)
- Atomic hydrogen (in Hα -- red)
- Molecular hydrogen $H_2$ (radio emission -- color coded)

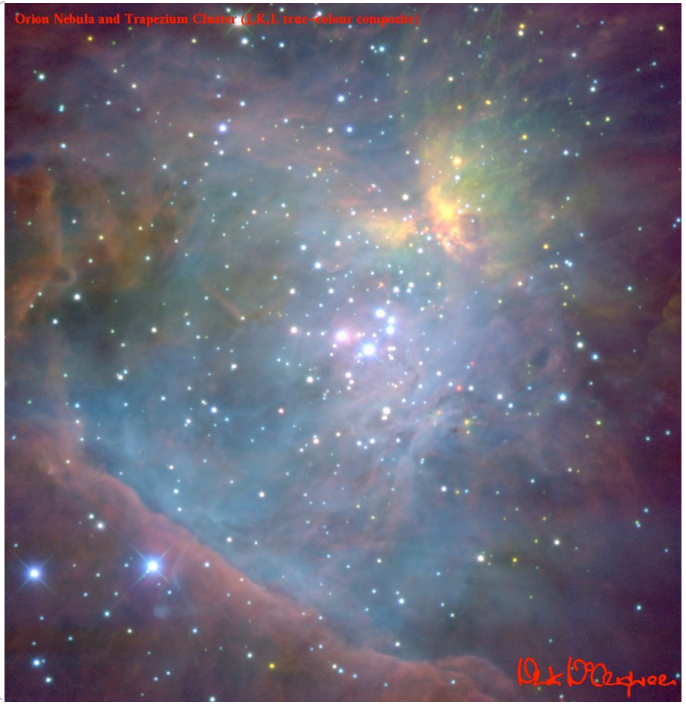
# Local star forming region: The Trapezium Cluster in Orion



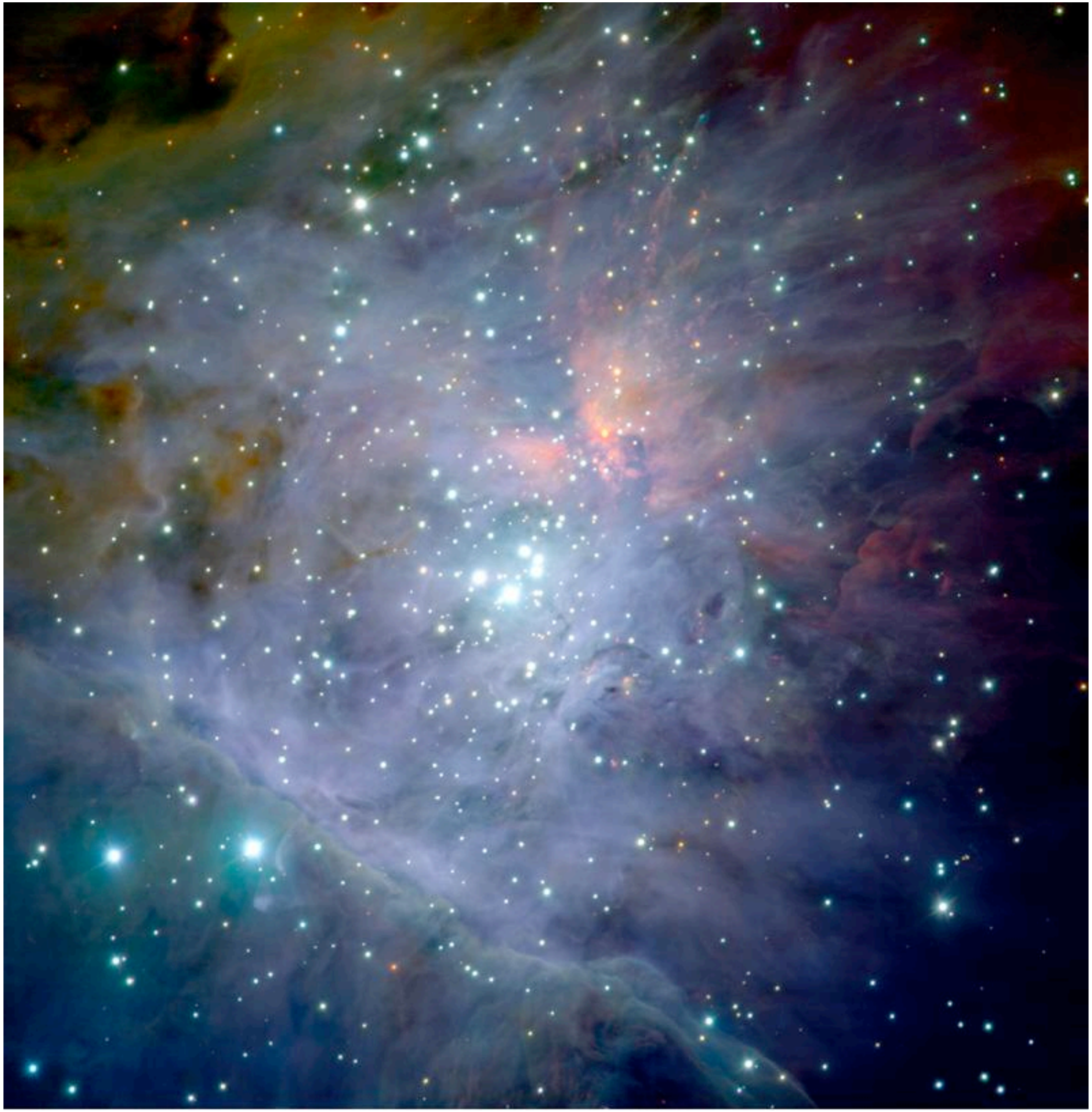
Orion molecular cloud

The Orion molecular cloud is the birth- place of several young embedded star clusters.

The Trapezium cluster is only visible in the IR and contains about 2000 newly born stars.




Trapezium cluster



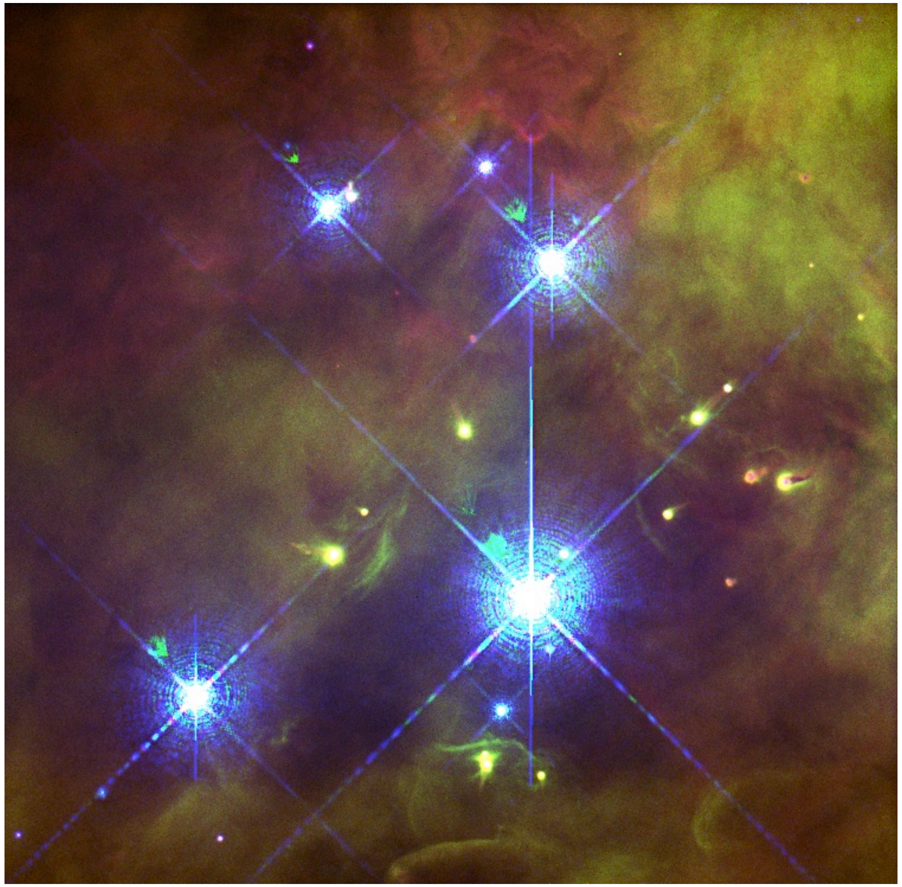
# Trapezium Cluster

(detail)

- stars form in clusters
- stars form in molecular clouds
- (proto)stellar feedback is important

(color composite J,H,K by M. McCaughrean, VLT, Paranal, Chile)

# Trapezium Cluster: Central Region

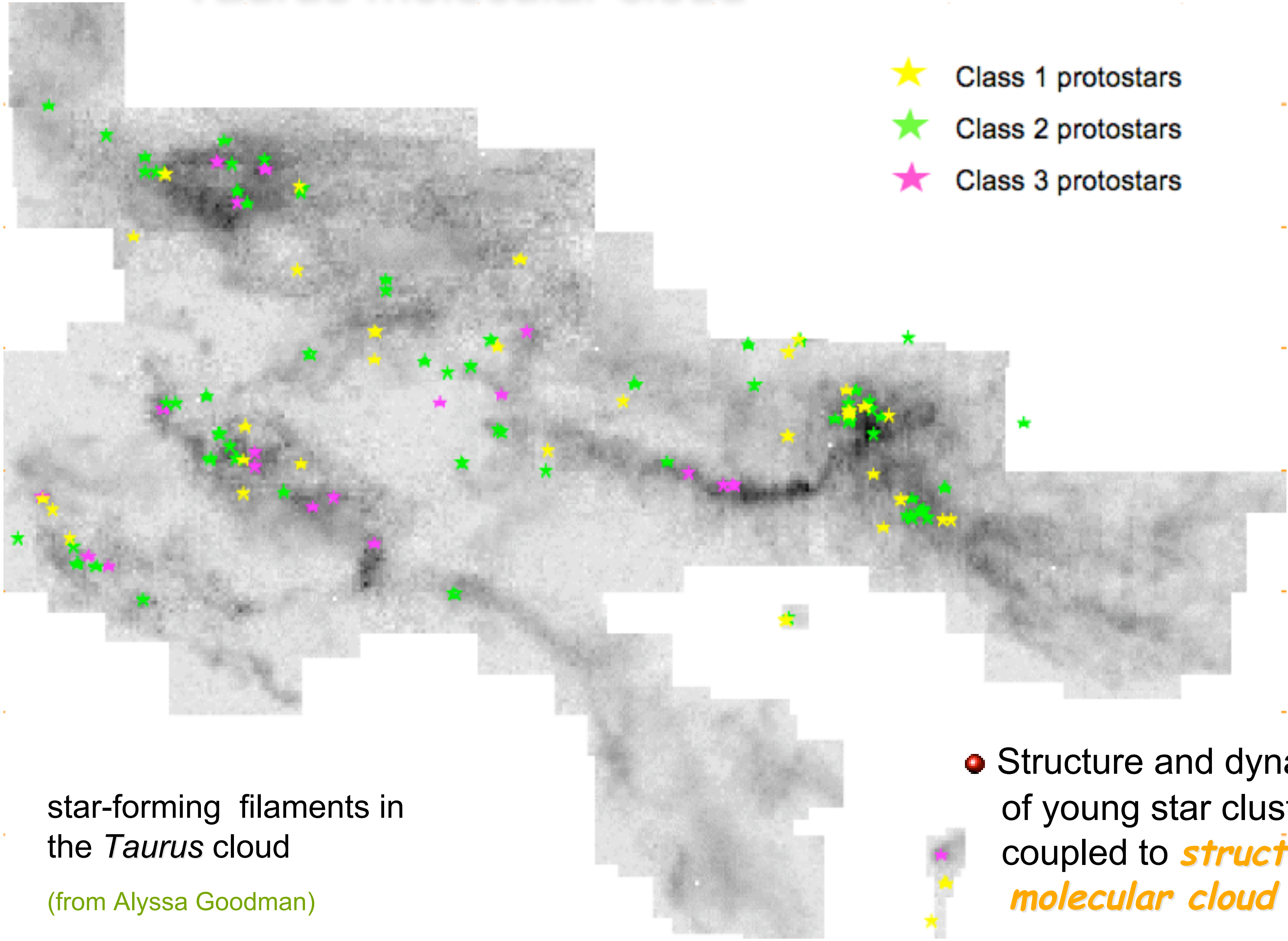


Ionizing radiation from central star
**Θ1C Orionis**

**Proplyds:** Evaporating ``protoplanetary´´ disks around young low-mass protostars

(images: Doug Johnstone et al.)

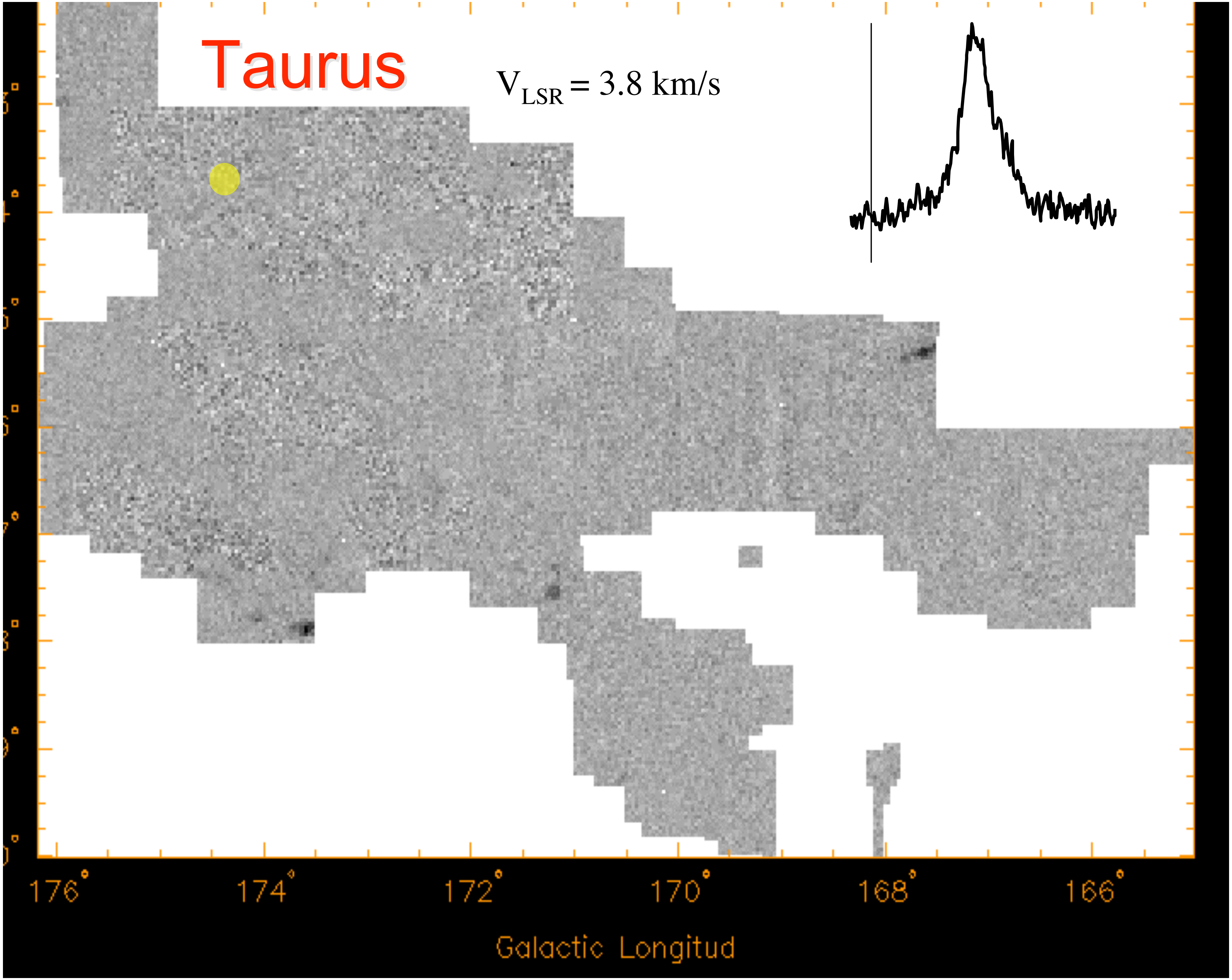
# Taurus molecular cloud



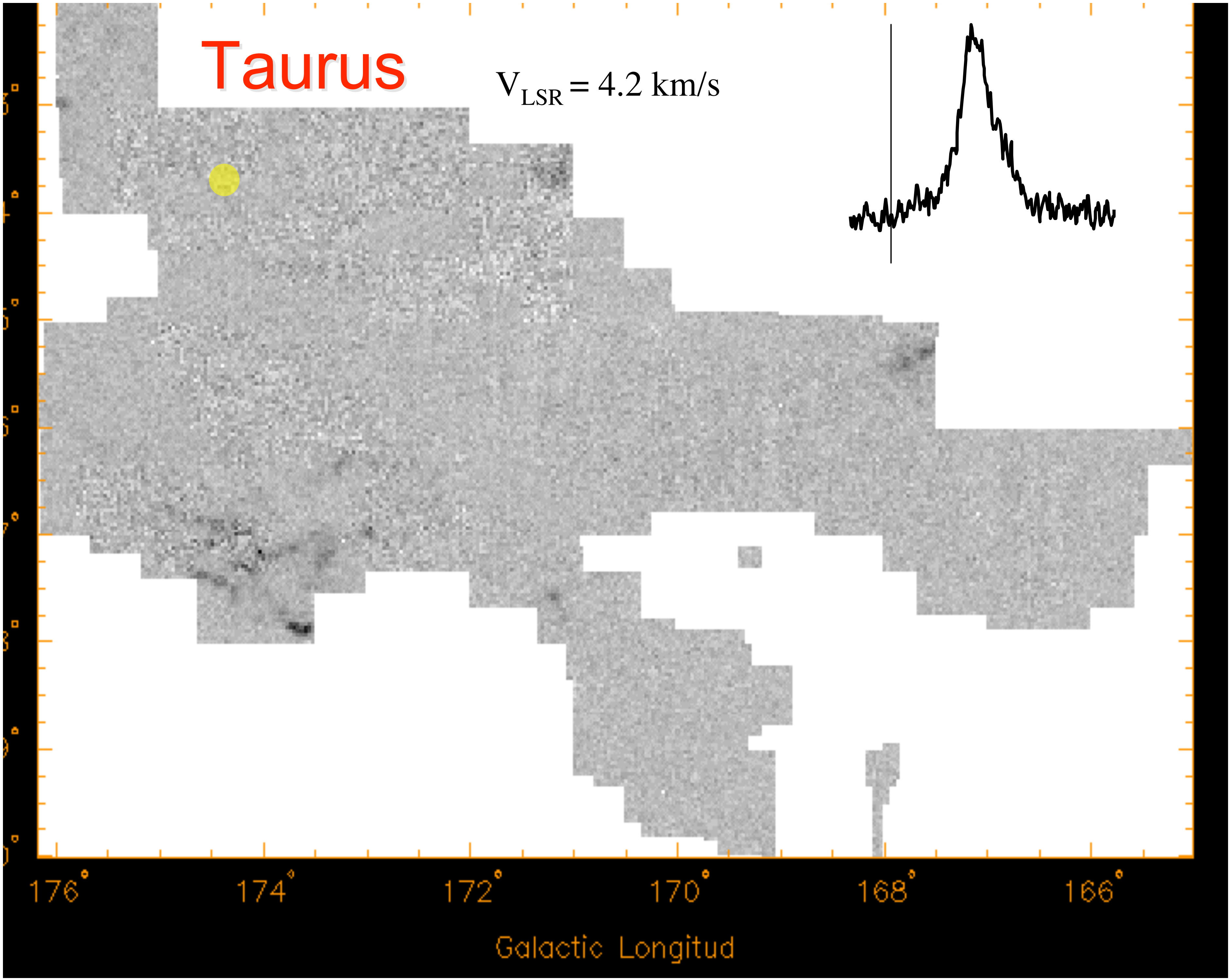

star-forming filaments in the *Taurus* cloud

(from Alyssa Goodman)

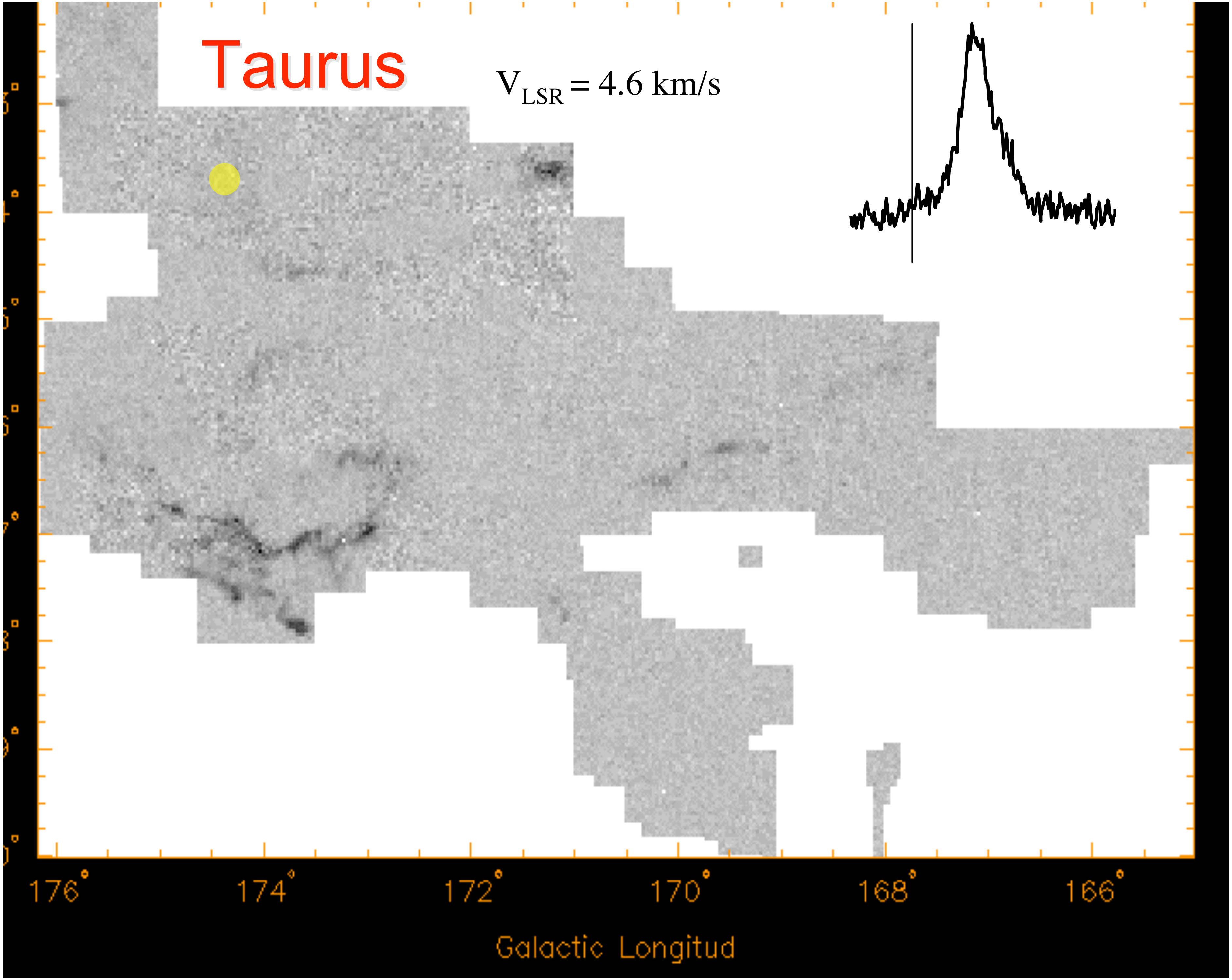
- Structure and dynamics of young star clusters is coupled to ***structure of molecular cloud***

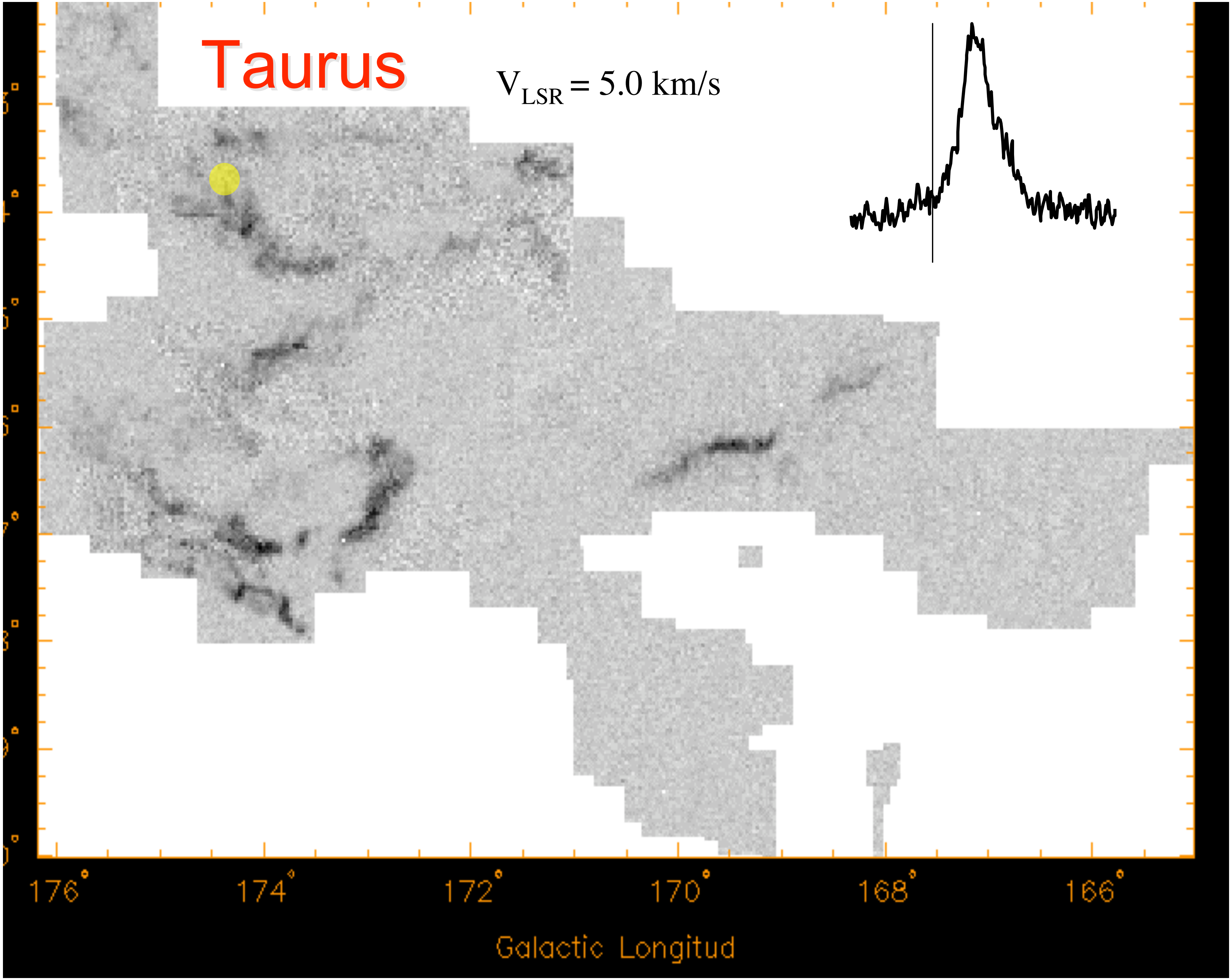


Taurus
V_LSR = 3.8 km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud

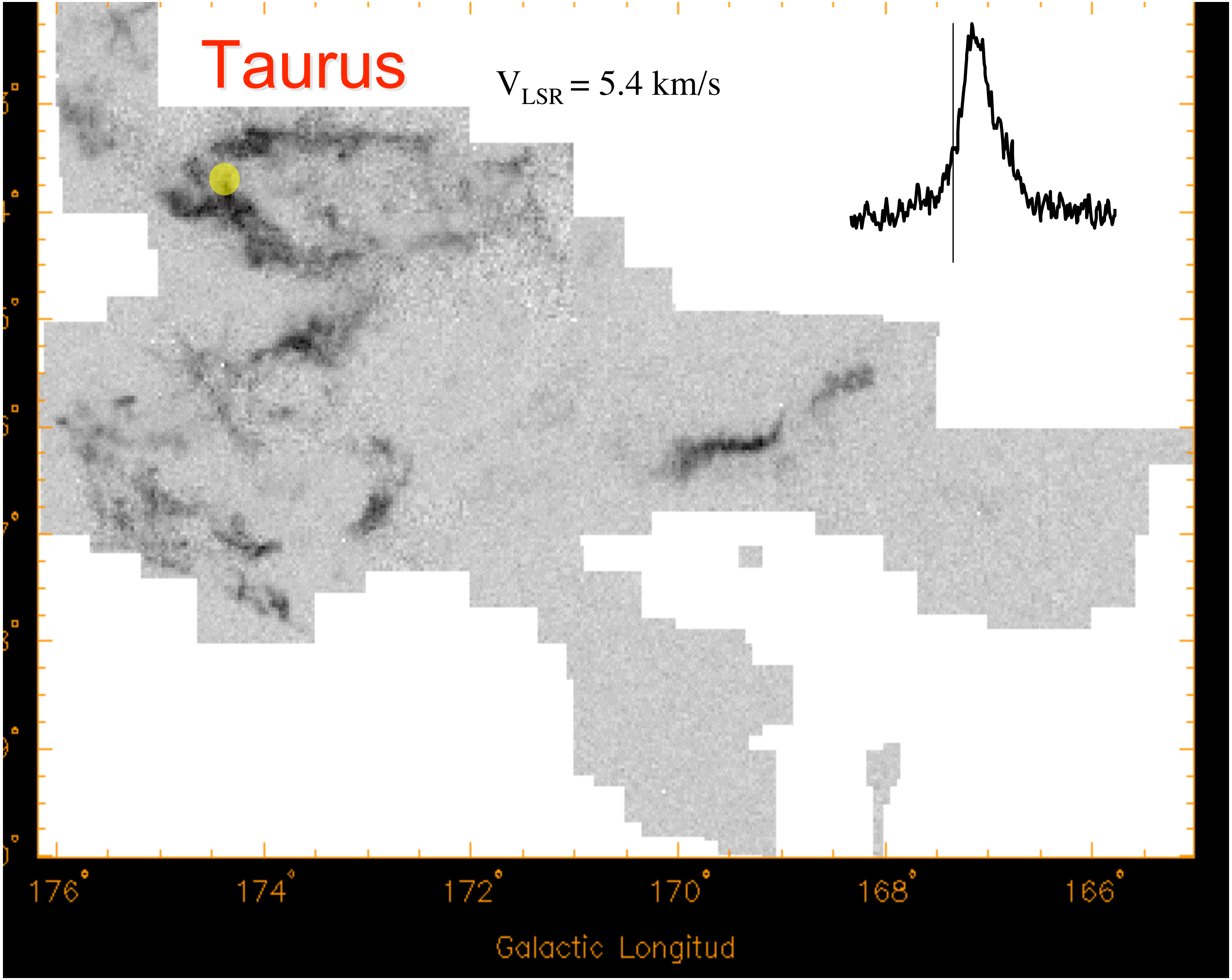


Taurus
V_LSR = 4.2 km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud

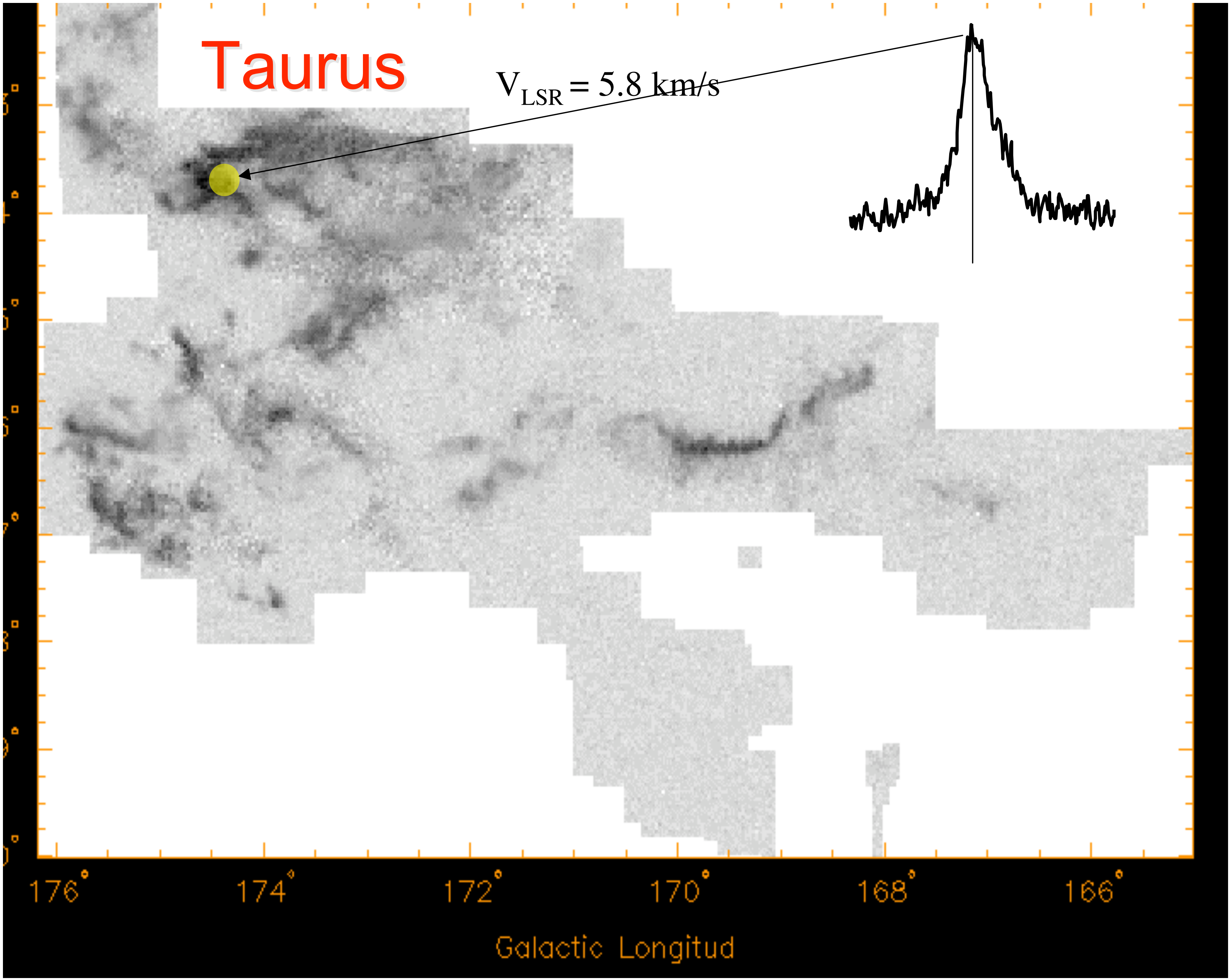

Taurus
VLSR = 4.6 km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud



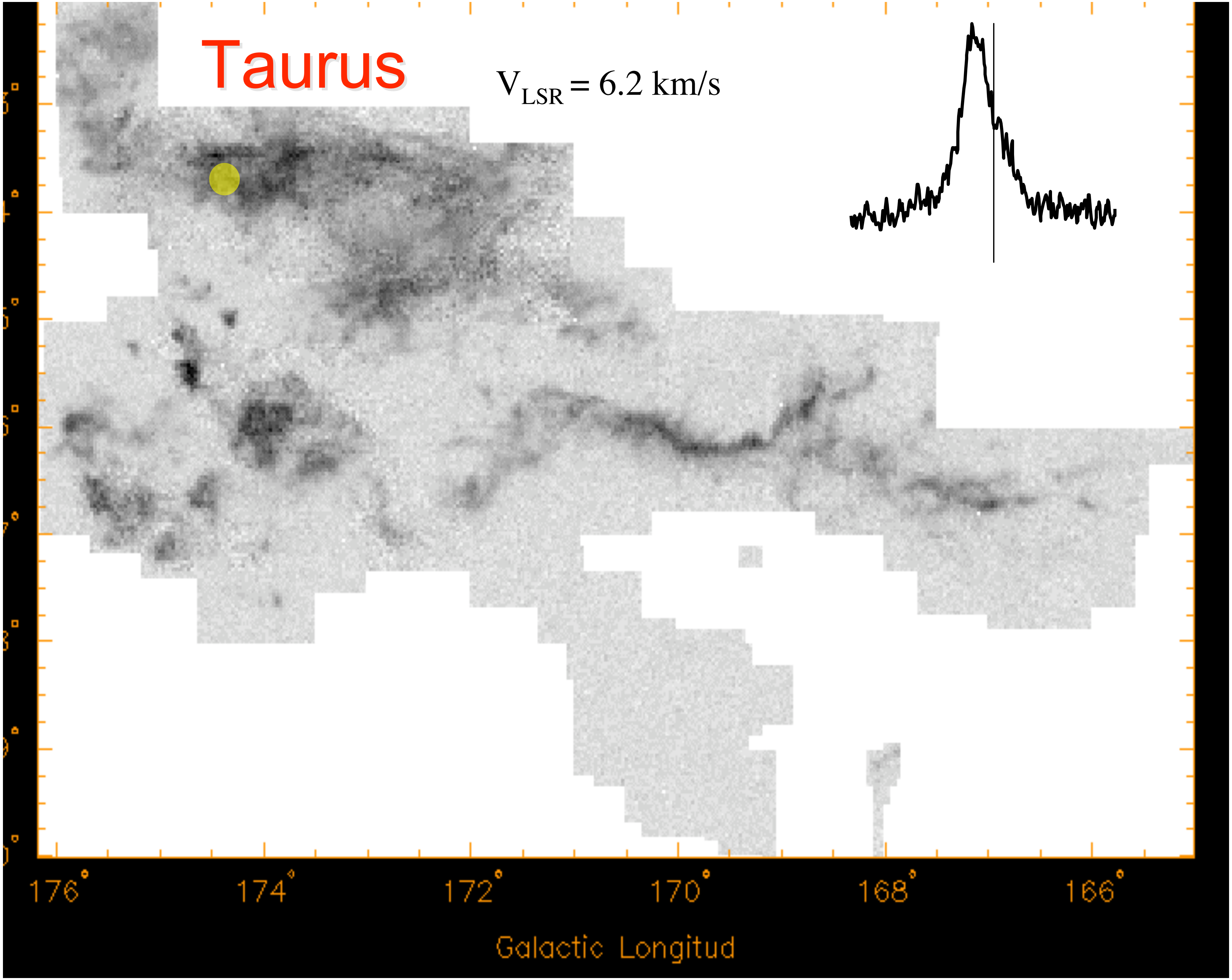
Taurus
V_LSR = 5.0 km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud

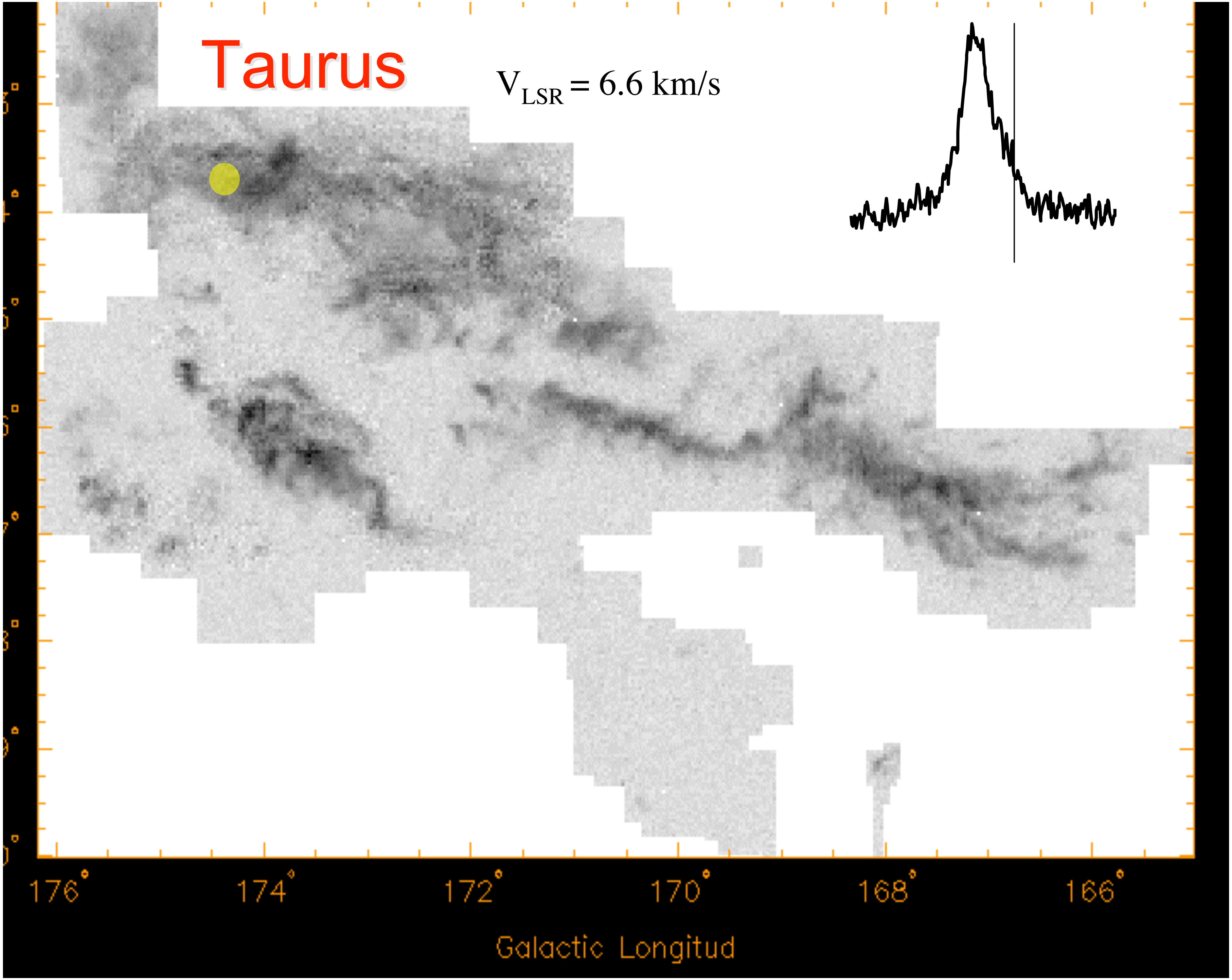


Taurus
V$_{LSR}$ = 5.4 km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud

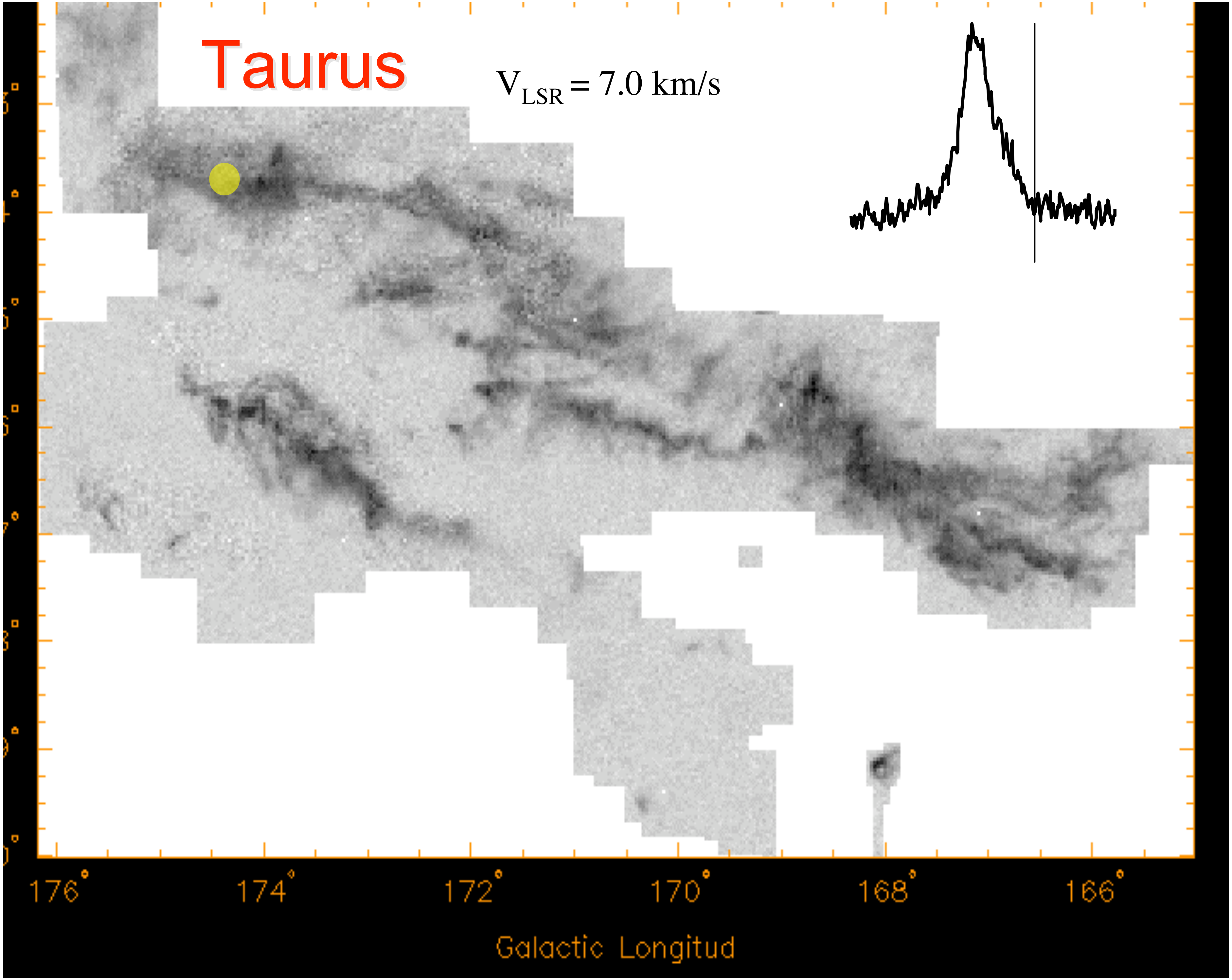


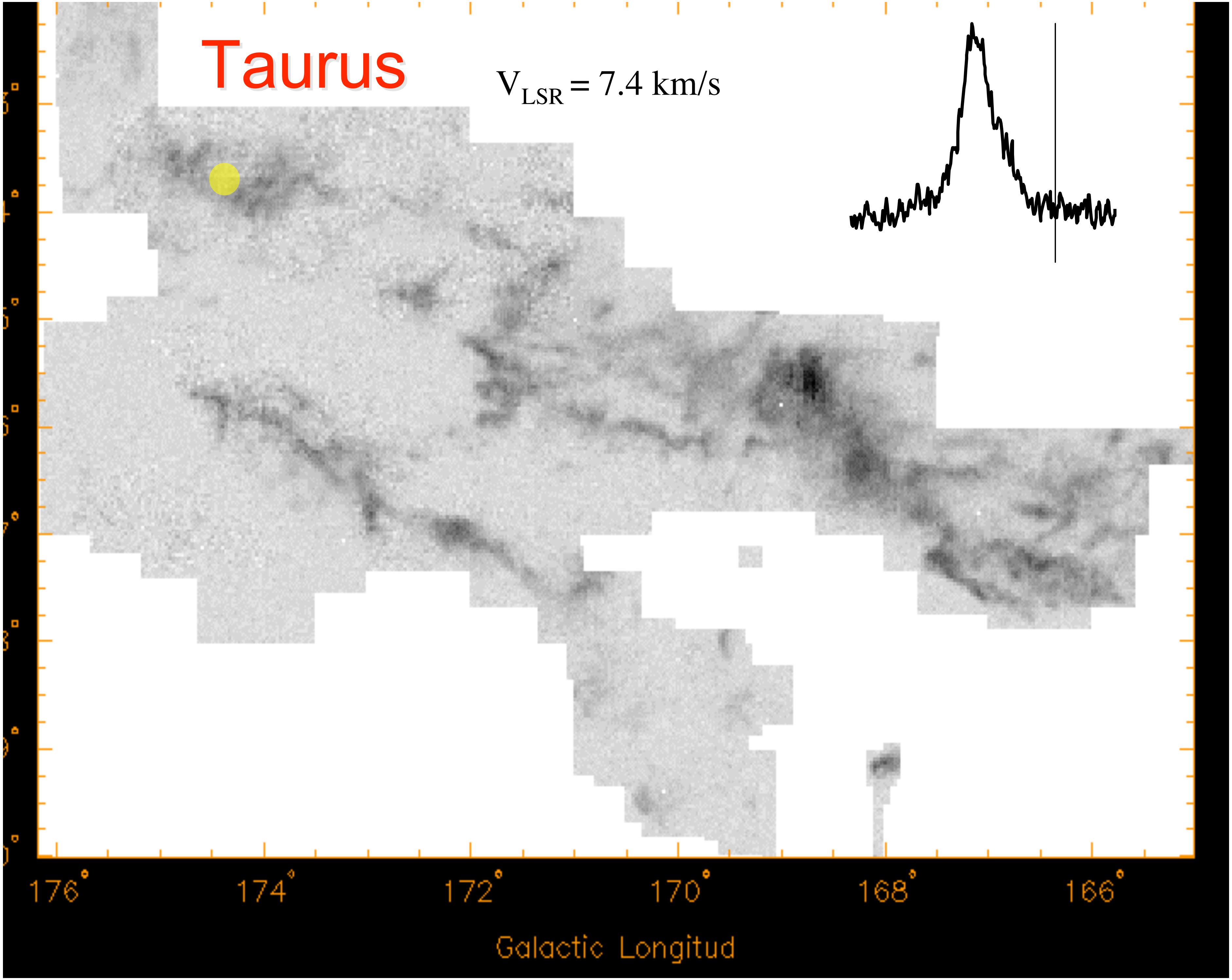
Taurus
$V_{LSR} = 5.8$ km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud


Taurus
$V_{LSR}$ = 6.2 km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud

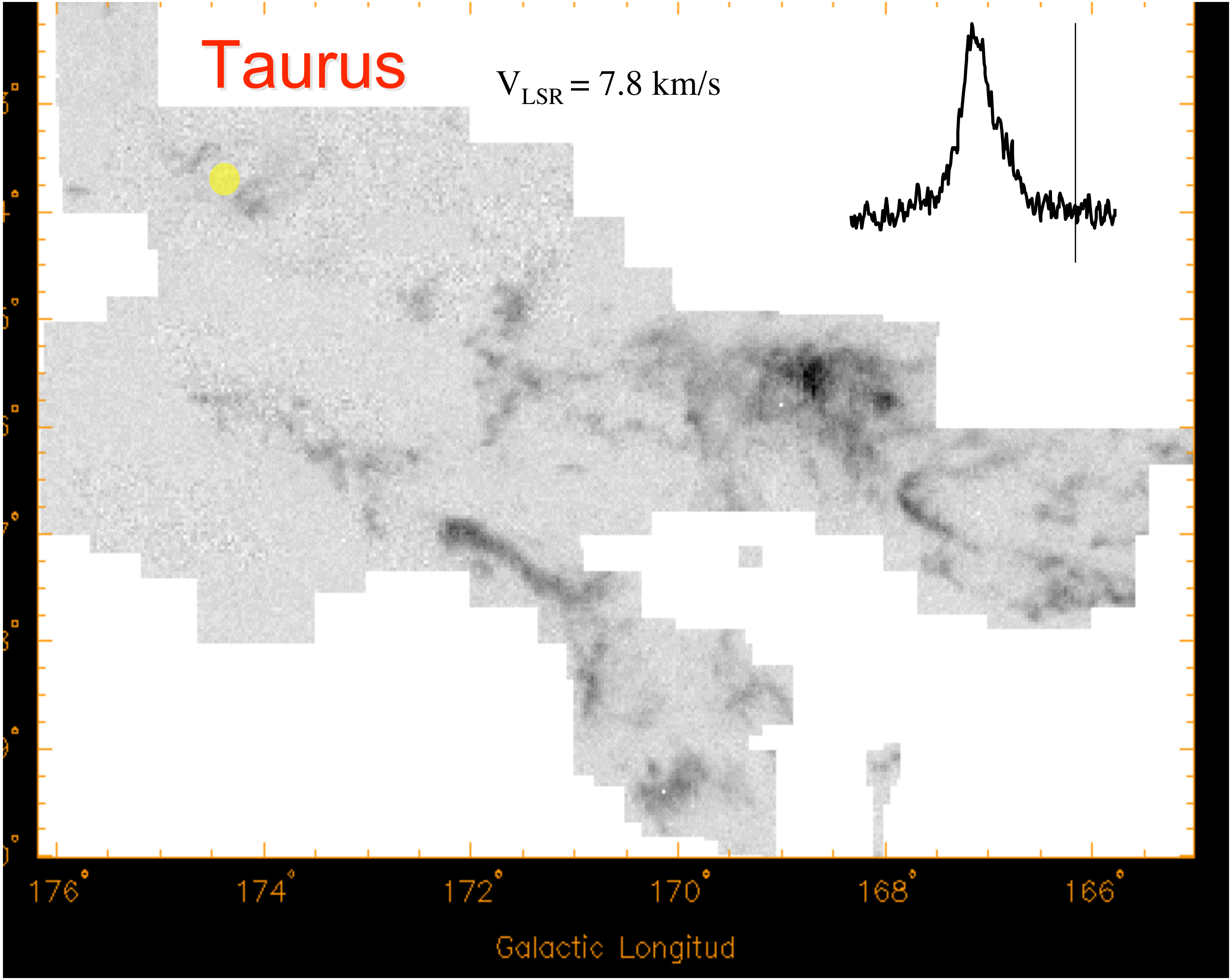


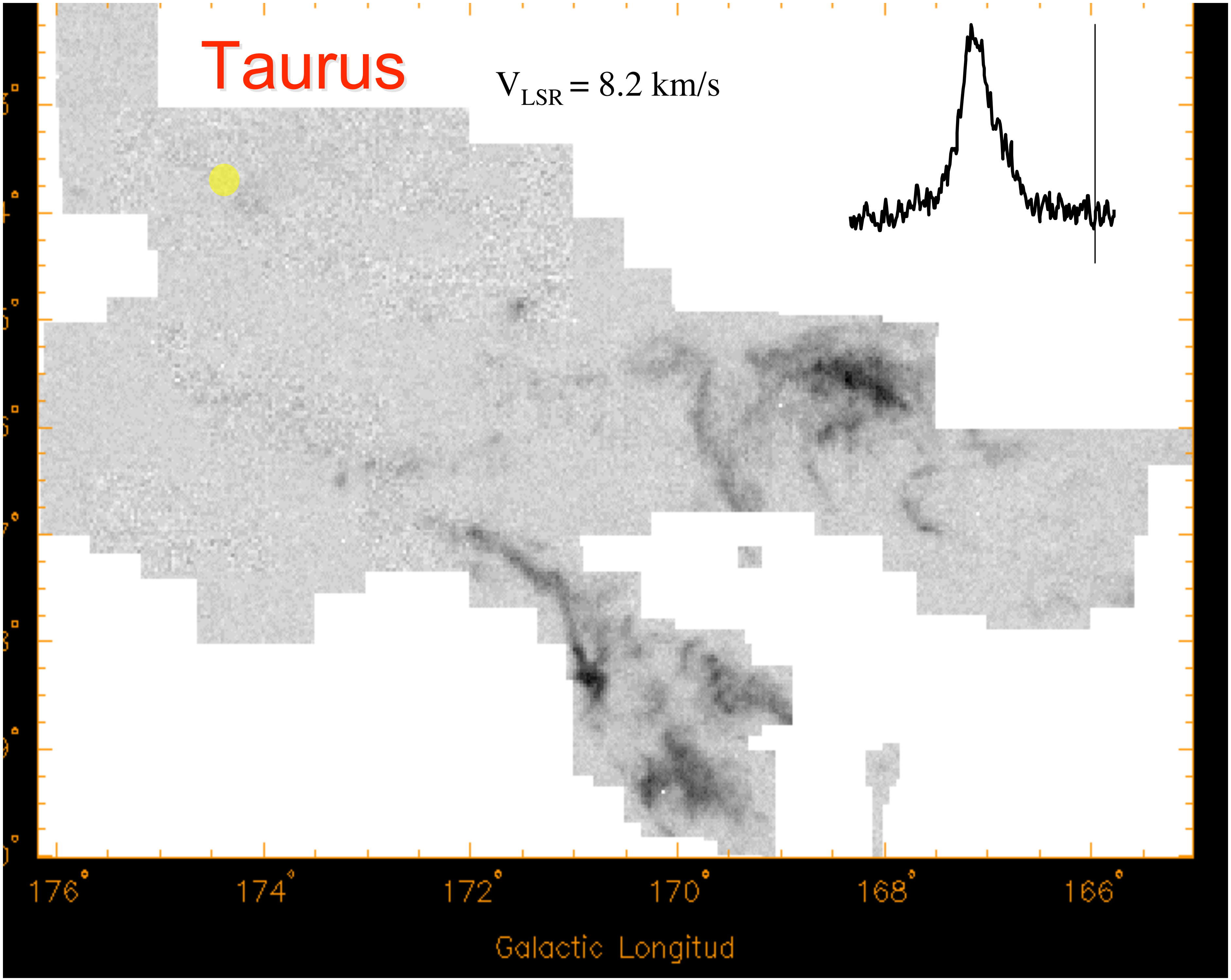
Taurus
$V_{LSR}$ = 6.6 km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud


Taurus
V_LSR = 7.0 km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud

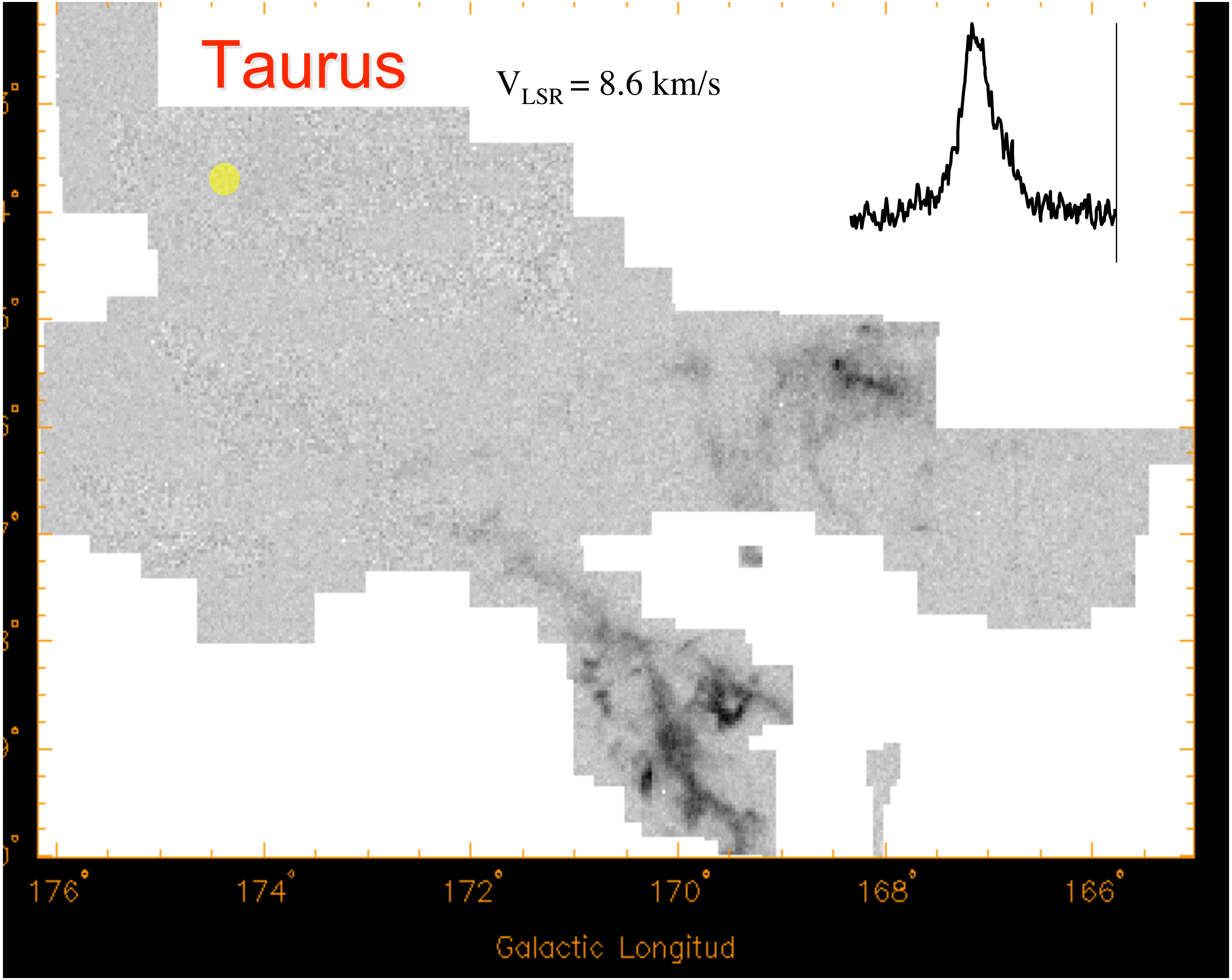


Taurus
V_LSR = 7.4 km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud

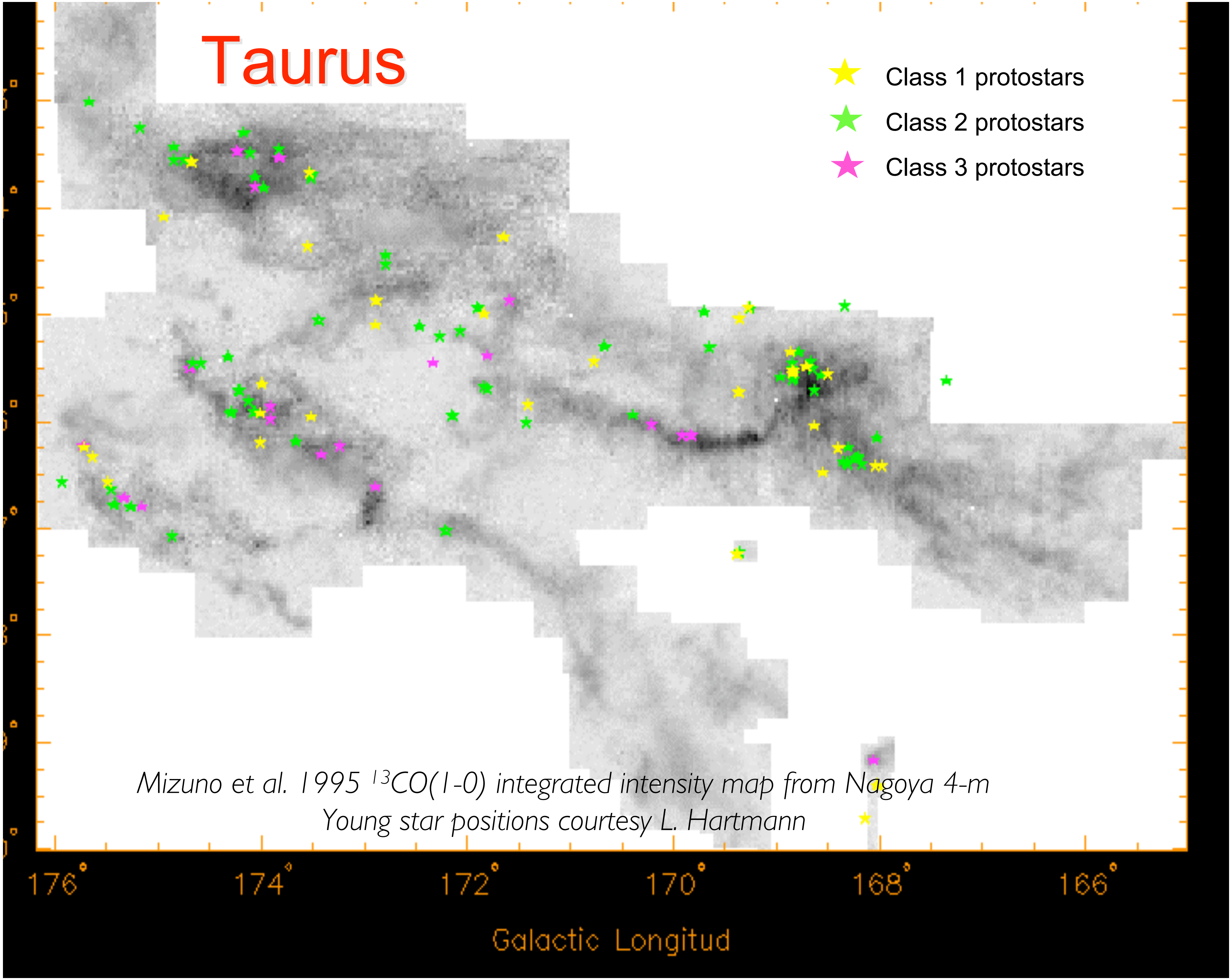

Taurus
V_LSR = 7.8 km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud



Taurus
$V_{LSR}$ = 8.2 km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud

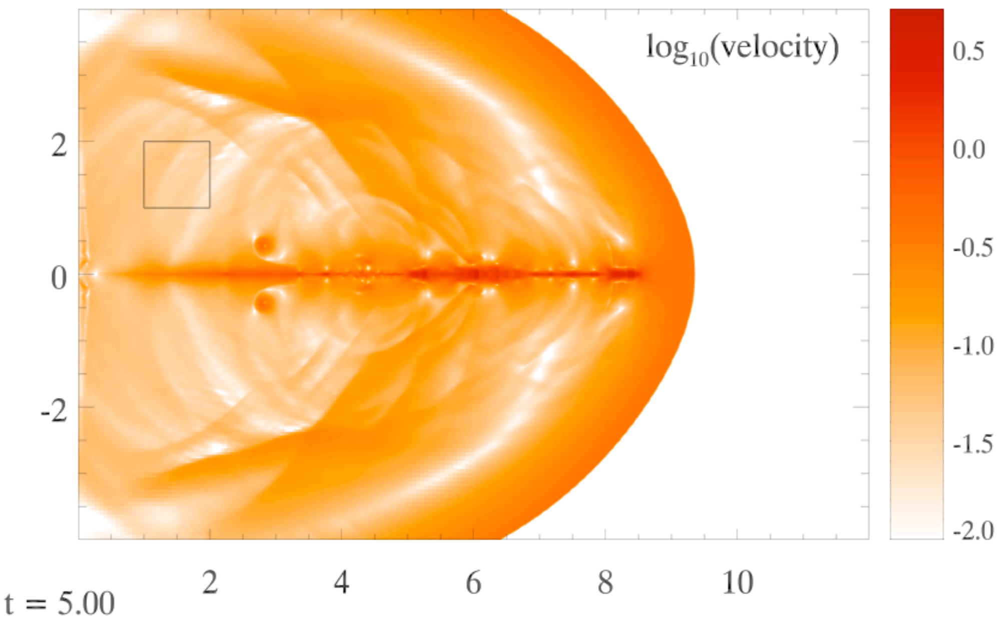


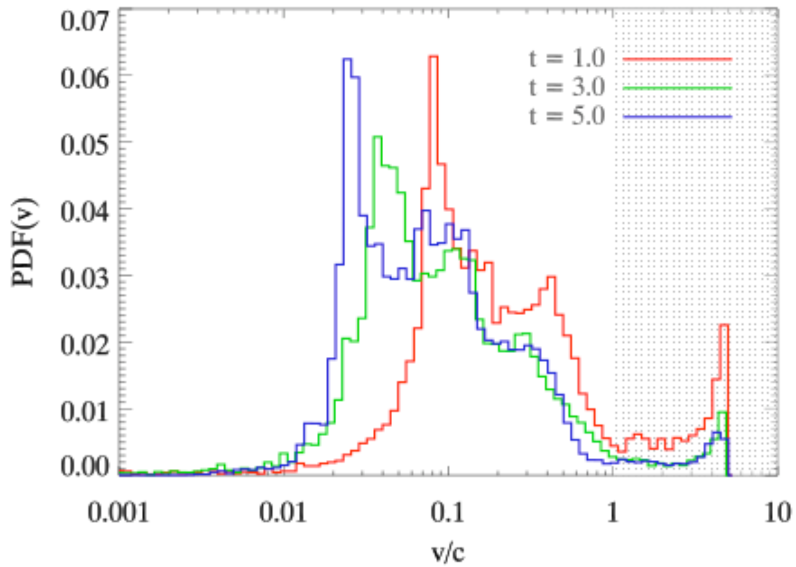
Taurus
V_LSR = 8.6 km/s
176°
174°
172°
170°
168°
166°
Galactic Longitud


Taurus
Class 1 protostars
Class 2 protostars
Class 3 protostars
176°
174°
172°
170°
168°
166°
Galactic Longitud
Mizuno et al. 1995 13CO(1-0) integrated intensity map from Nagoya 4-m
Young star positions courtesy L. Hartmann

# local feedback

- can outflows drive turbulence locally?

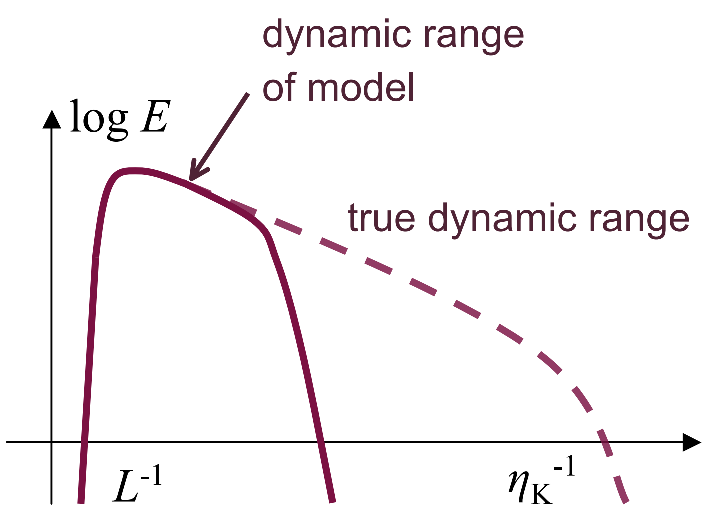



Banerjee, Klessen, & Fendt (2007)

# Large-eddy simulations

- We use ***LES*** to model the large-scale dynamics
- Principal problem: only large scale flow properties
  - Reynolds number: Re = LV/ν ***($Re_{nature}$ >> $Re_{model}$)***
  - dynamic range much smaller than true physical one
  - need *subgrid model* (in our case simple: only dissipation)
  - but what to do for more complex when processes on subgrid scale determine large-scale dynamics
    (chemical reactions, nuclear burning, etc)
  - Turbulence is "space filling" --> difficulty for AMR (don't know what criterion to use for refinement)
- How *large* a Reynolds number do we need to catch basic dynamics right?



theoretical approach

# Gravoturbulent star formation

- Idea:

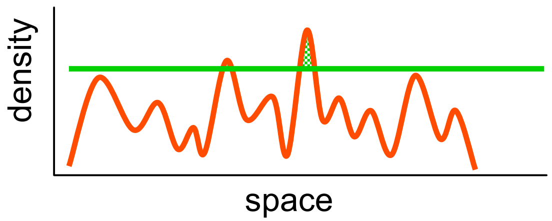
> ***Star formation is controlled by interplay between gravity and supersonic turbulence!***

- Dual role of turbulence:
  - *stability on large scales*
  - *initiating collapse on small scales*

(e.g., Larson, 2003, Rep. Prog. Phys, 66, 1651;
or Mac Low & Klessen, 2004, Rev. Mod. Phys., 76, 125)

# Gravoturbulent star formation

- interstellar gas is highly *inhomogeneous*
  - *thermal instability*
  - *gravitational instability*
  - *turbulent compression* (in shocks $\delta\rho/\rho \propto \mathcal{M}^2$; in atomic gas: $\mathcal{M} \approx 1...3$)



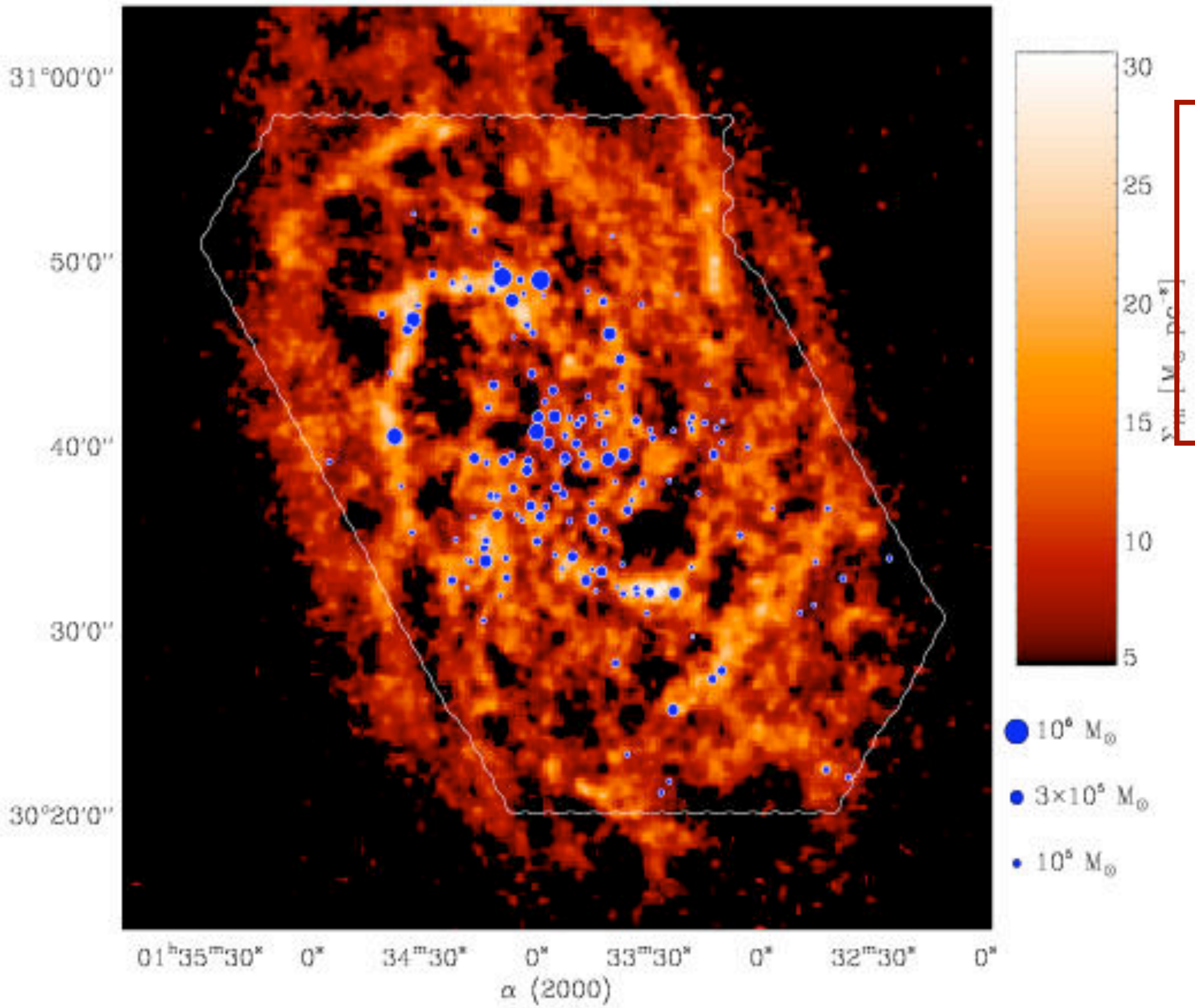

- cold *molecular clouds* can form rapidly in high-density regions at *stagnation points* of *convergent large-scale flows*
  - chemical *phase transition*: atomic → molecular
  - process is *modulated* by large-scale *dynamics* in the galaxy
- inside *cold clouds:* turbulence is highly supersonic ($\mathcal{M} \approx 1...20$)
  → *turbulence* creates large density contrast,
  *gravity* selects for collapse

  ⟶ ***GRAVOTUBULENT FRAGMENTATION***

- *turbulent cascade:* local compression *within* a cloud provokes collapse
  → formation of individual *stars* and *star clusters*

(e.g. Mac Low & Klessen, 2004, Rev. Mod. Phys., 76, 125-194)

# molecular cloud formation

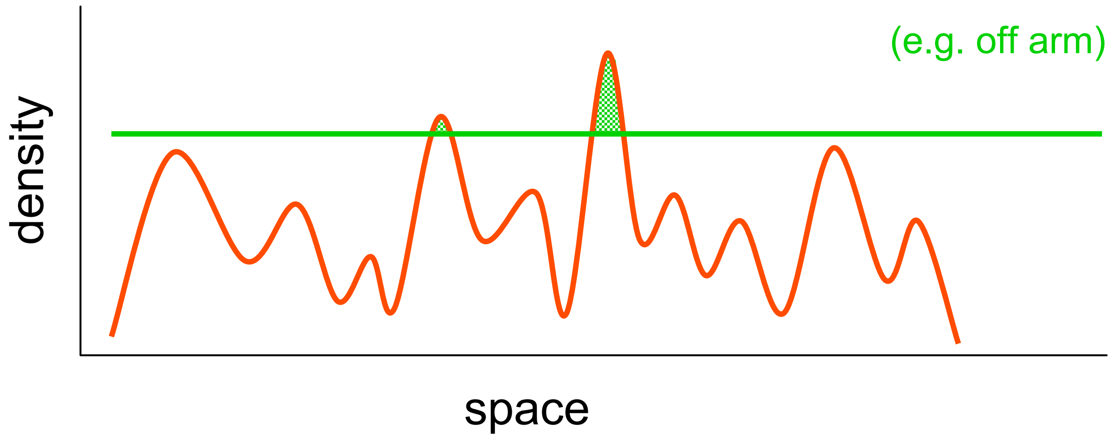


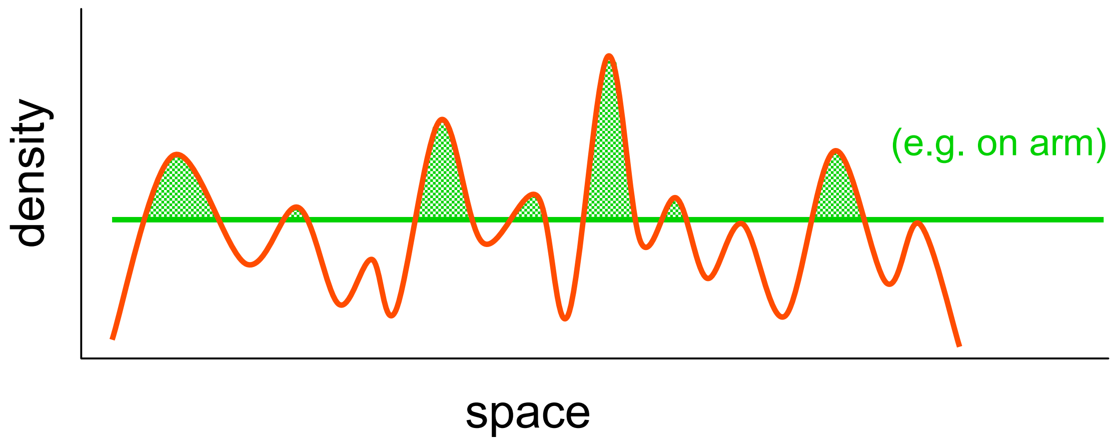

Thesis:

Molecular clouds form at *stagnation points* of large-scale convergent flows, mostly triggered by global (or external) perturbations.

(Deul & van der Hulst 1987, Blitz et al. 2004)

# correlation with large-scale perturbations




*density/temperature fluctuations* in warm atomar ISM are caused by *thermal/gravitational instability* and/or *supersonic turbulence*

some fluctuations are *dense* enough to *form $H_2$* within "*reasonable time*"

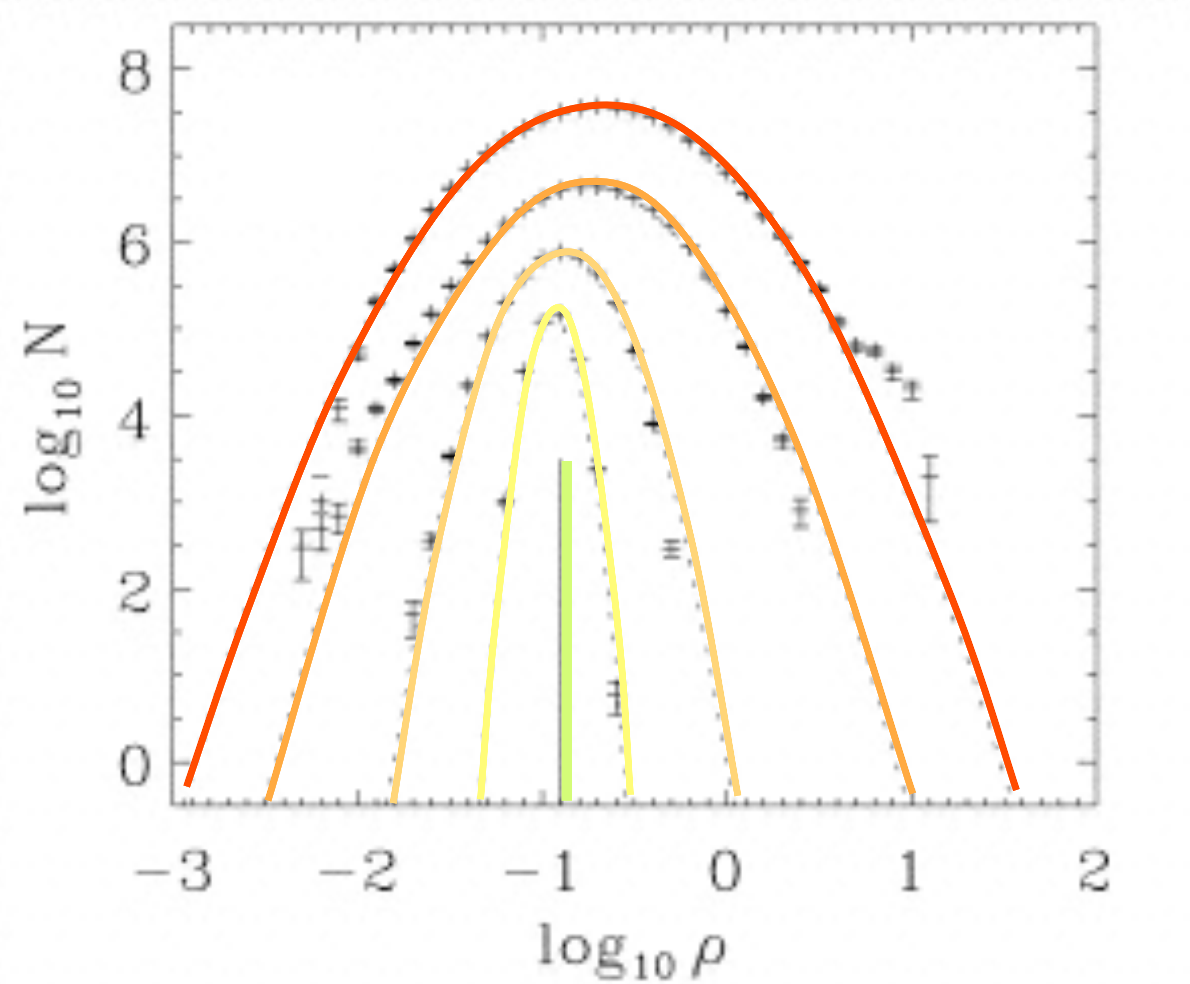
→ *molecular cloud*

(Glover & Mac Low 2007a,b)

*external perturbations* (i.e. potential changes) *increase* likelihood

(Dobbs & Bonnell 2006)

# star formation on *global* scales




mass weighted ρ-pdf, each shifted by ΔlogN=1

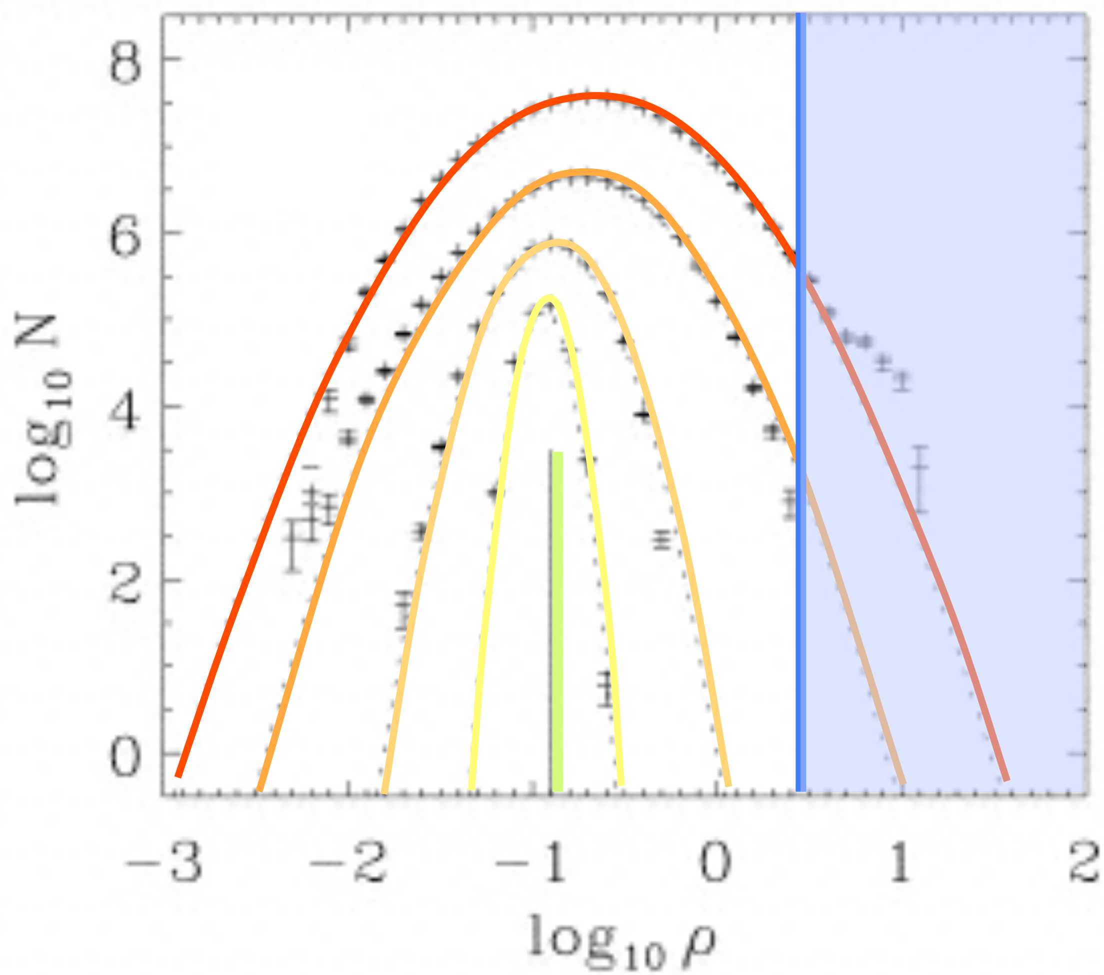
probability distribution function of the density *(ρ-pdf)*

*varying rms Mach numbers:*

**M1 > M2 > M3 > M4 > 0**

(from Klessen, 2001; also Gazol et al. 2005, Mac Low et al. 2005)

# star formation on *global* scales




mass weighted ρ-pdf, each shifted by ΔlogN=1

(rate from Hollenback, Werner, & Salpeter 1971)

$H_2$ formation rate:

$$\tau_{H_2} \approx \frac{1.5\,\mathrm{Gyr}}{n_H / 1\mathrm{cm}^{-3}}$$

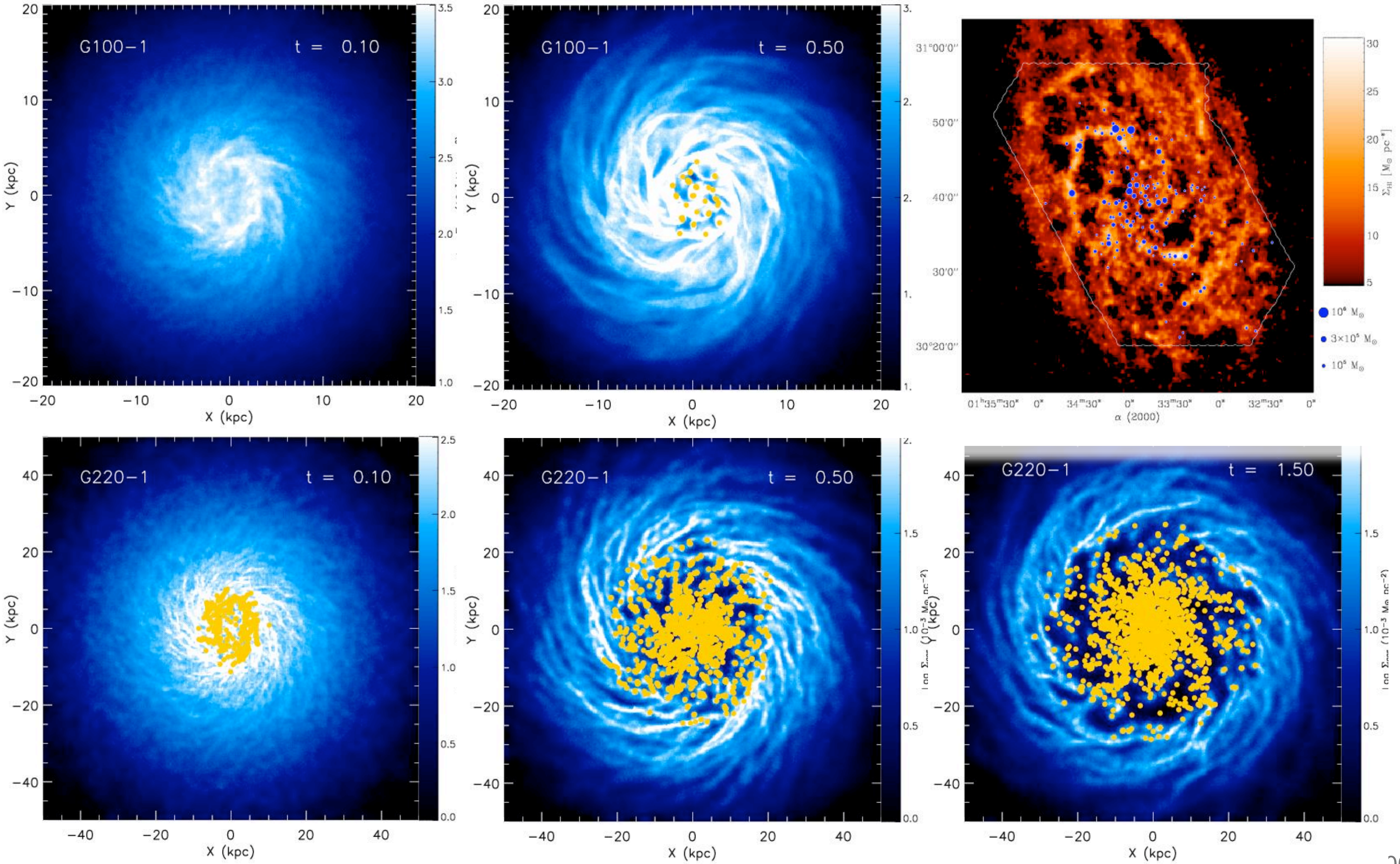
for $n_H \geq 100\ \mathrm{cm}^{-3}$, $H_2$ forms within 10Myr, this is about the lifetime of typical MC's.

in turbulent gas, the $H_2$ fraction can become very high on short timescale

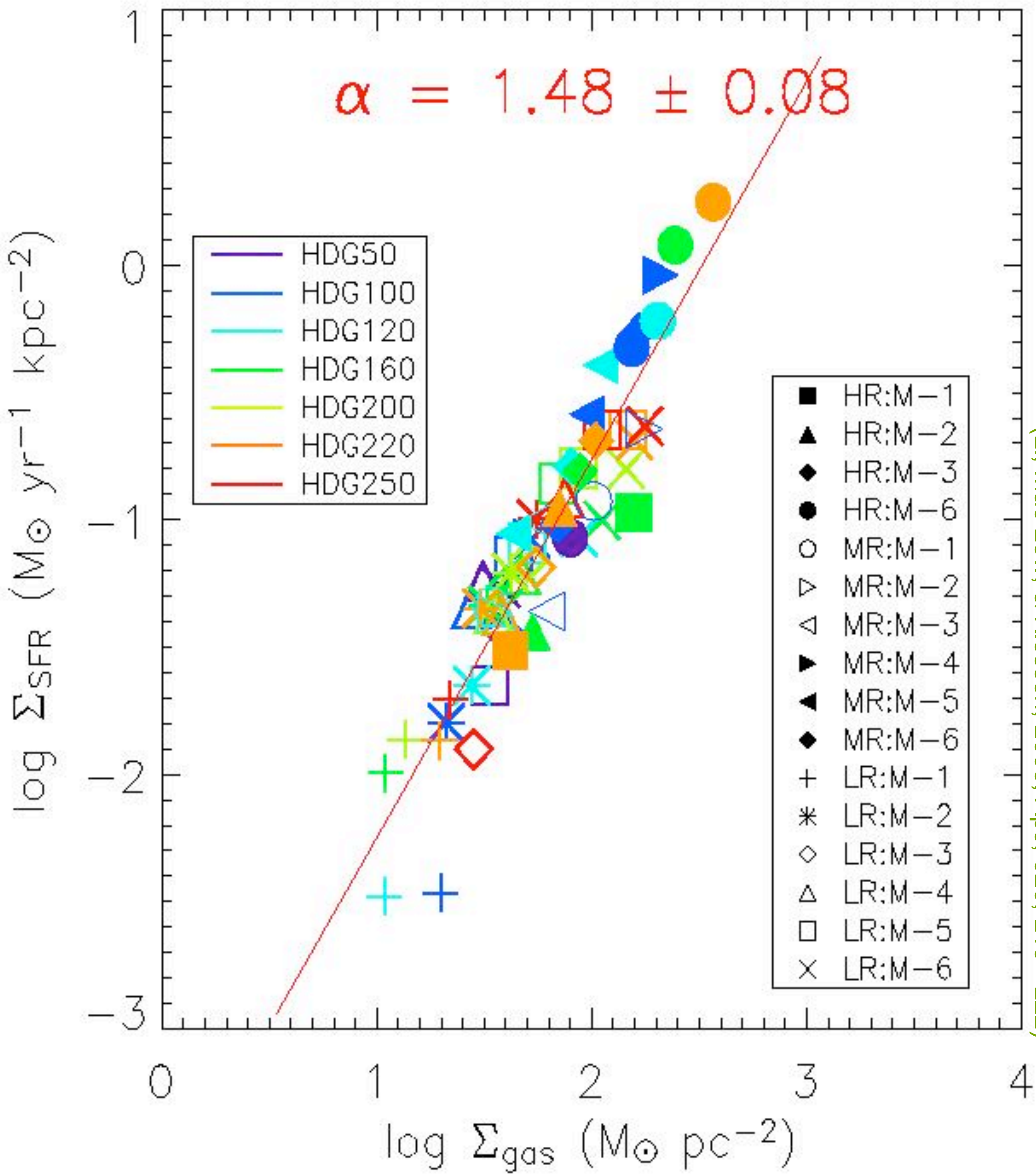
(for models with coupling between cloud dynamics and time-dependent chemistry, see Glover & Mac Low 2007a,b)

# modeling galactic SF

SPH calculations of self-gravitating disks of stars and (isothermal) gas in dark-matter potential, sink particles measure local collapse --> star formation




(Li, Mac Low, & Klessen, 2005, ApJ, 620,L19 - L22)



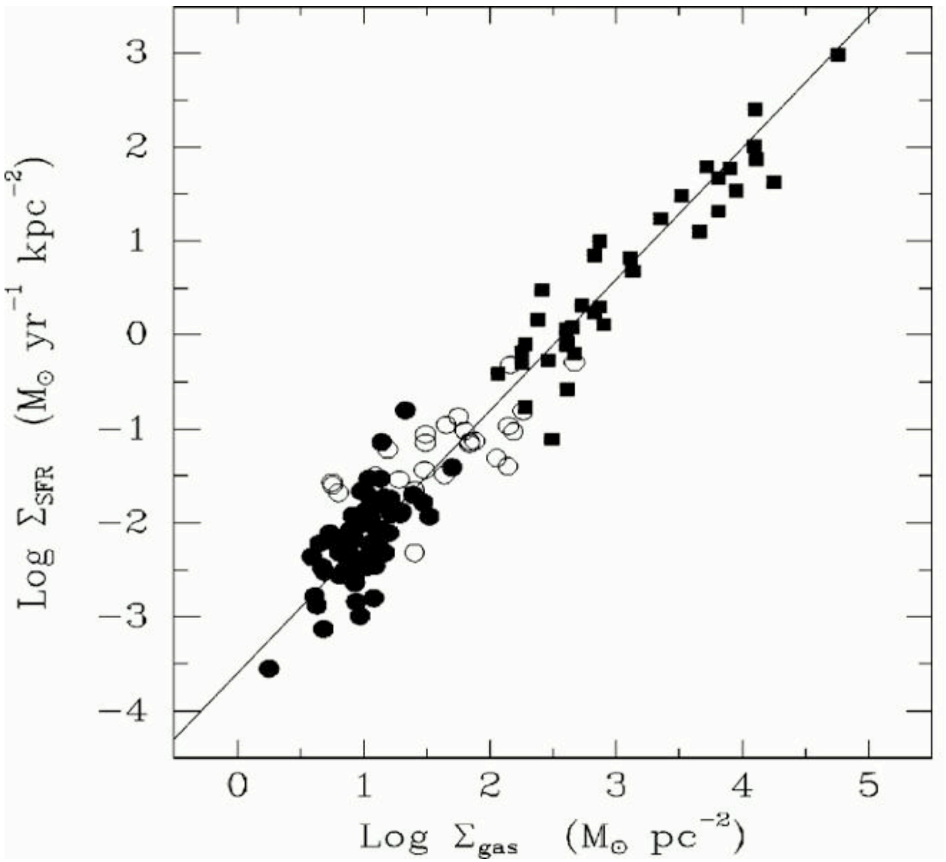

(Li, Mac Low, & Klessen, 2005, ApJ, 620, L19 - L22)

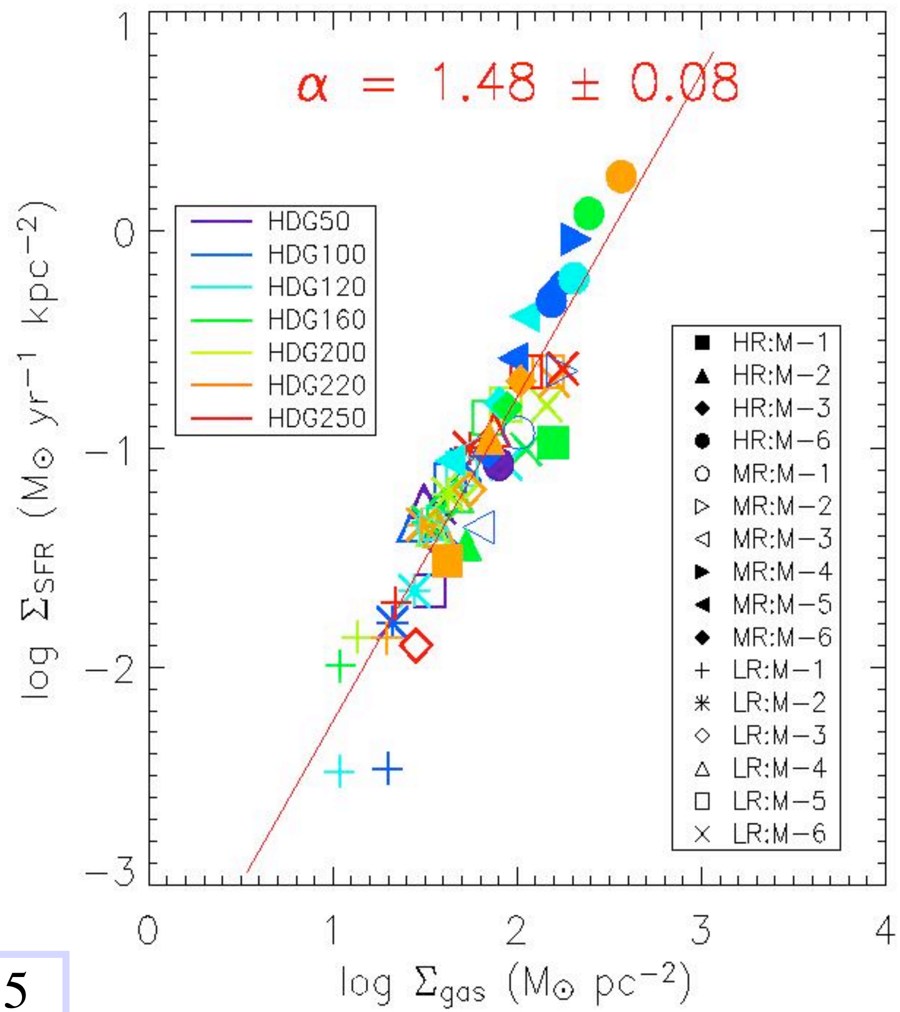
We find correlation between *star formation rate* and *gas surface density:*

$$\Sigma_{SFR} \propto \Sigma_{gas}^{1.5}$$

*global Schmidt law*

# observed Schmidt law



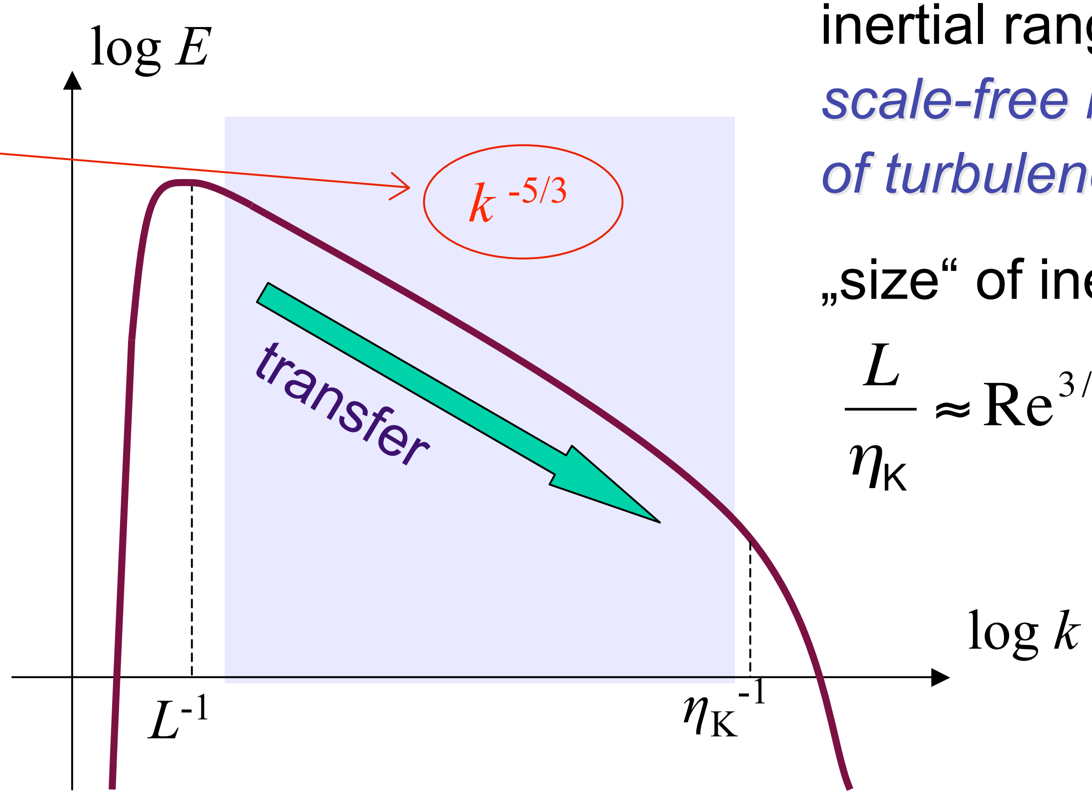

in both cases: $\Sigma_{\mathrm{SFR}} \propto \Sigma_{\mathrm{gas}}^{1.5}$

(from Kennicutt 1998)

# turbulence

# Turbulent cascade

Kolmogorov (1941) theory
incompressible turbulence




energy
input
scale

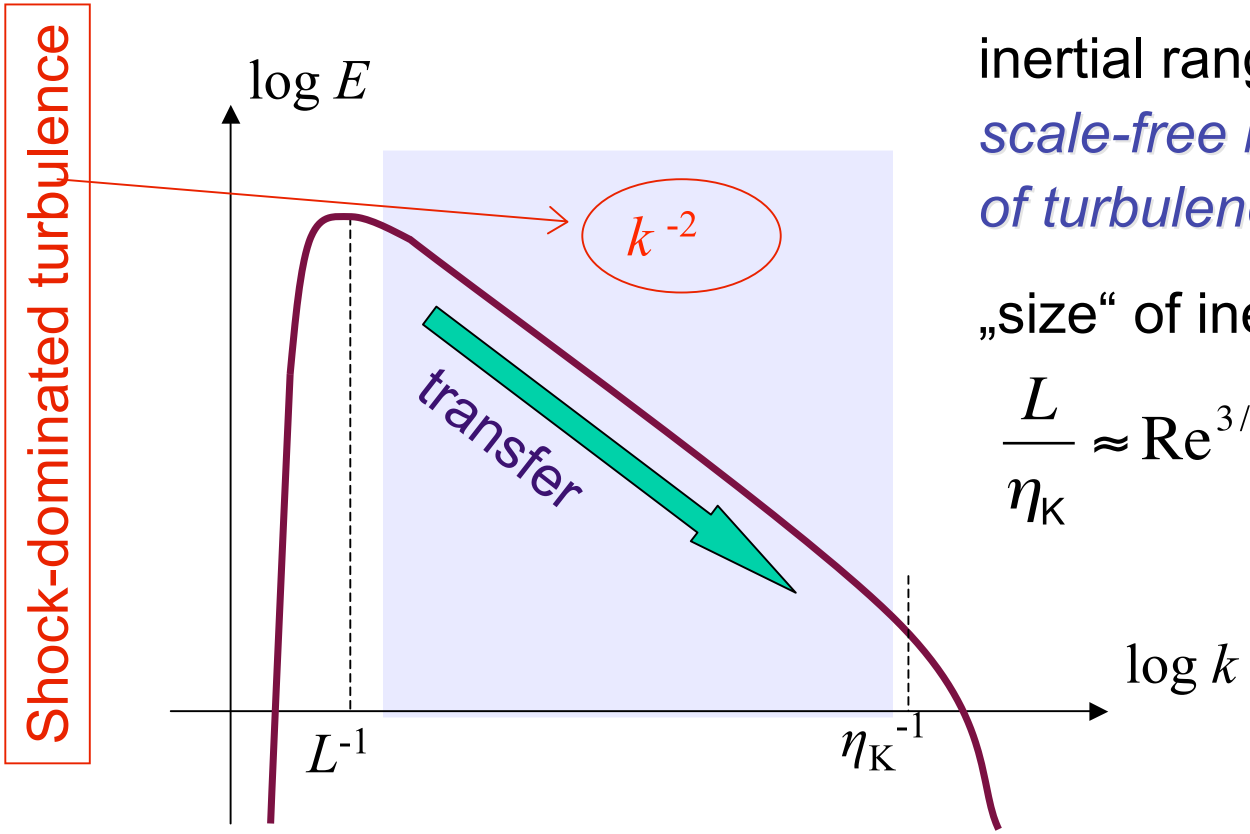
energy
dissipation
scale

inertial range:
*scale-free behavior of turbulence*

„size“ of inertial range:

$$\frac{L}{\eta_K} \approx \mathrm{Re}^{3/4}$$

# Turbulent cascade




energy
input
scale

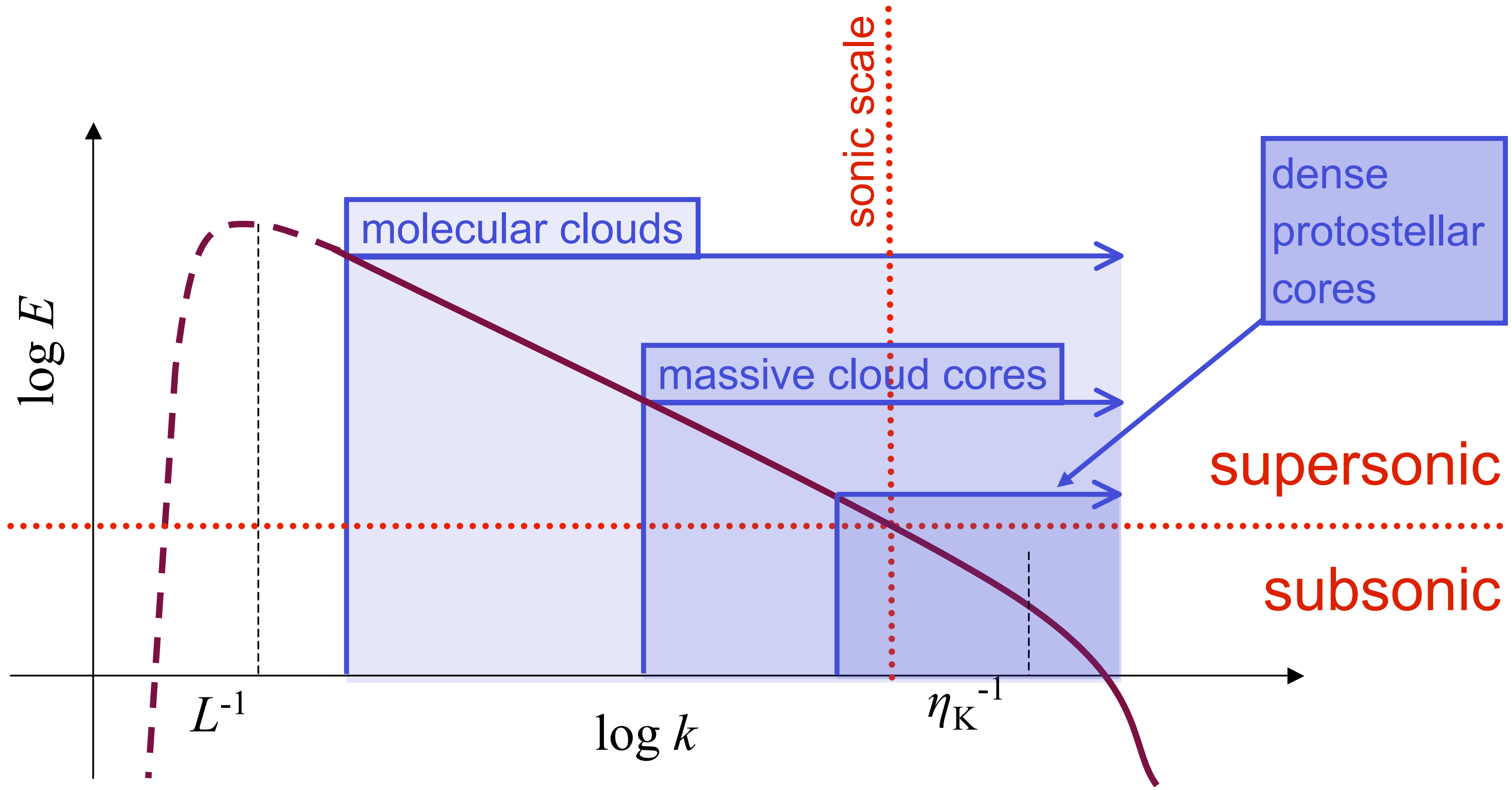
energy
dissipation
scale

inertial range:
*scale-free behavior of turbulence*

„size“ of inertial range:

$$\frac{L}{\eta_K} \approx \mathrm{Re}^{3/4}$$

# Turbulent cascade in ISM

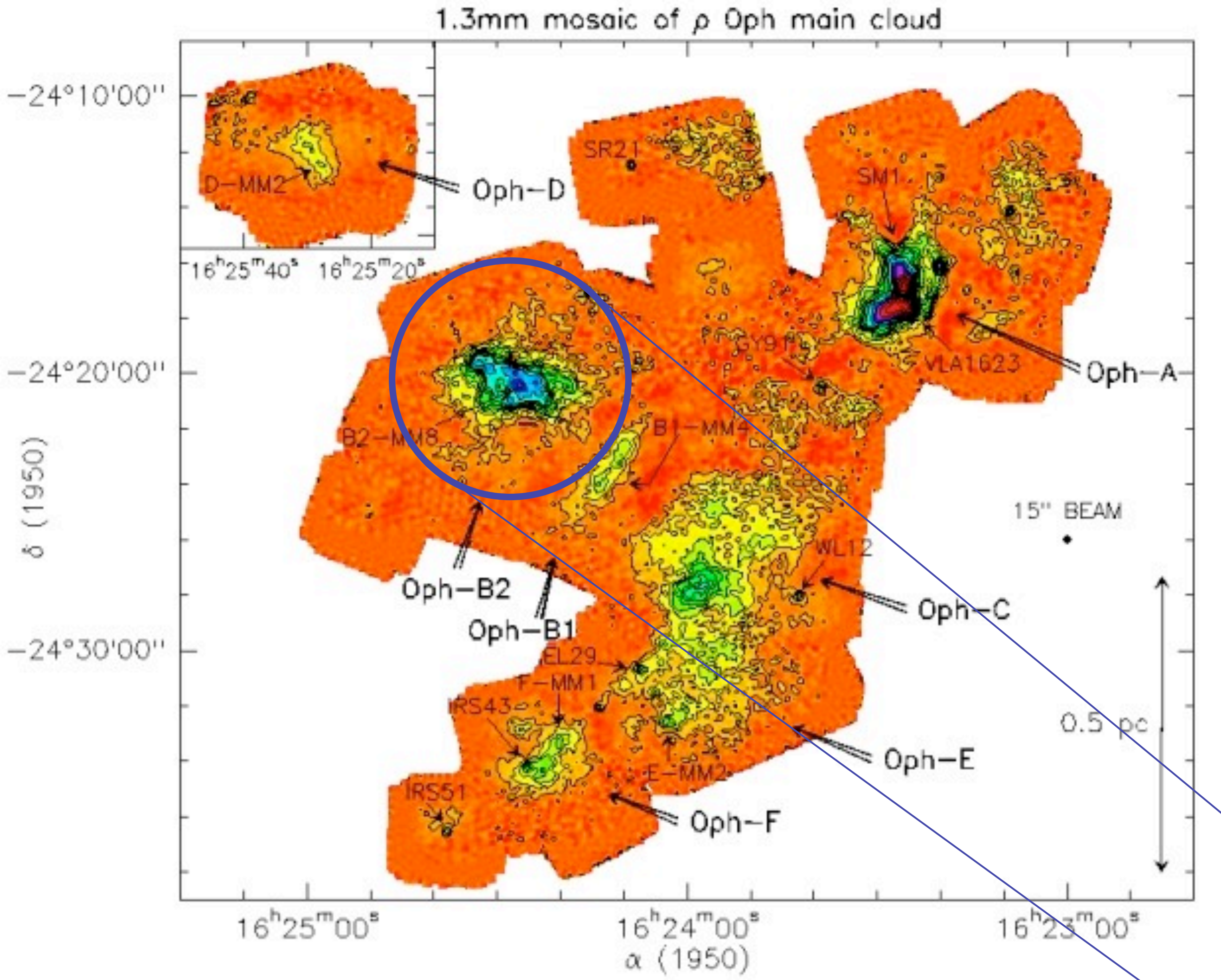



energy source & scale *NOT known* (supernovae, winds, spiral density waves?)

$\sigma_{rms} << 1$ km/s
$M_{rms} \leq 1$
$L \approx 0.1$ pc

dissipation scale not known (ambipolar diffusion, molecular diffusion?)

# Density structure of MC's




(Motte, André, & Neri 1998)

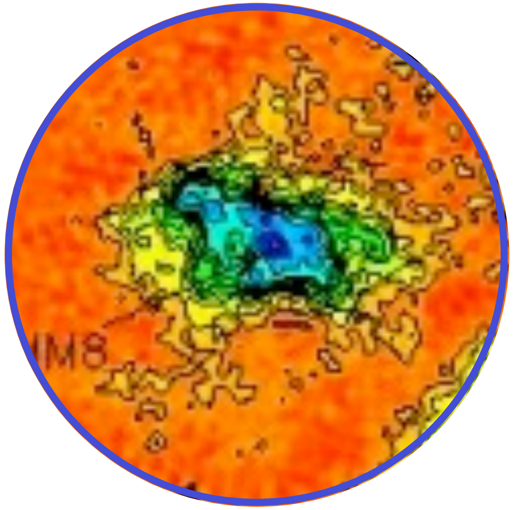
molecular clouds are highly inhomogeneous

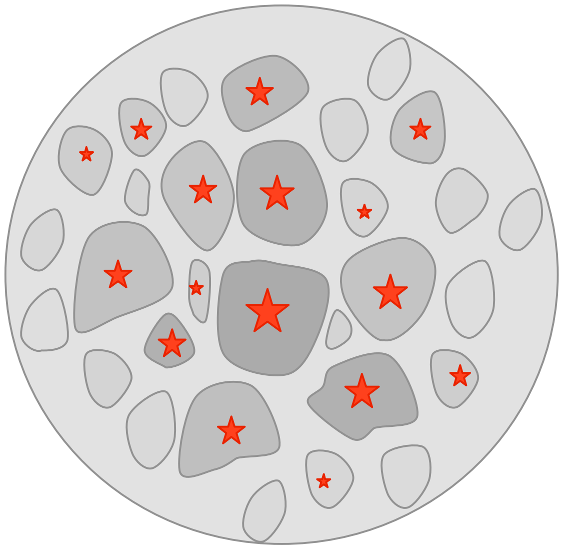
stars form in the densest and coldest parts of the cloud

ρ-Ophiuchus cloud seen in dust emission

let's focus on a cloud core like this one

# Evolution of cloud cores

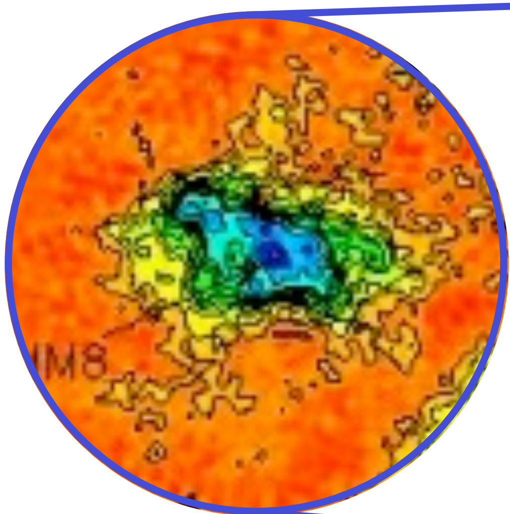


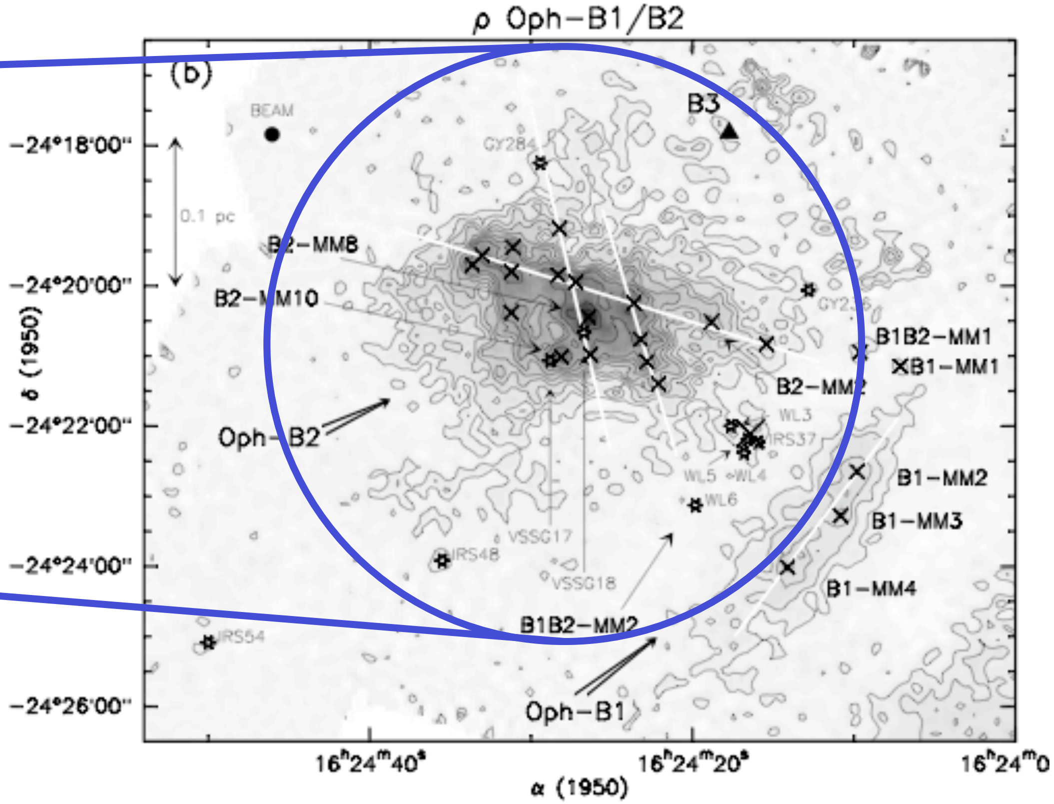



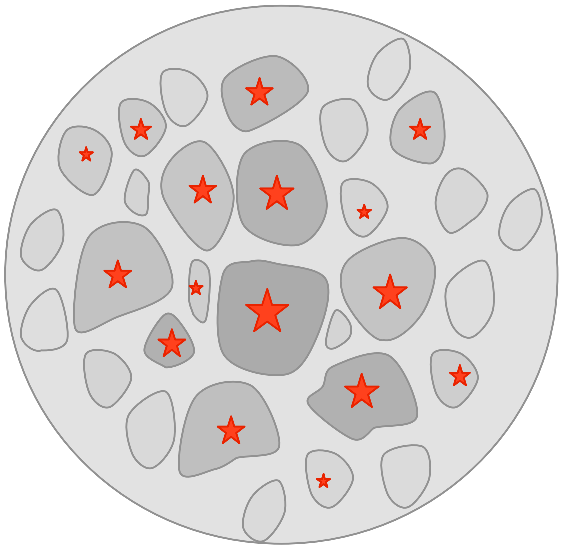
- How does this core evolve?
  Does it form one single massive star
  or cluster with mass distribution?

- Turbulent cascade „goes through" cloud core
  --> NO *scale separation* possible
  --> NO *effective sound speed*
- Turbulence is supersonic!
  --> produces strong density contrasts:
  $\delta\rho/\rho \approx \mathcal{M}^2$
  --> with typical $\mathcal{M} \approx 10$ --> $\delta\rho/\rho \approx 100$!
- many of the shock-generated fluctuations are Jeans unstable and go into collapse
- --> expectation: *core breaks up and forms a cluster of stars*

# Evolution of cloud cores



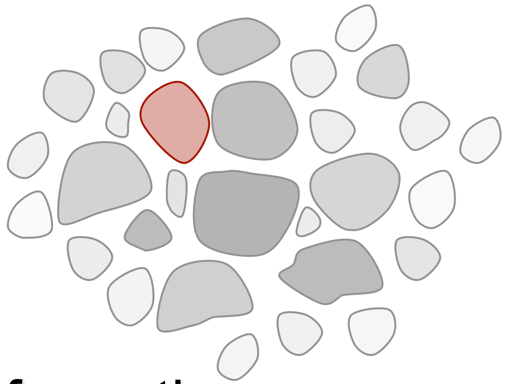


indeed ρ-Oph B1/2 contains several cores ("starless" cores are denoted by ×, cores with embedded protostars by ☆)

(Motte, André, & Neri 1998)

# Formation and evolution of cores



- protostellar cloud cores form at the *stagnation points* of *convergent turbulent flows*
- if $M > M_{Jeans} \propto \rho^{-1/2}\, T^{3/2}$: collapse and star formation
- if $M < M_{Jeans} \propto \rho^{-1/2}\, T^{3/2}$: reexpansion after external compression fades away

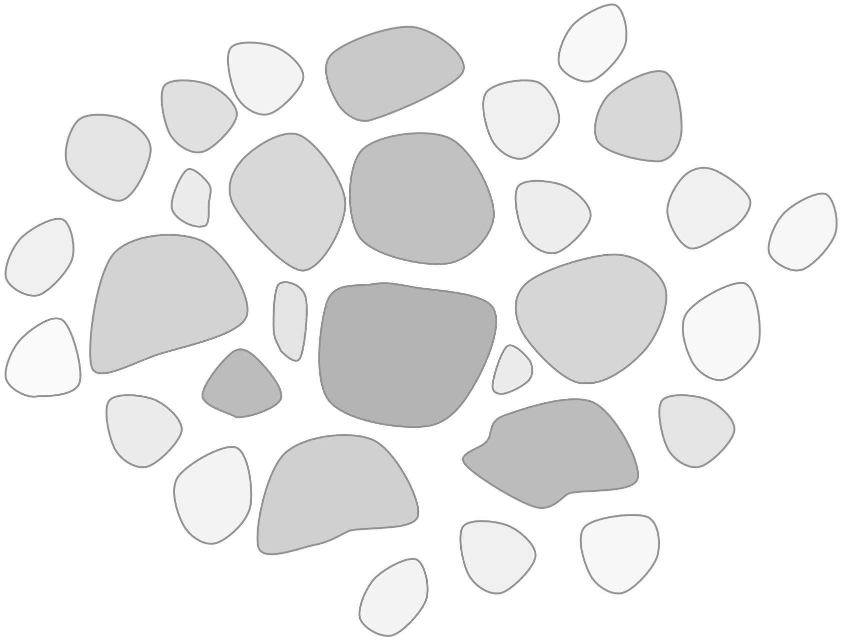
  (e.g. Vazquez-Semadeni et al 2005)

- typical timescales: $t \approx 10^4 \ldots 10^5$ yr
- because *turbulent* ambipolar diffusion time is *short*, this time estimate still holds for the presence of magnetic fields, in *magnetically critical cores*

  (e.g. Fatuzzo & Adams 2002, Heitsch et al. 2004)

# Formation and evolution of cores

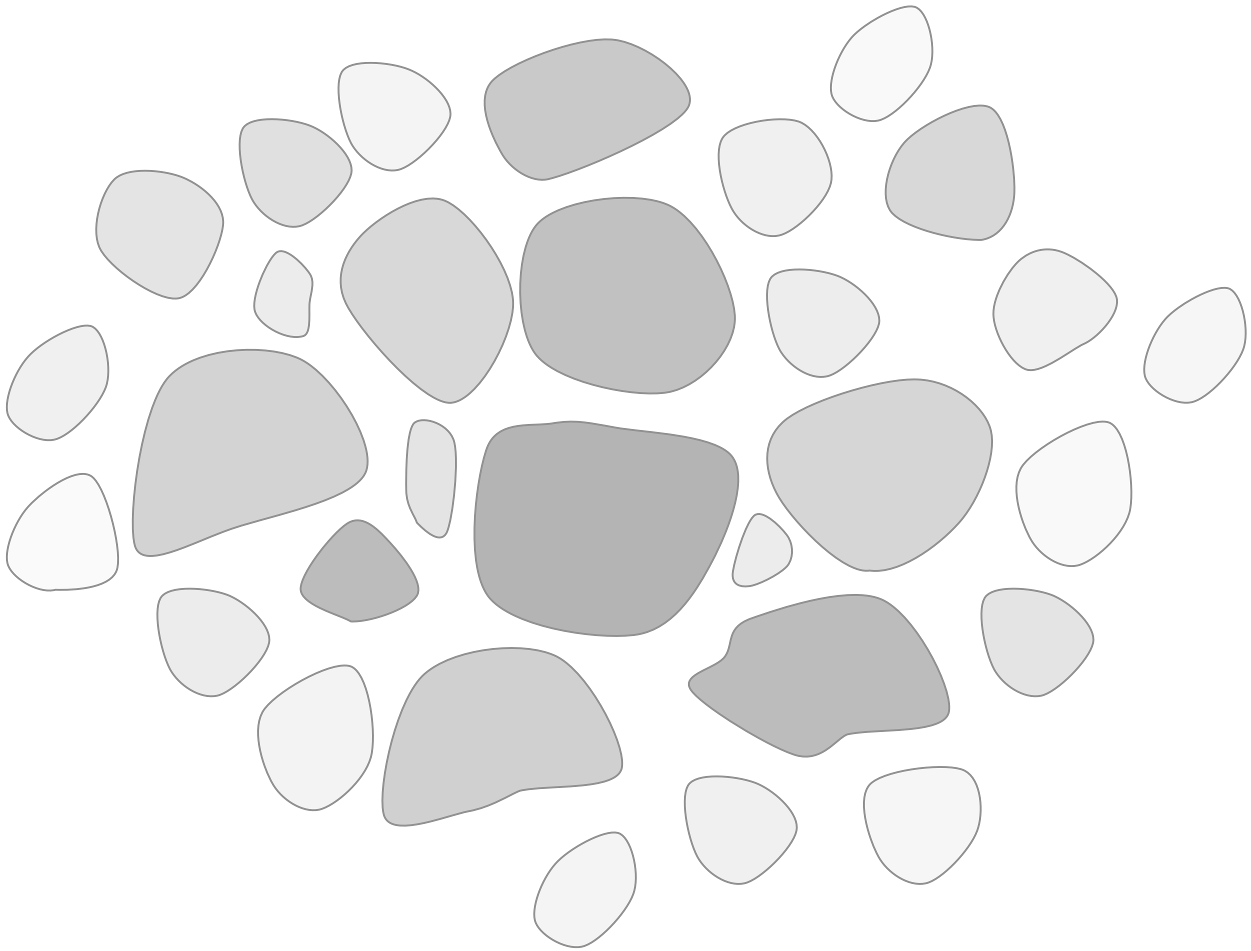
What happens to distribution of cloud cores?



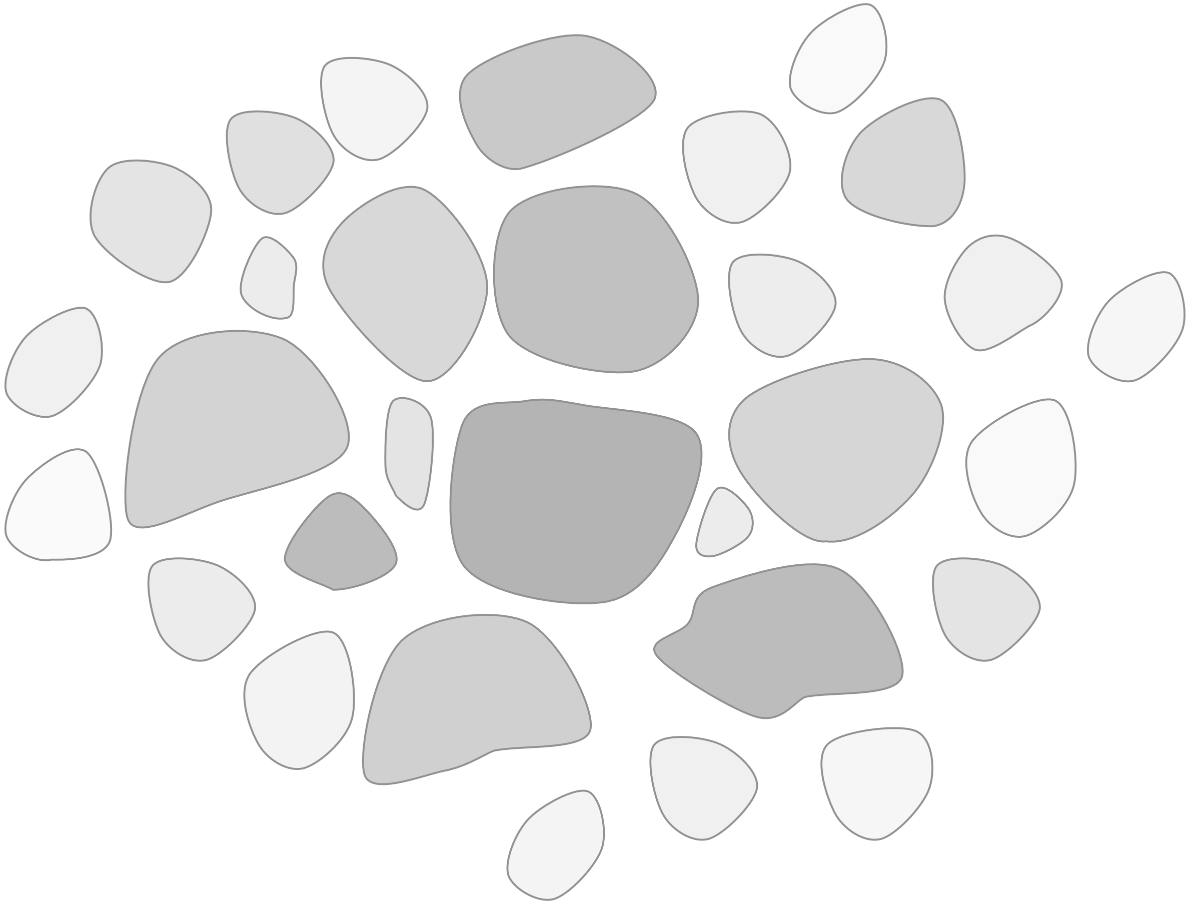
Two exteme cases:

(1) turbulence dominates energy budget:
$\alpha = E_{kin}/|E_{pot}| > 1$
--> individual cores do *not* interact
--> *collapse* of *individual* cores dominates *stellar mass growth*
--> *loose cluster of low-mass stars*

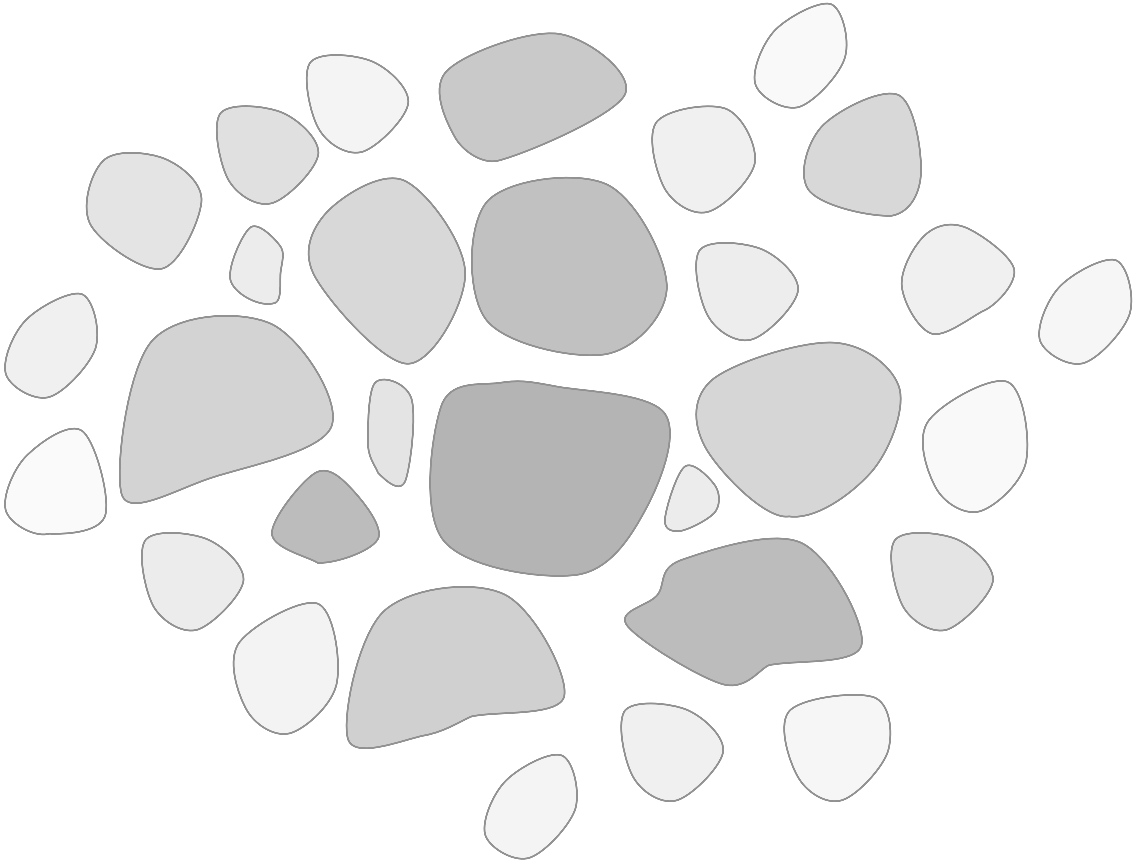
(2) turbulence decays, i.e. gravity dominates: $\alpha = E_{kin}/|E_{pot}| < 1$
--> *global contraction*
--> core do *interact* while collapsing
--> *competition* influences *mass growth*
--> *dense cluster with high-mass stars*



turbulence creates a hierarchy of clumps



as turbulence decays locally, contraction sets in



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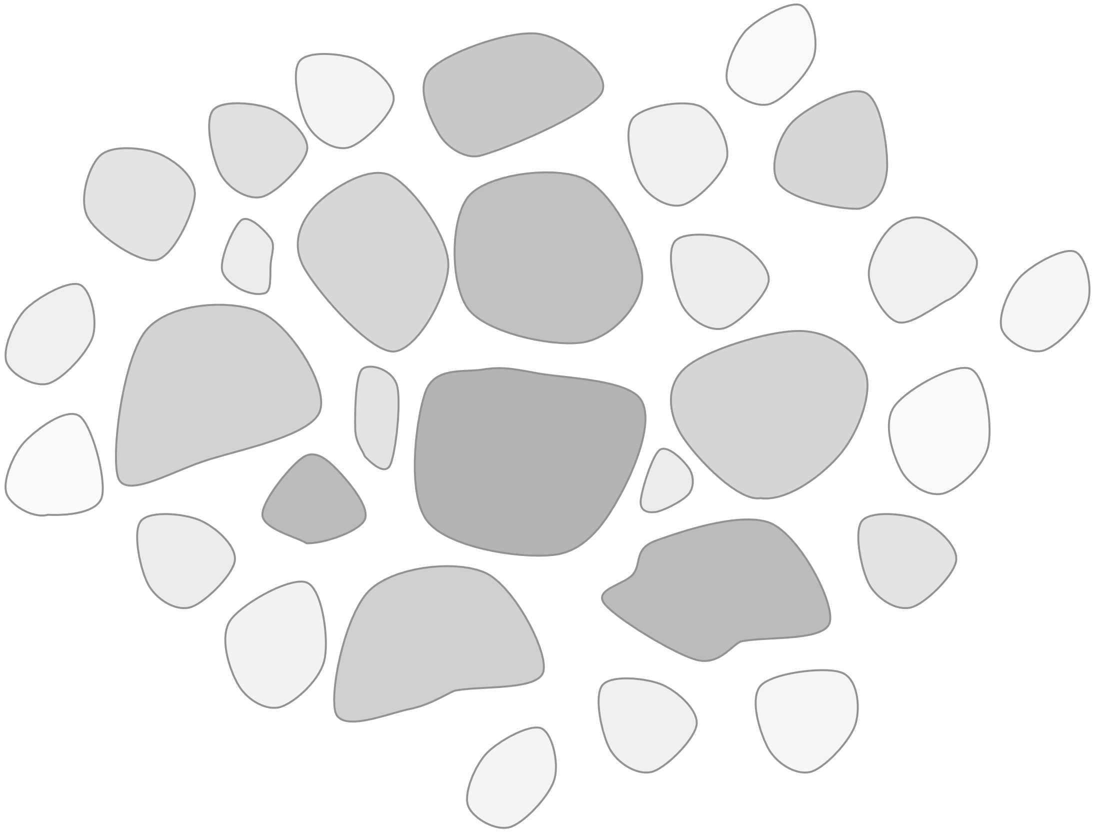
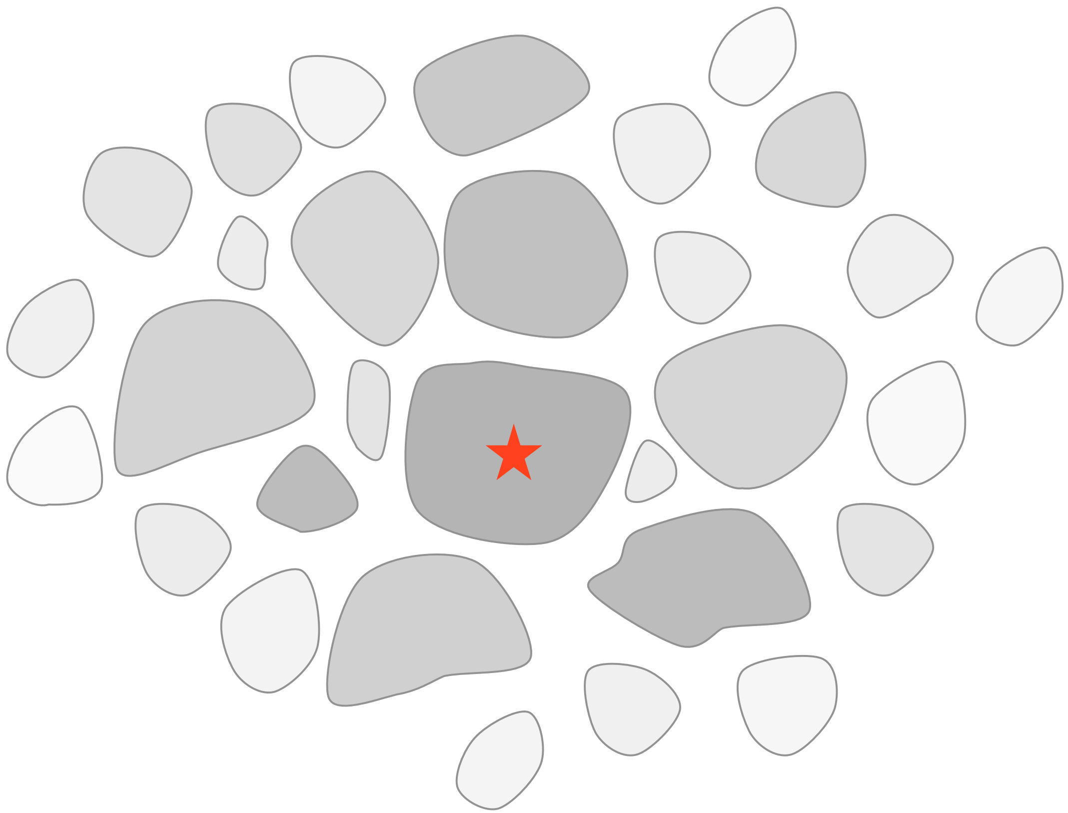


while region contracts, individual clumps collapse to form stars



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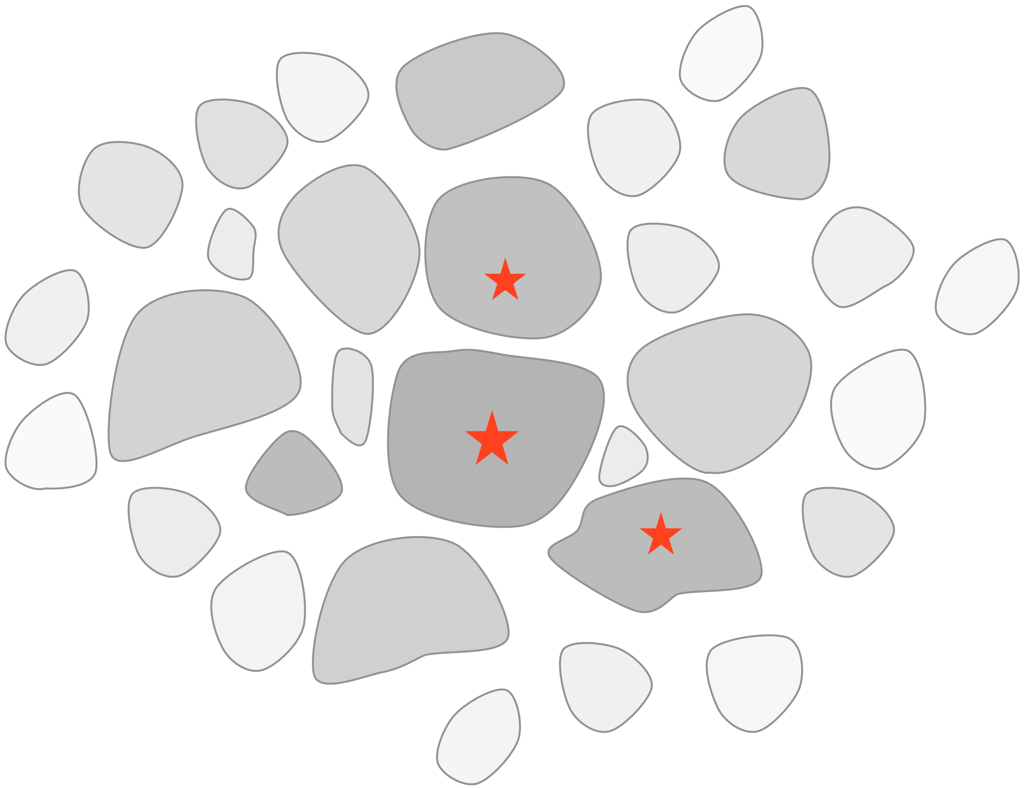
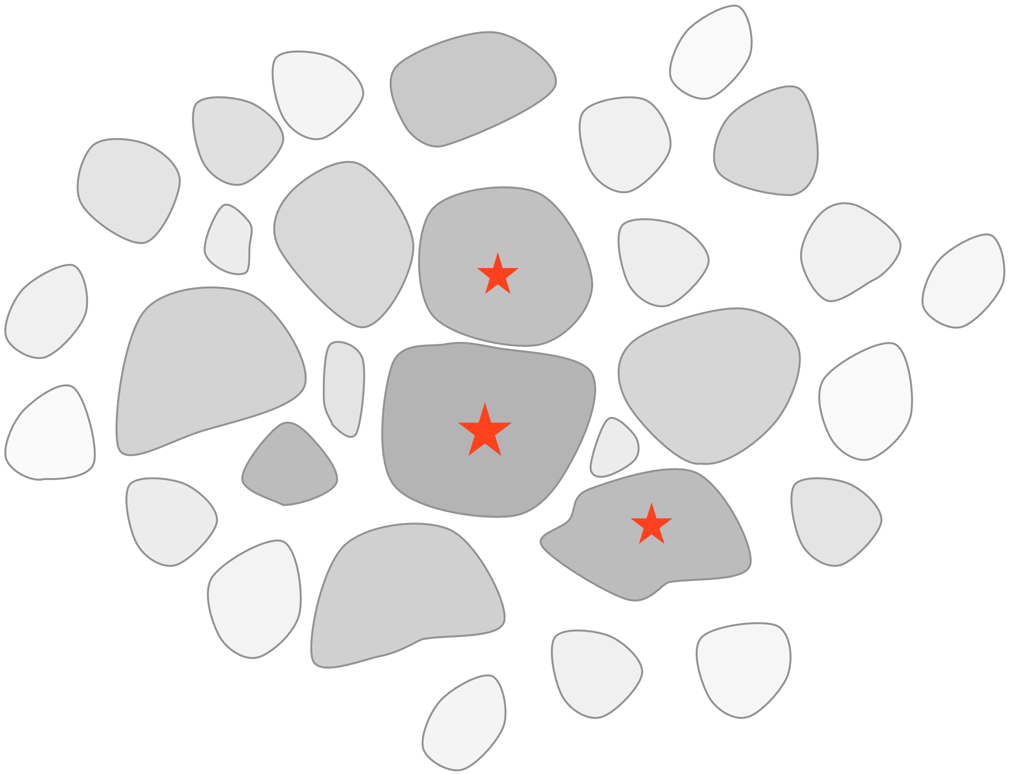
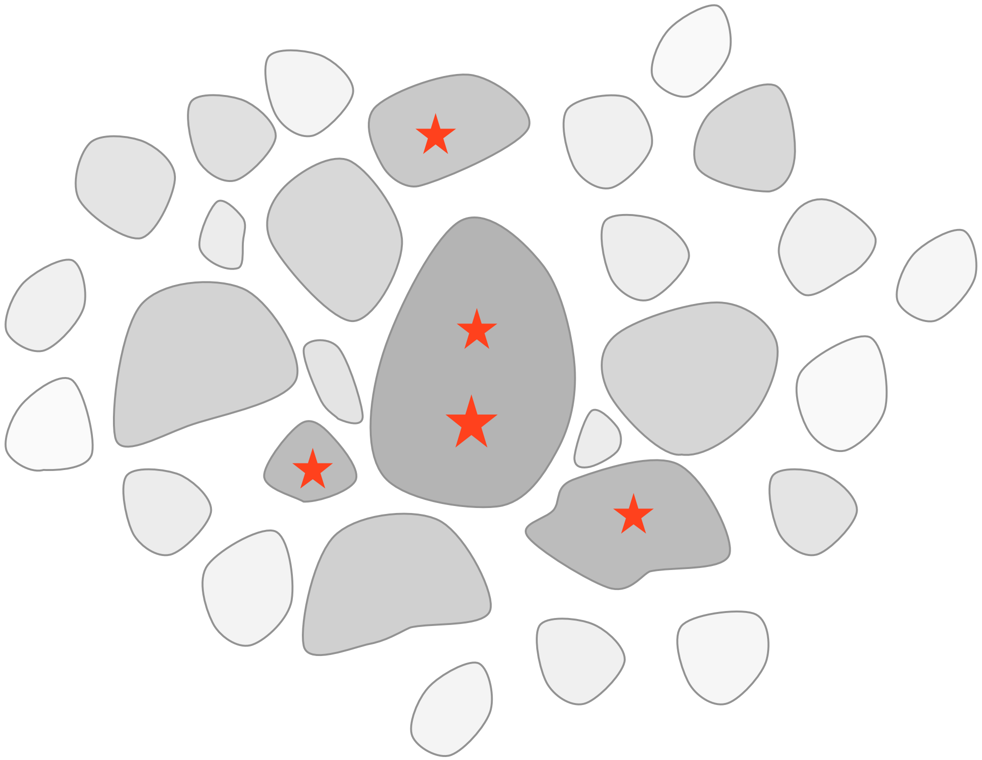


individual clumps collapse to form stars

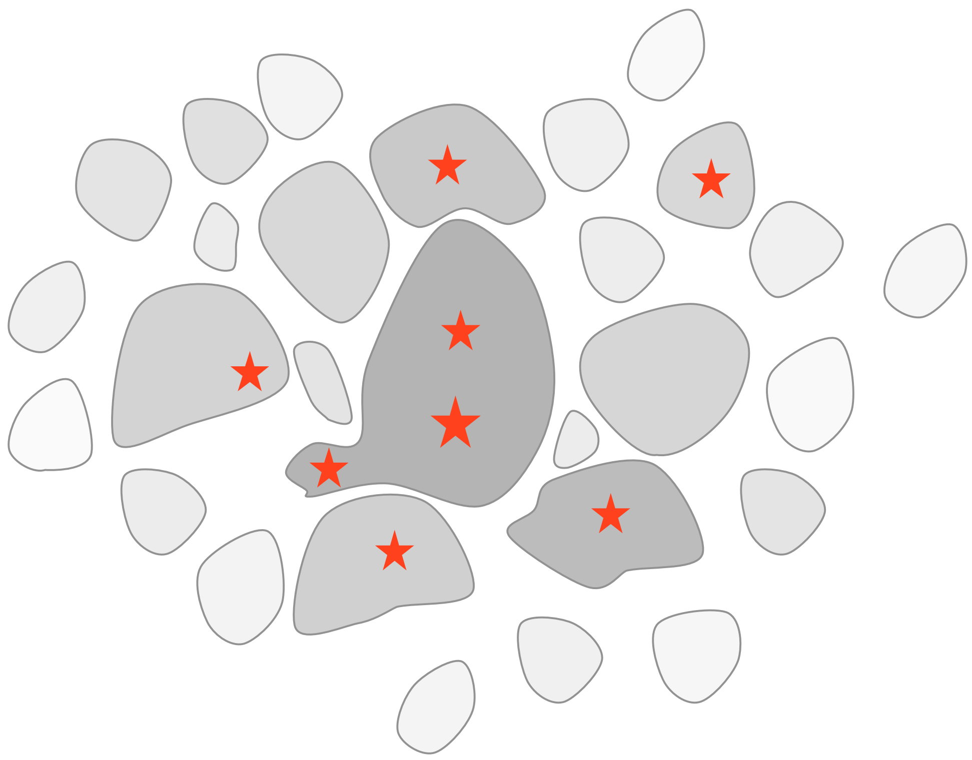


individual clumps collapse to form stars



$\alpha = E_{kin}/|E_{pot}| < 1$

in *dense clusters*, clumps may merge while collapsing
--> then contain multiple protostars



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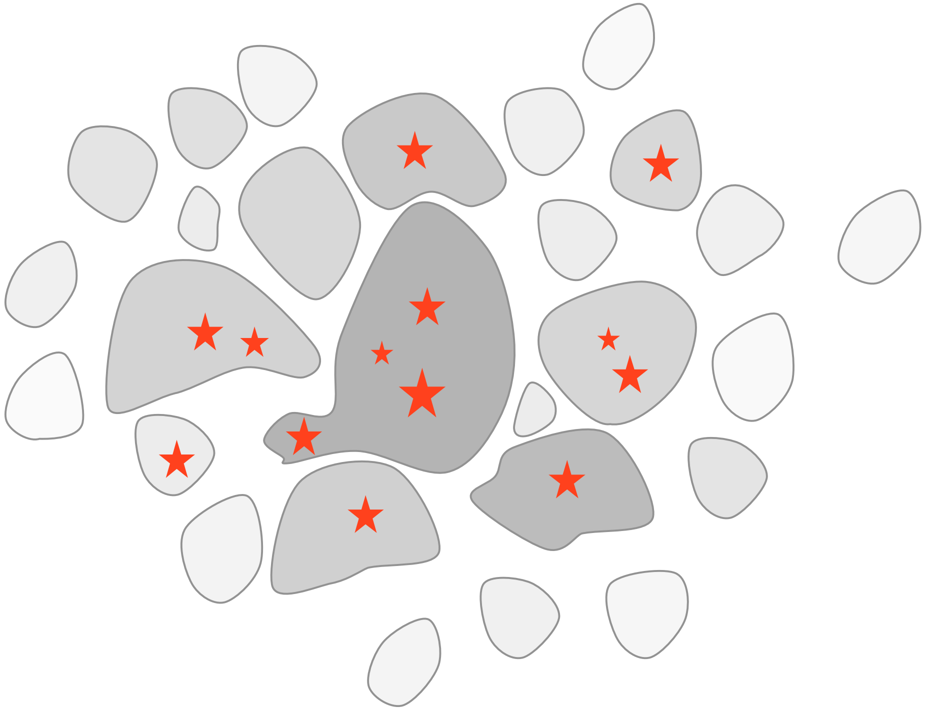

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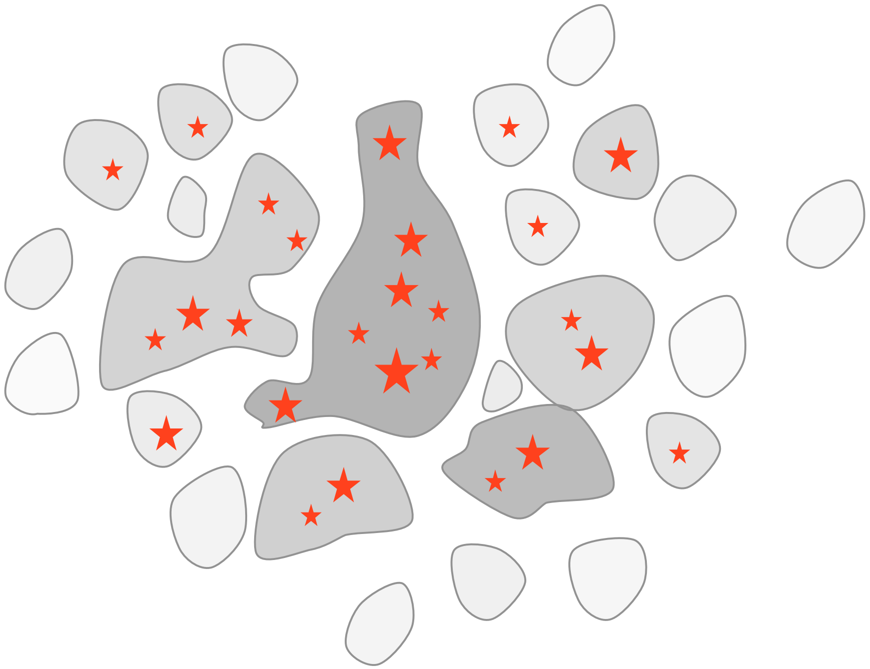
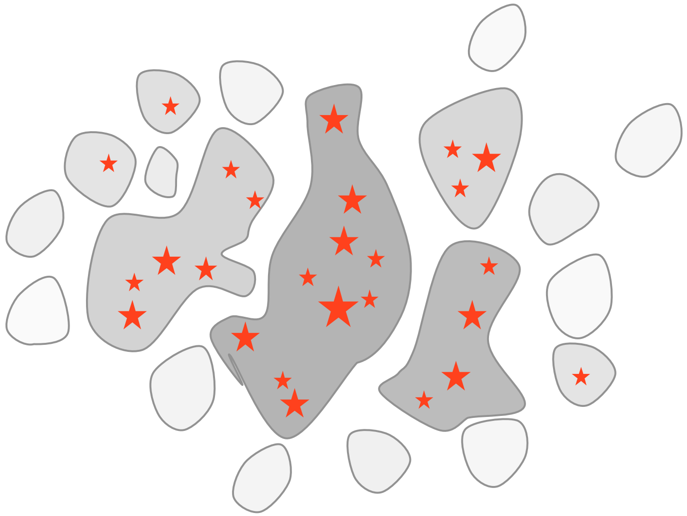
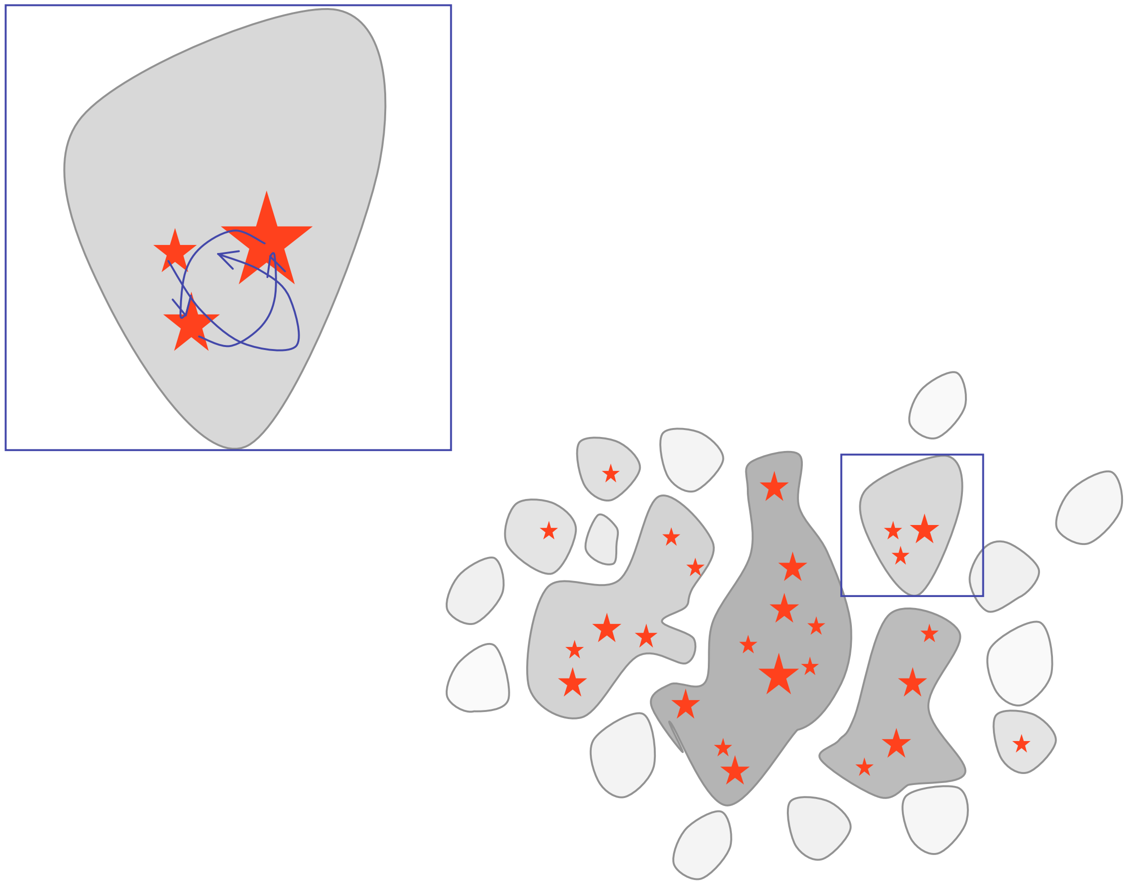


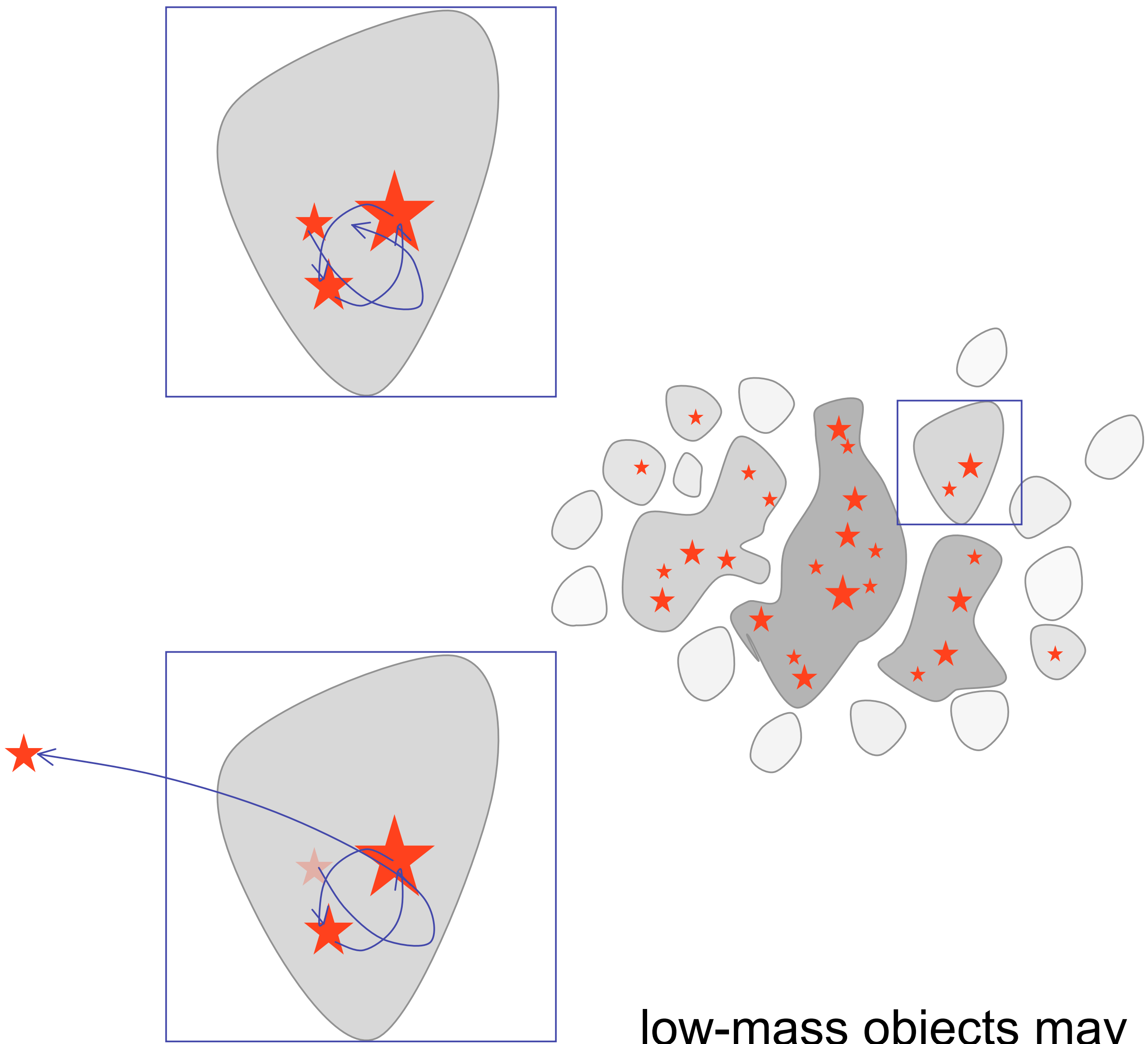
in *dense clusters*, competitive mass growth becomes important



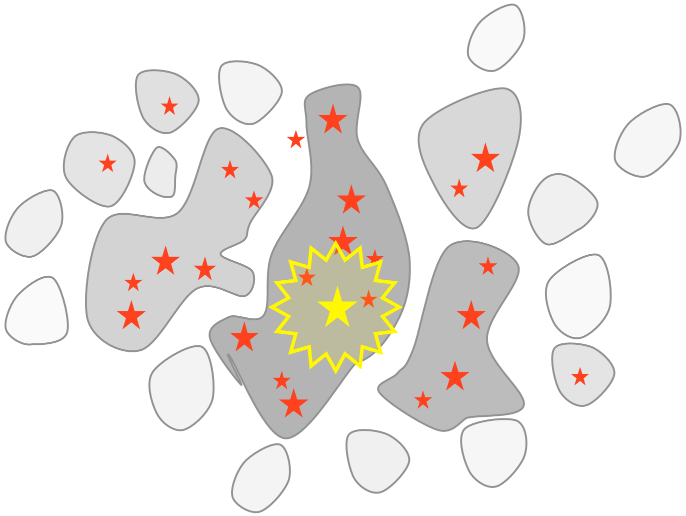
in *dense clusters*, competitive mass growth becomes important



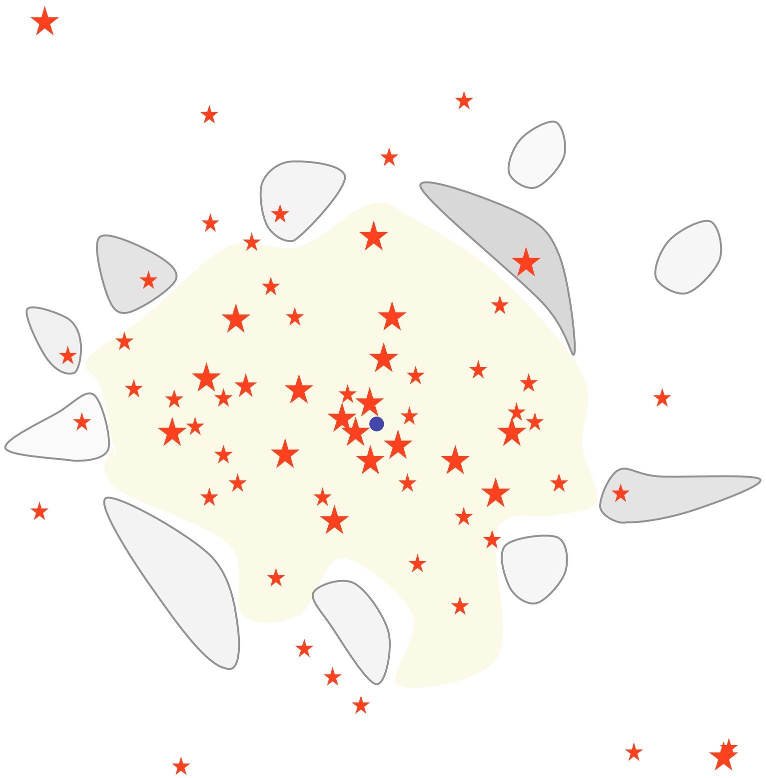
in *dense clusters*, *N*-body effects influence mass growth



low-mass objects may
become ejected --> accretion stops



feedback terminates star formation



result: *star cluster*, possibly with HII region

# predictions

# Predictions

- *global properties* (statistical properties)
  - SF efficiency and timescale
  - stellar mass function -- IMF
  - dynamics of young star clusters
  - description of self-gravitating turbulent systems (pdf's, $\Delta$-*var.)*
  - chemical mixing properties
- *local properties* (properties of individual objects)
  - properties of individual clumps (e.g. shape, radial profile, lifetimes)
  - accretion history of individual protostars (dM/dt vs. t, j vs. t)
  - binary (proto)stars (eccentricity, mass ratio, etc.)
  - SED's of individual protostars
  - dynamic PMS tracks: $T_{bol}$-$L_{bol}$ evolution

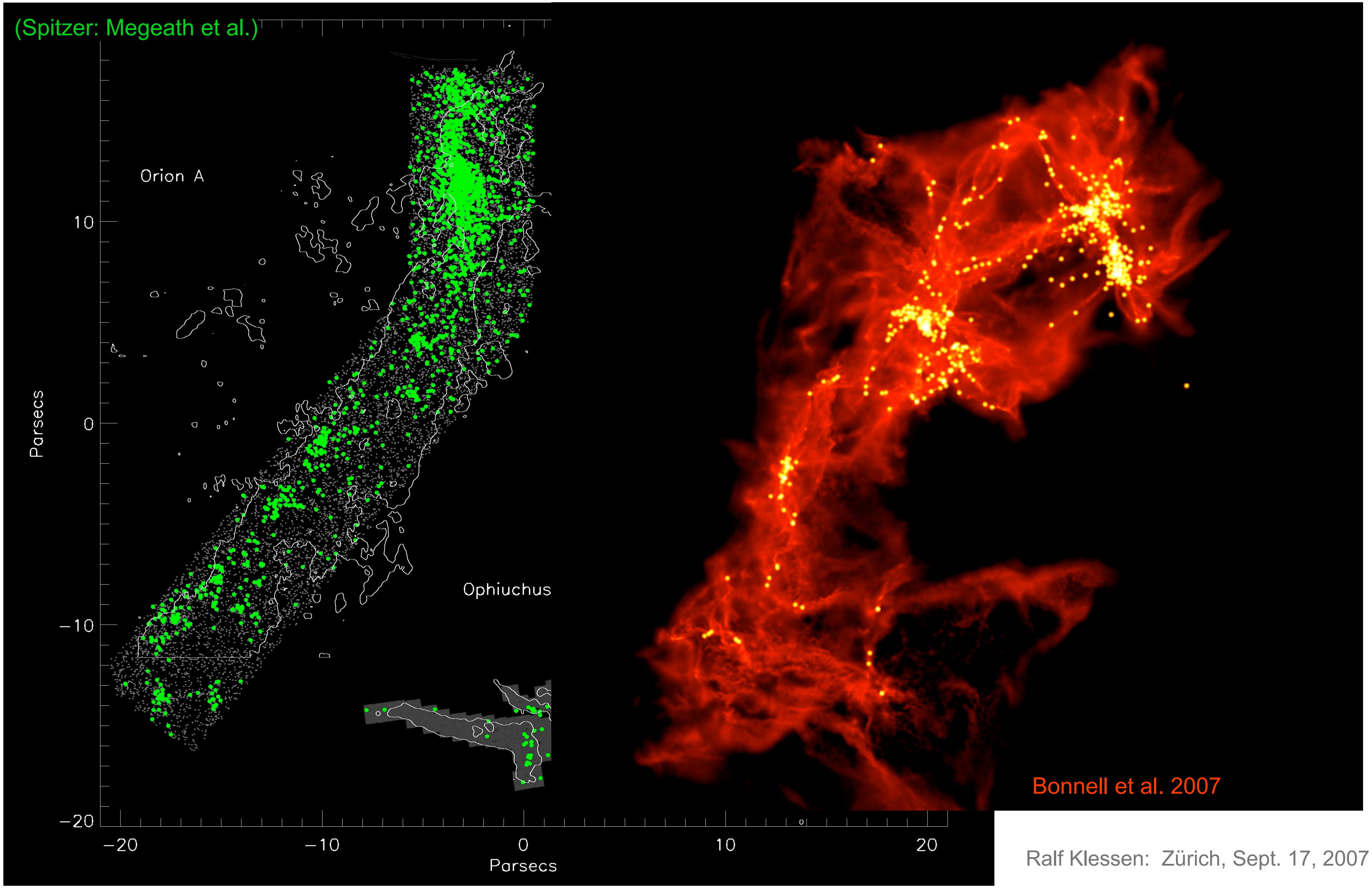
# Examples and predictions

*example 1:* star cluster formation: *dynamics*

*example 2:* star cluster formation: *thermodynamics*
--> speculations on the origin of the stellar mass spectrum (IMF)

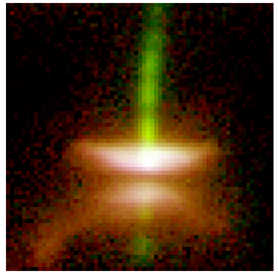
example 1

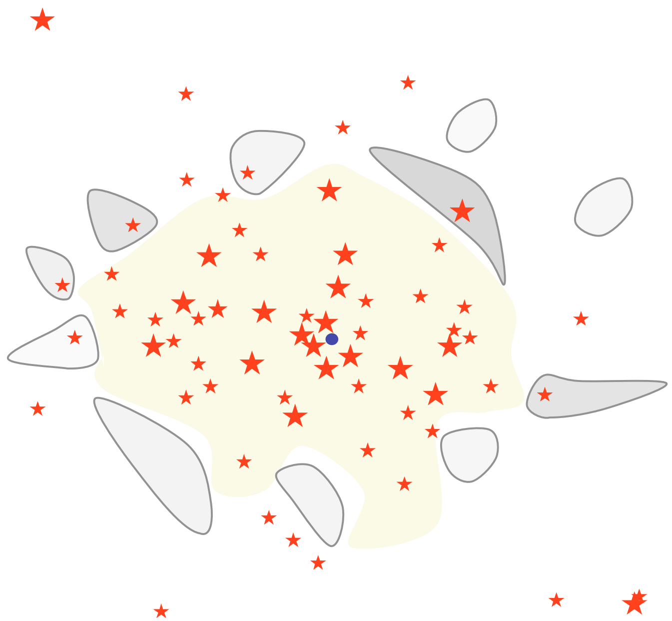
# Example: model of Orion cloud



# Models of star cluster formation

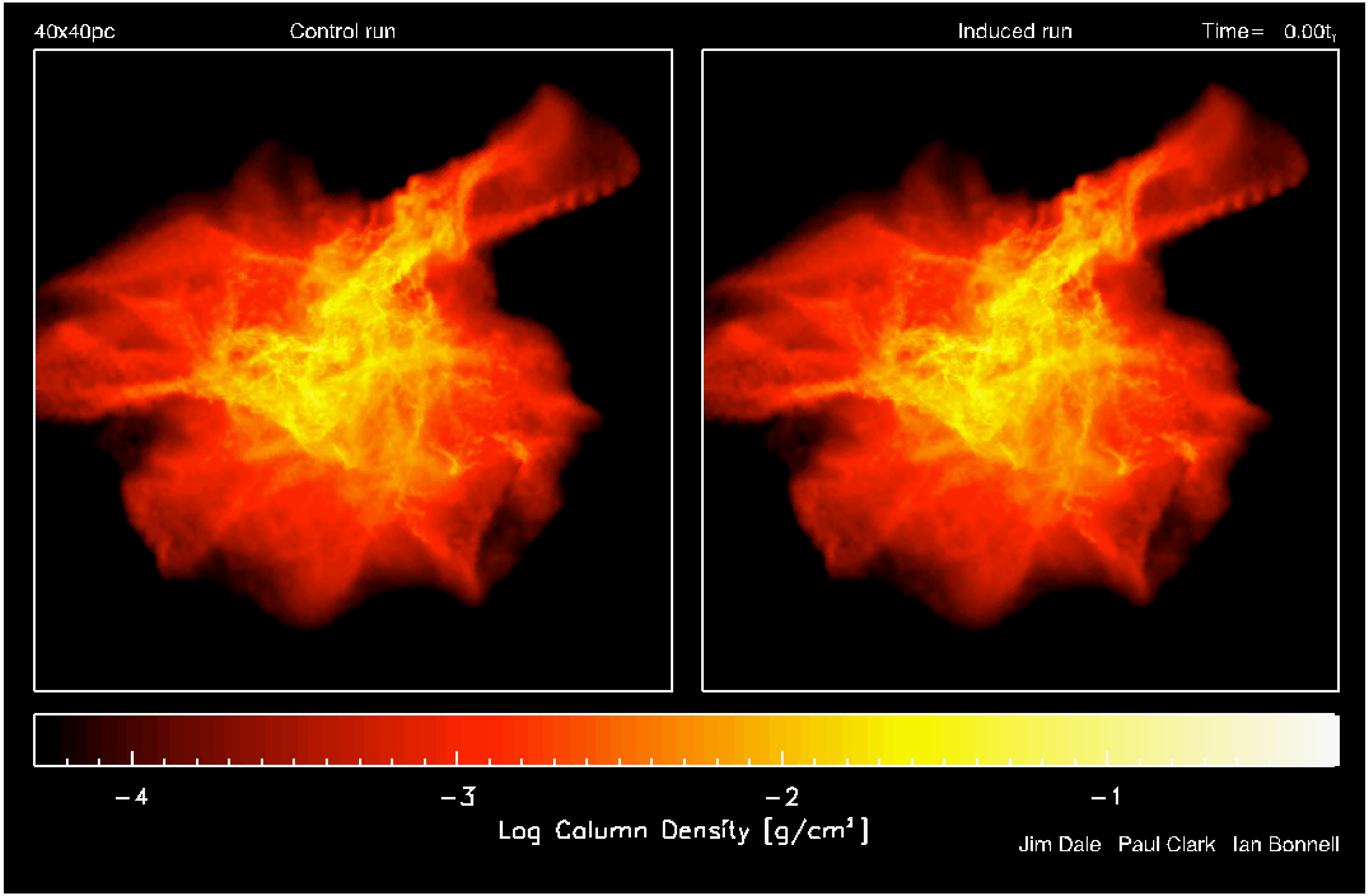
- dynamics:
  basic properties are probably okay
- BUT: no feedback
  (outflows, radiation, etc.)
- *how much detail are we missing?*
  - how does that change properties like *IMF, boundedness, efficiency*?





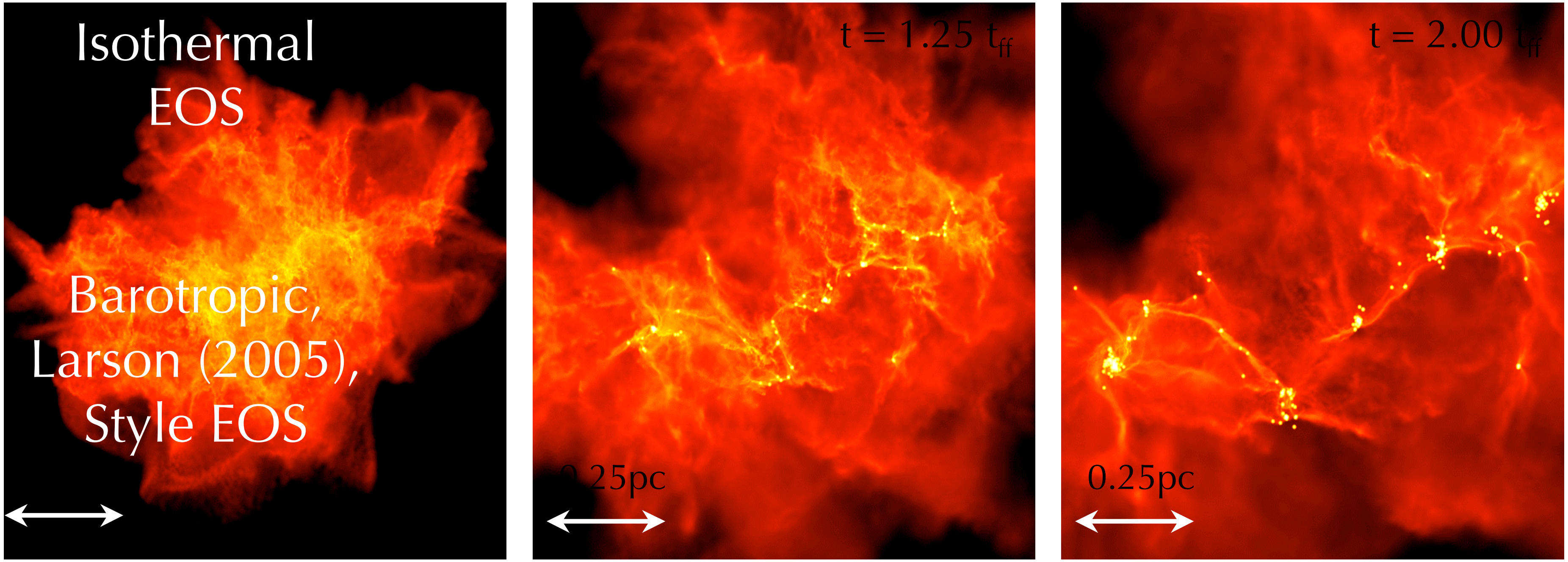
# Model with ionizing feedback

SPH with radiation feedback:  first calculations of star-cluster formation with ionization



# Unbound clouds

KE = 2 x PE (initially), 1000 solar masses, 0.5pc




**No global collapse**:

local $t_{ff}$ < global interaction time-scale

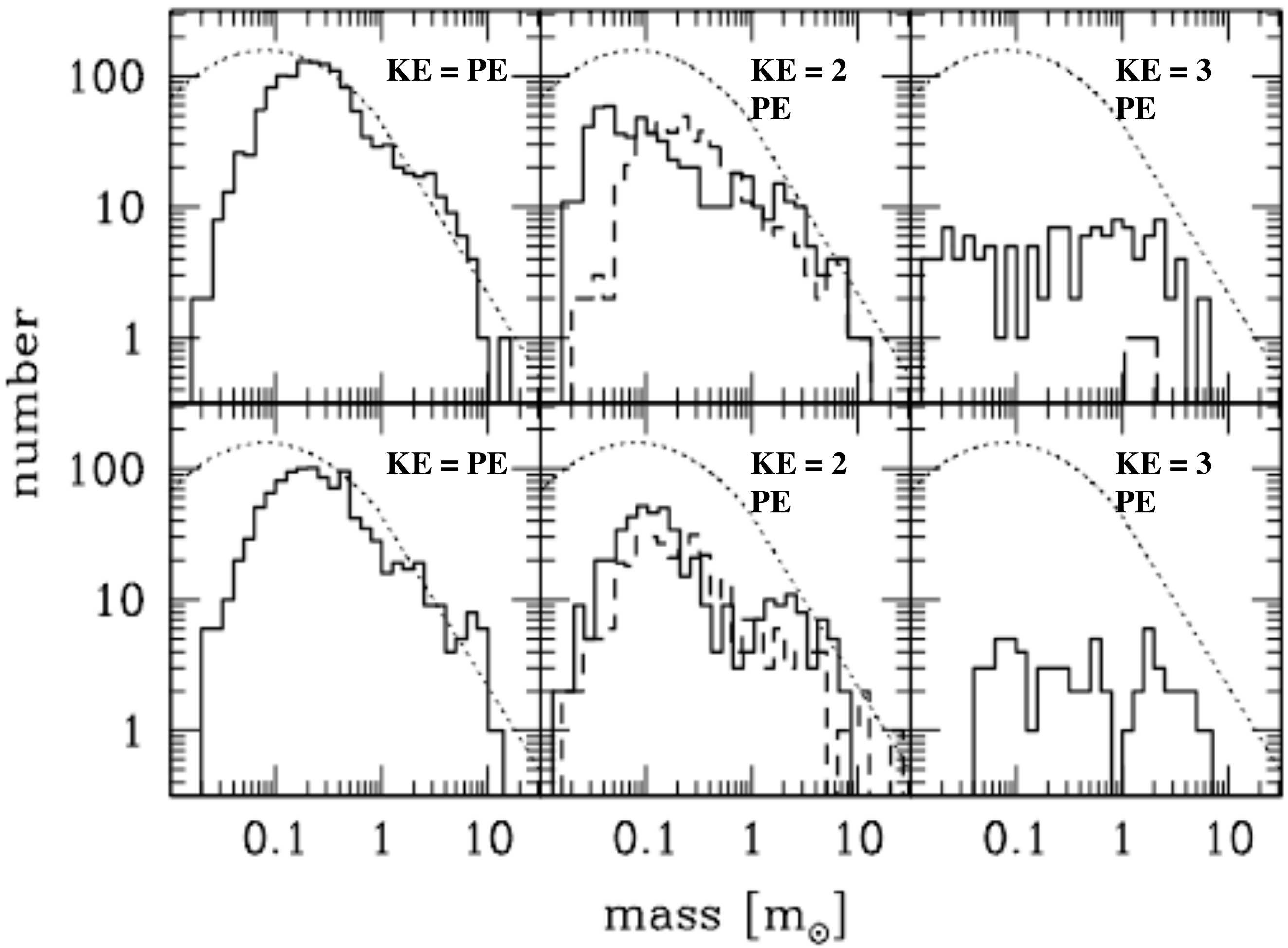
$t_{ff} \sim 2 \times 10^5$ years

Clark, Bonnell & Klessen (2007)

# Mass functions

Isothermal EOS

Barotropic, Larson (2005), Style EOS




Clark, Bonnell & Klessen (2007)

# example 2

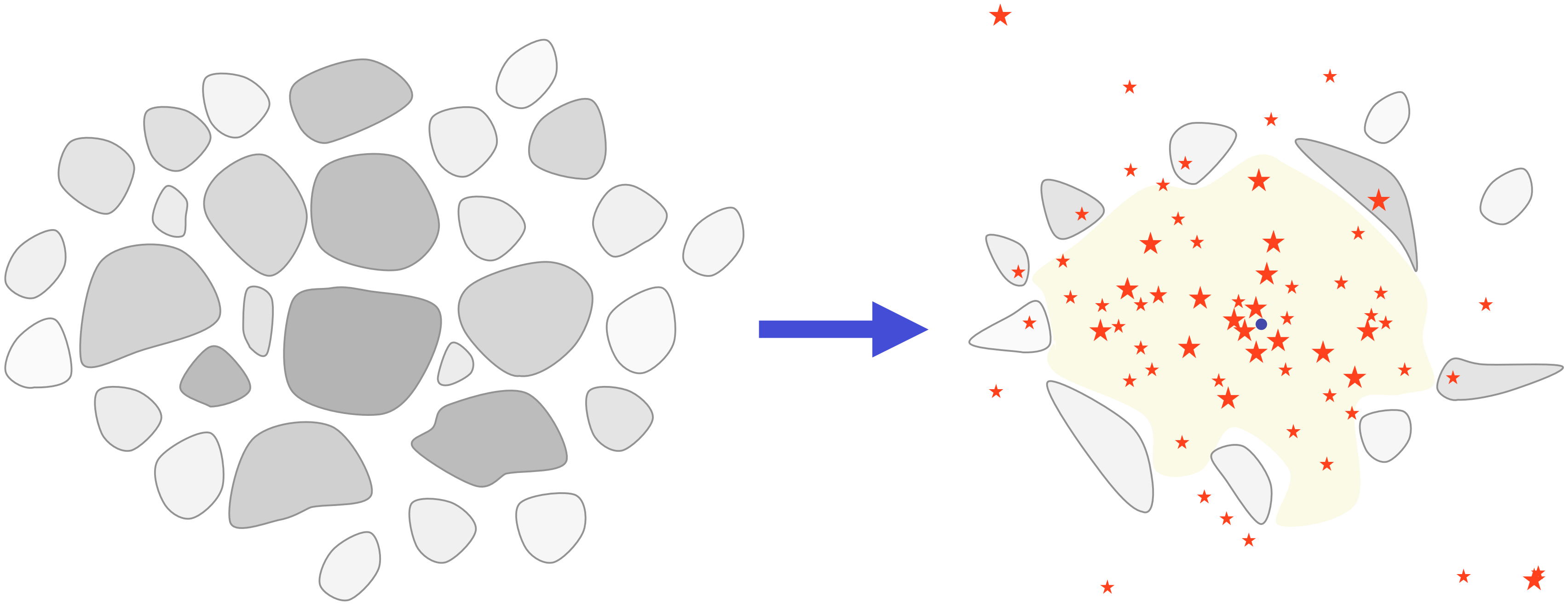
# IMF

- distribution of stellar masses depends on
  - turbulent initial conditions
    --> mass spectrum of prestellar cloud cores
  - collapse and interaction of prestellar cores
    --> competitive accretion and *N*-body effects
  - thermodynamic properties of gas
    --> balance between heating and cooling
    --> EOS (determines which cores go into collapse)
  - (proto) stellar feedback terminates star formation
    ionizing radiation, bipolar outflows, winds, SN

(e.g. Larson 2003, Prog. Rep. Phys.; Mac Low & Klessen, 2004, Rev. Mod. Phys, 76, 125 - 194)

# Star cluster formation

Most stars form in clusters → *star formation = cluster formation*



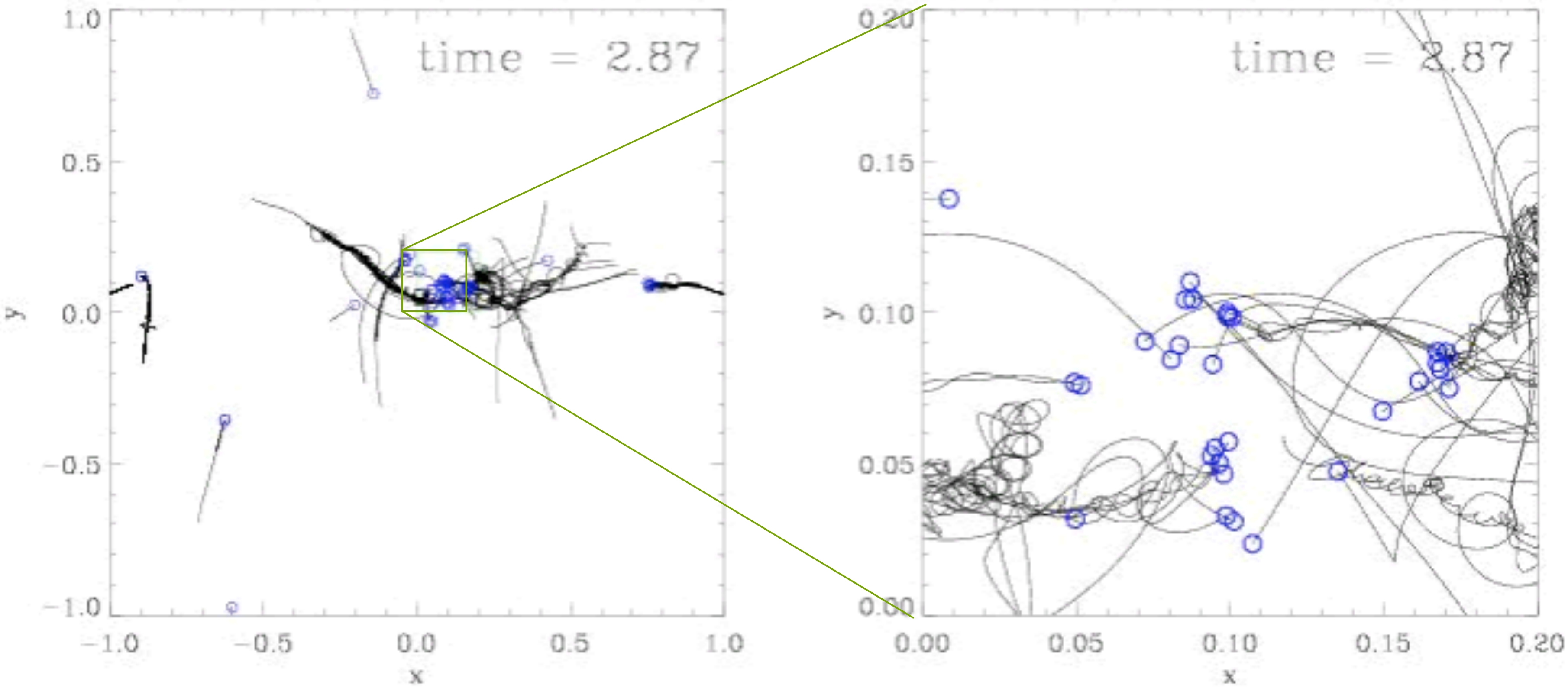
How to get from cloud cores to star clusters?

How do the stars acquire mass?

(e.g. Larson 2003, Prog. Rep. Phys.; Mac Low & Klessen, 2004, Rev. Mod. Phys, 76, 125 - 194)

# Dynamics of nascent star cluster

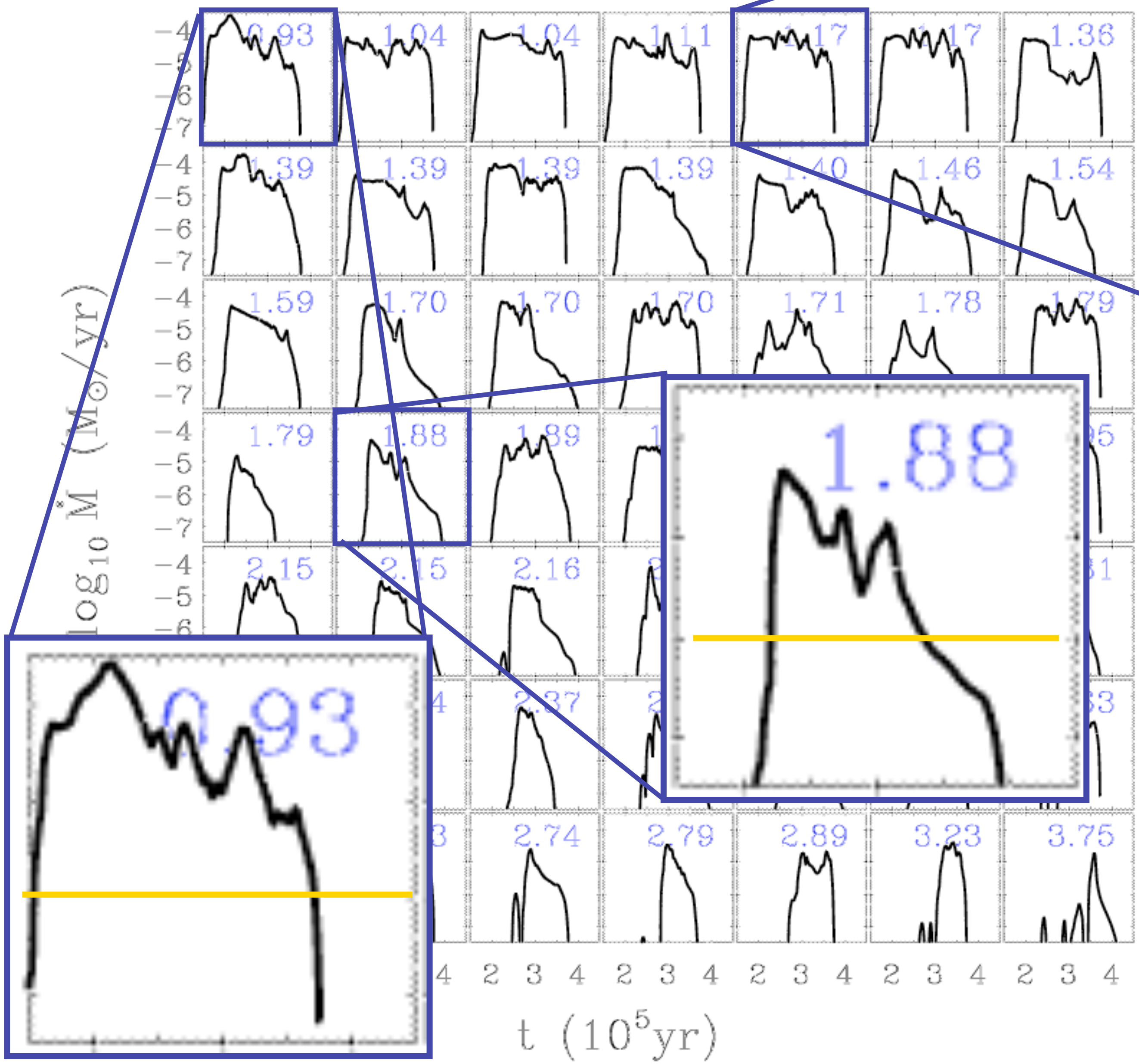
in dense clusters protostellar interaction may be come important!

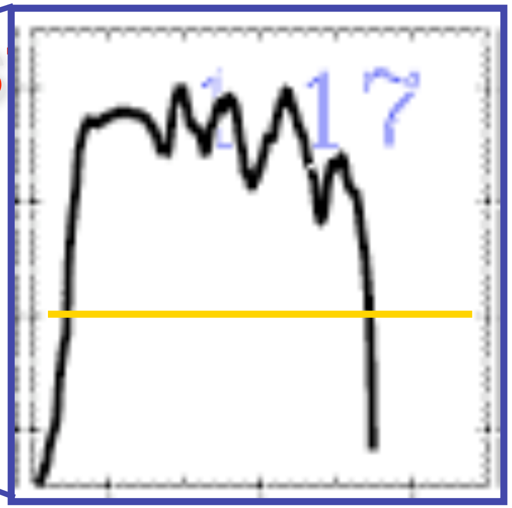



Trajectories of protostars in a nascent dense cluster created by gravoturbulent fragmentation

(from Klessen & Burkert 2000, ApJS, 128, 287)

# Accretion rates in clus




Mass accretion rates *vary with time* and are strongly *influenced* by the *cluster environment*.

(Klessen 2001, ApJ, 550, L77; also Schmeja & Klessen, 2004, A&A, 419, 405)

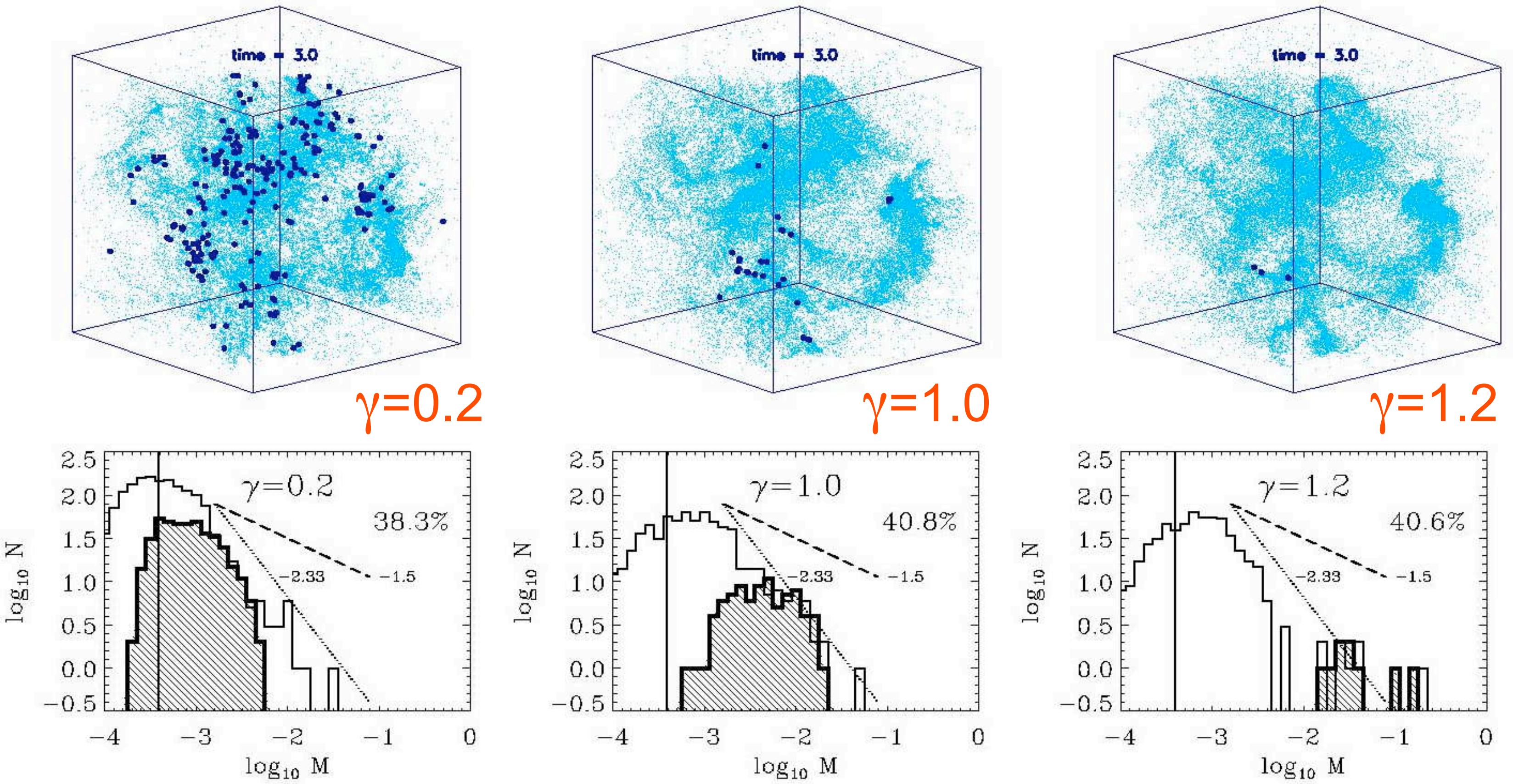
# Dependency on EOS

- degree of fragmentation depends on *EOS!*

- polytropic EOS: $p \propto \rho^{\gamma}$
- $\gamma<1$: dense cluster of low-mass stars
- $\gamma>1$: isolated high-mass stars

(see Li, Klessen, & Mac Low 2003, ApJ, 592, 975; also Kawachi & Hanawa 1998, Larson 2003)

# Dependency on EOS




for γ<1 fragmentation is enhanced → *cluster of low-mass stars*

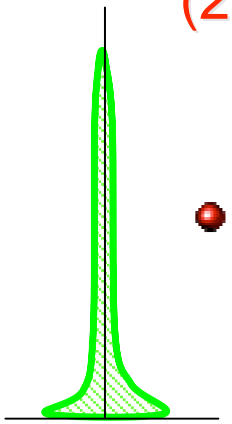
for γ>1 it is suppressed → formation of *isolated massive stars*

(from Li, Klessen, & Mac Low 2003, ApJ, 592, 975)

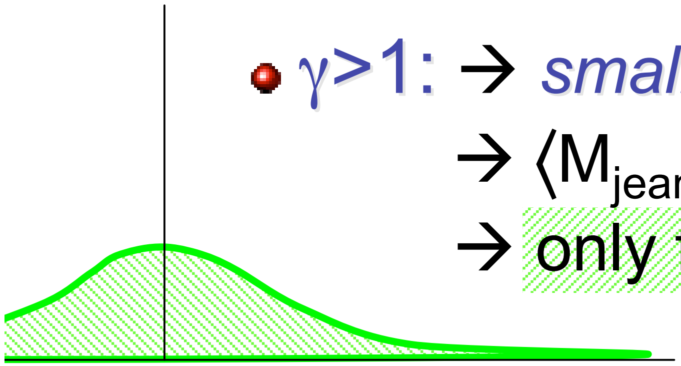
# How does that work?

(1) $p \propto \rho^{\gamma} \rightarrow \rho \propto p^{1/\gamma}$

(2) $M_{jeans} \propto \gamma^{3/2} \rho^{(3\gamma-4)/2}$



- $\gamma<1$: → *large* density excursion for given pressure
  → $\langle M_{jeans} \rangle$ becomes small
  → number of fluctuations with $M > M_{jeans}$ is large

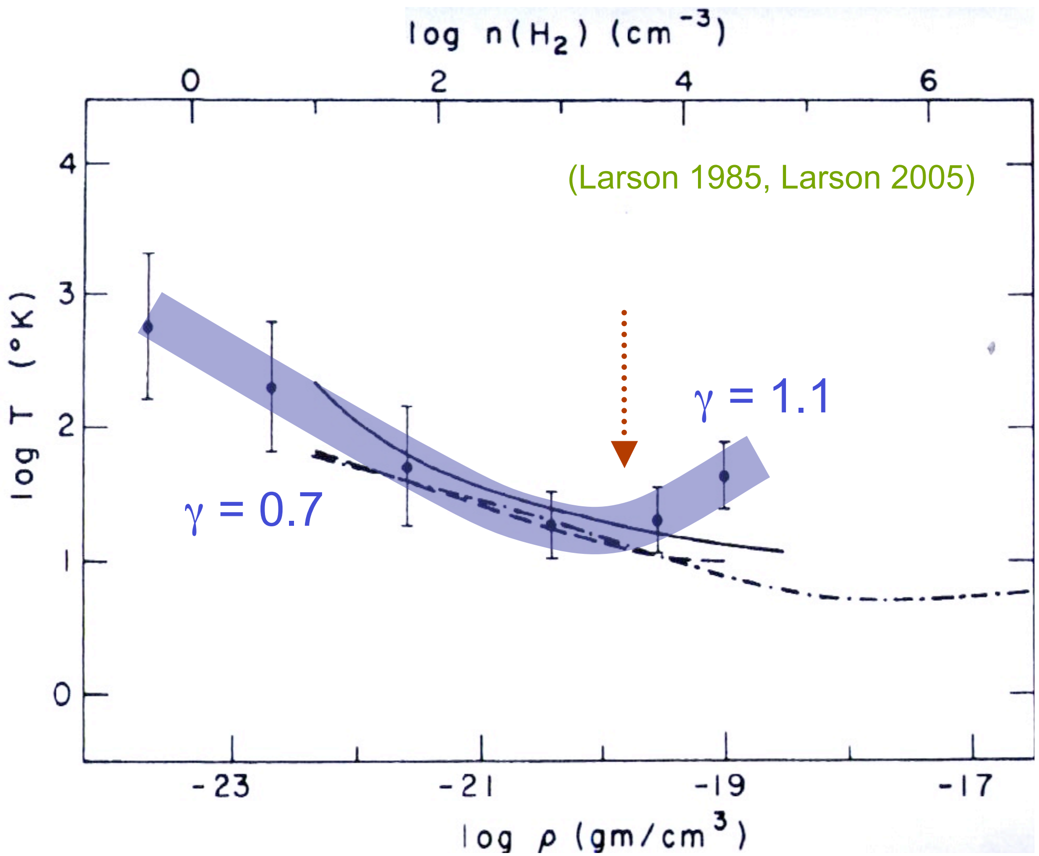


- $\gamma>1$: → *small* density excursion for given pressure
  → $\langle M_{jeans} \rangle$ is large
  → only few and massive clumps exceed $M_{jeans}$

# EOS for solar neighborhood

below $10^{-18}$ gcm$^{-3}$: ρ ⬆ ⇒ T ⬇

above $10^{-18}$ gcm$^{-3}$: ρ ⬆ ⇒ T ⬆




$P \propto \rho^\gamma$

$P \propto \rho T$

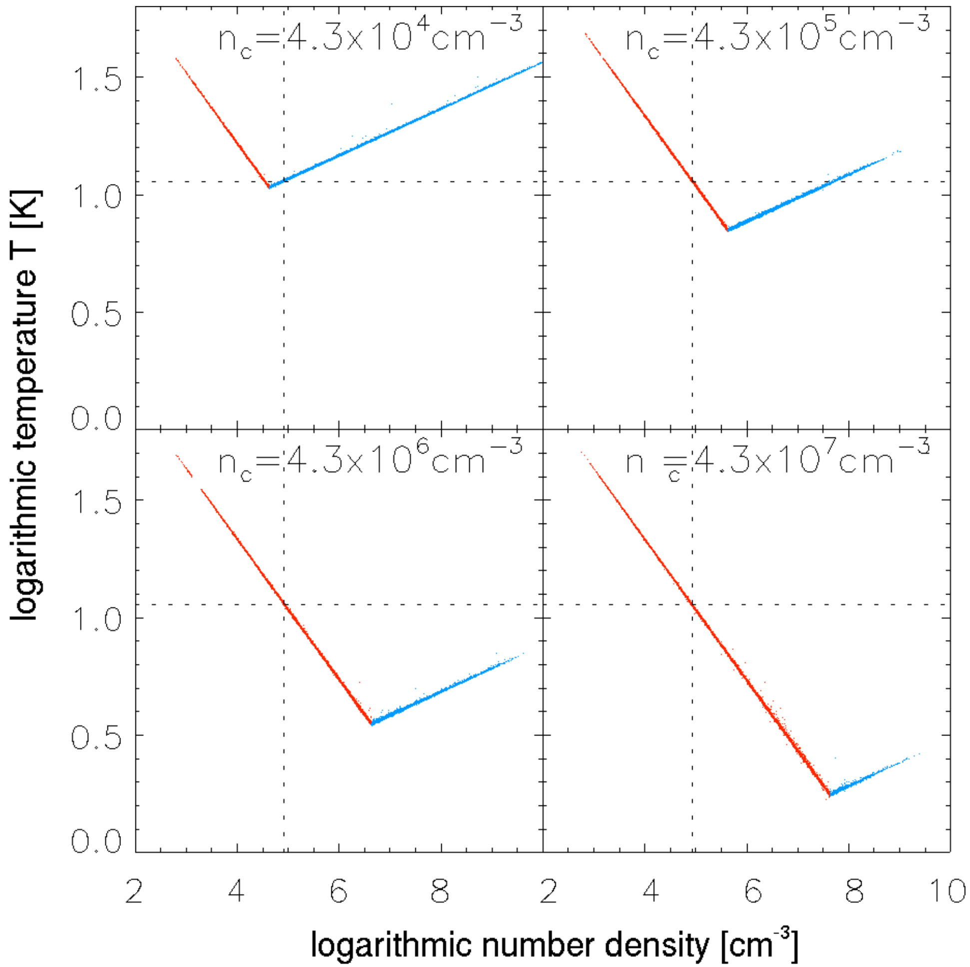
$\rightarrow \gamma = 1 + d\ln T / d\ln \rho$

# IMF from simple piece-wise polytropic EOS

$\gamma_1 = 0.7$

$\gamma_2 = 1.1$

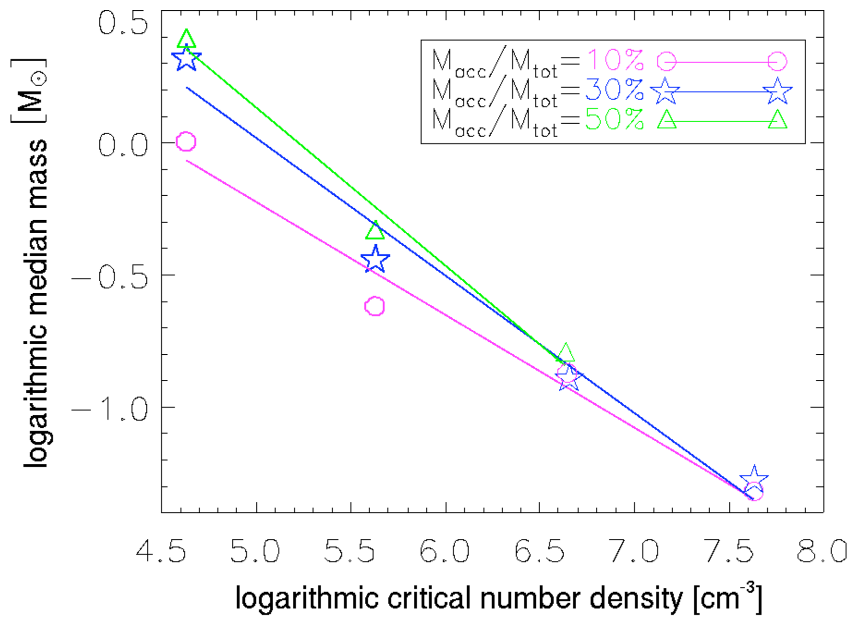
$$T \sim \rho^{\gamma-1}$$

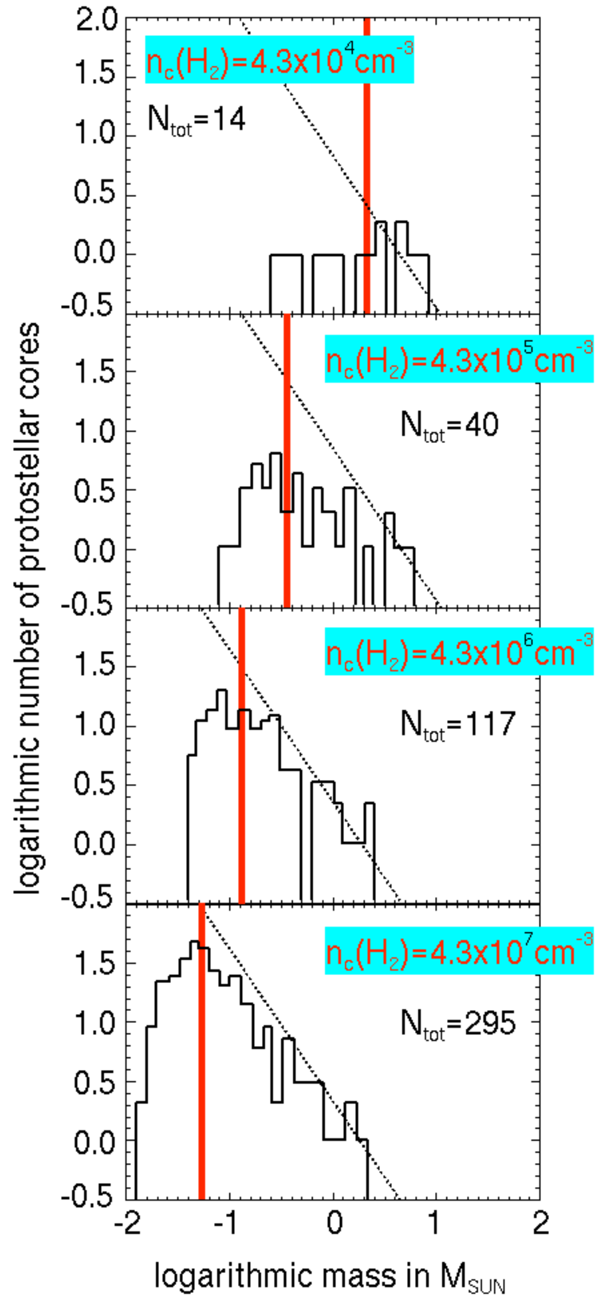



(Jappsen et al. 2005)

# IMF from simple piece-wise polytropic EOS

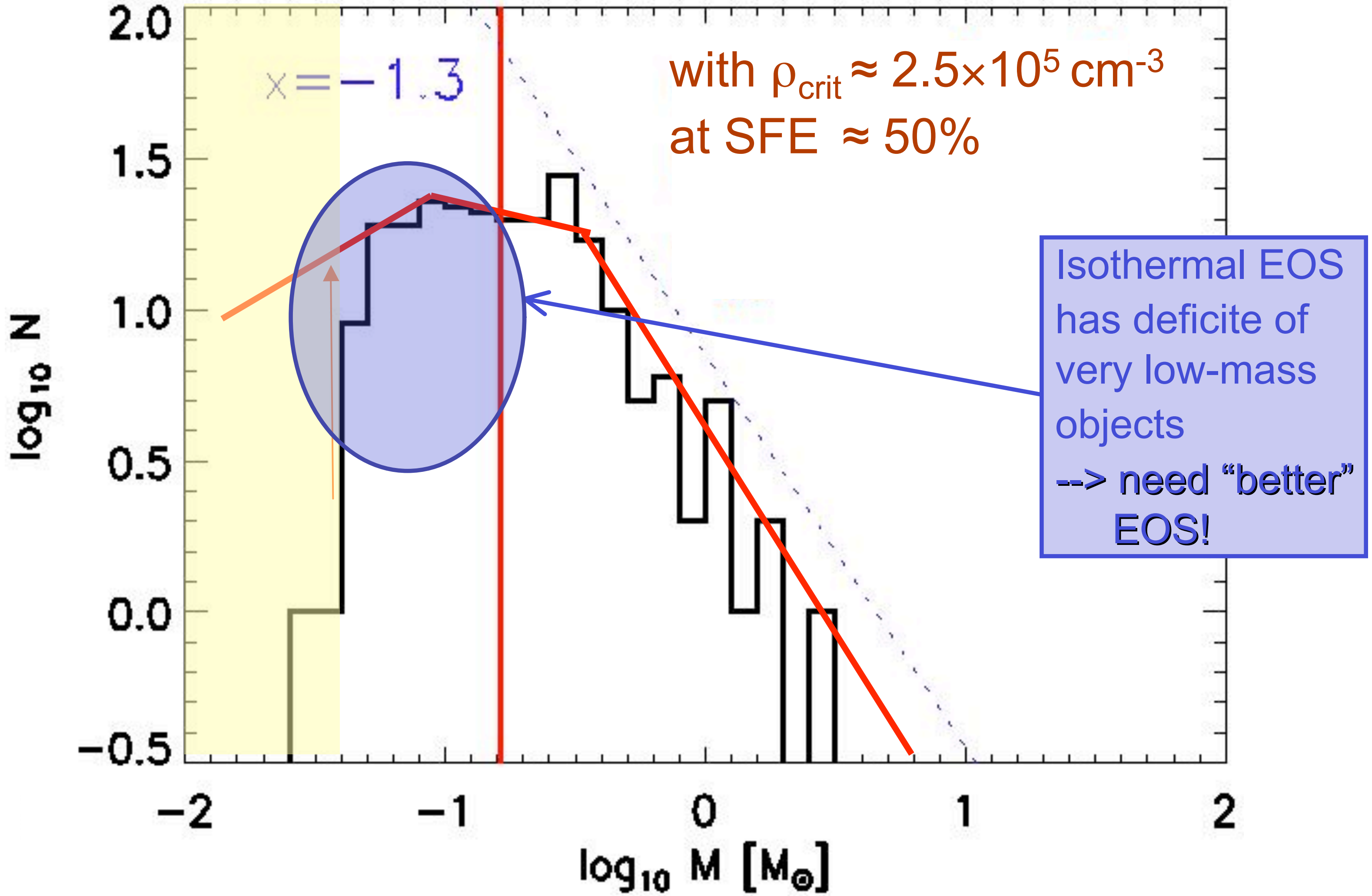
critical density ⬆ ⇨ median mass ⬇



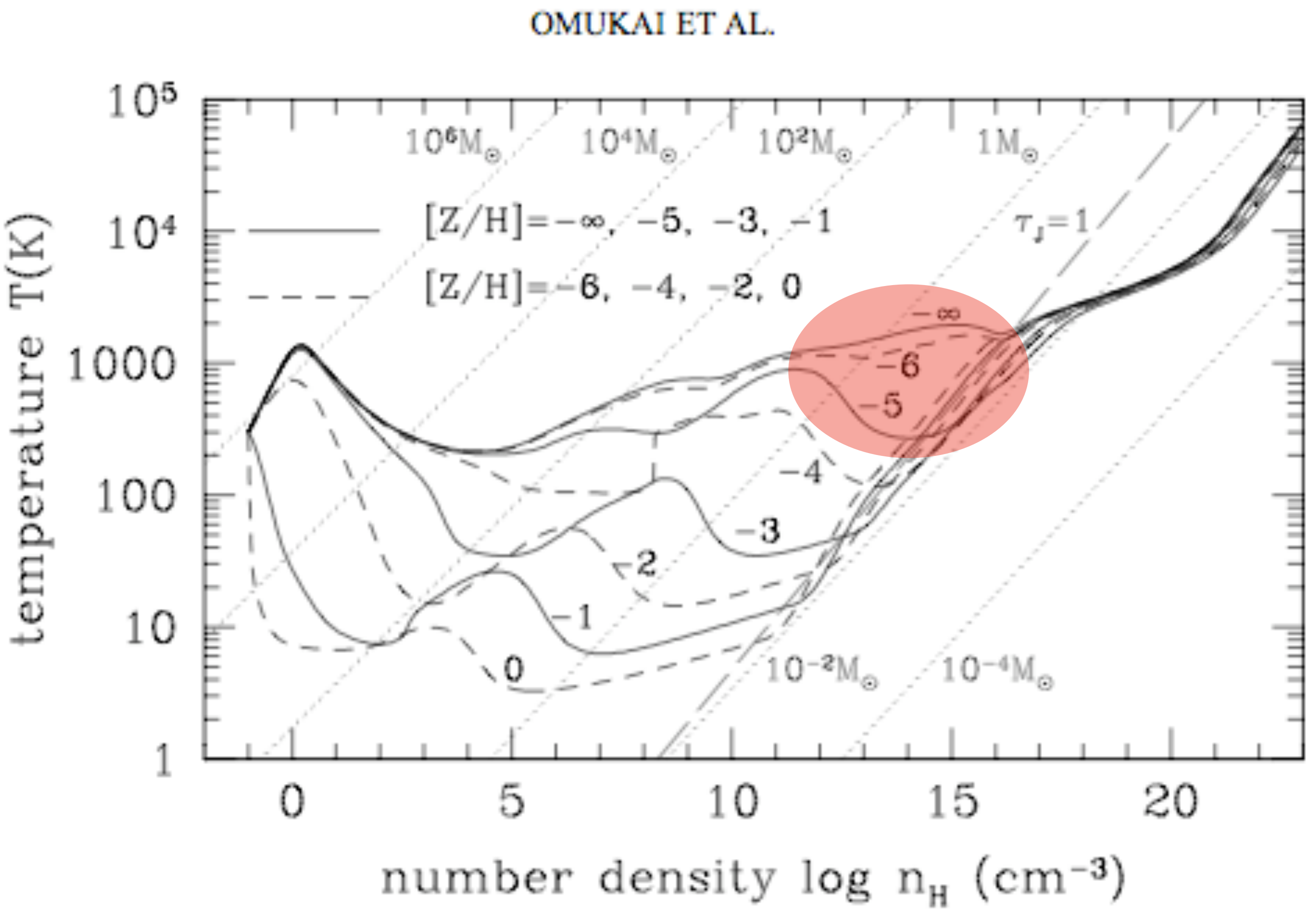

(Jappsen et al. 2005)

# IMF in nearby molecular clouds



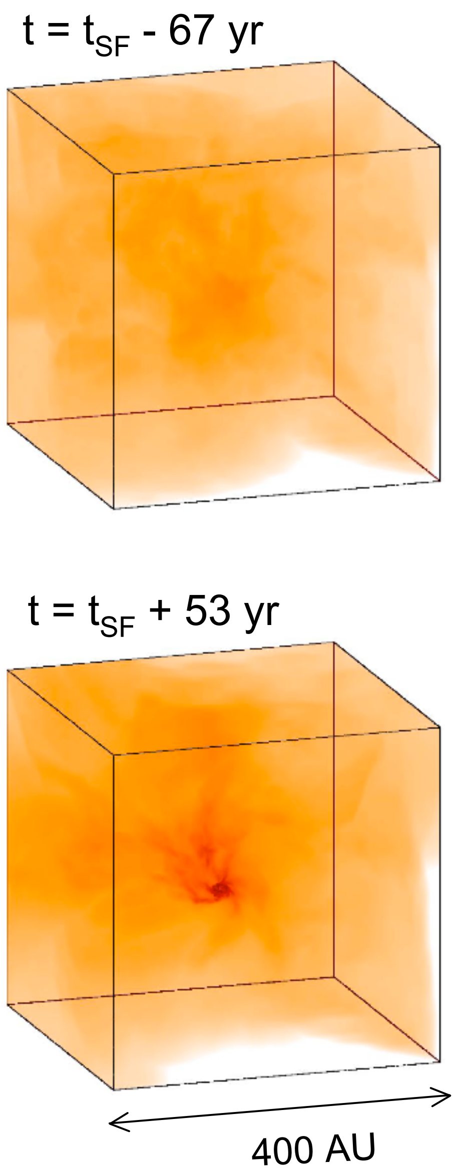

(Jappsen et al. 2005, A&A, 435, 611)c

# transition: Pop III to Pop II.5




(Omukai et al. 2005)

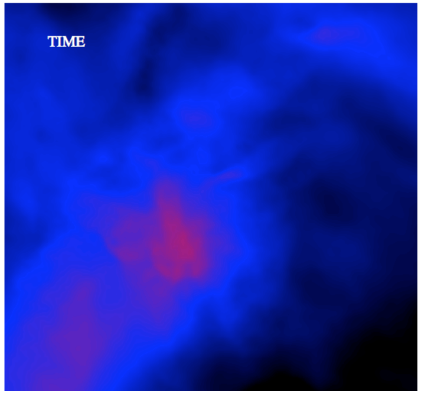
# dust induced fragmentation at $Z=10^{-5}$



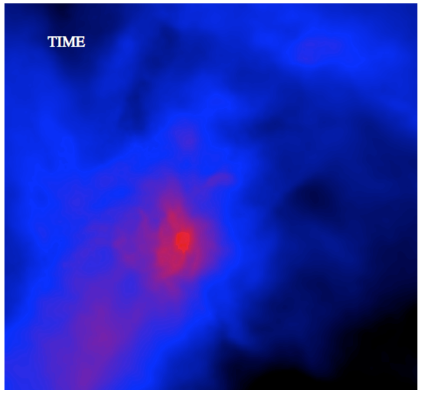

(Clark et al. 2007)

# dust induced fragmentation at $Z=10^{-5}$

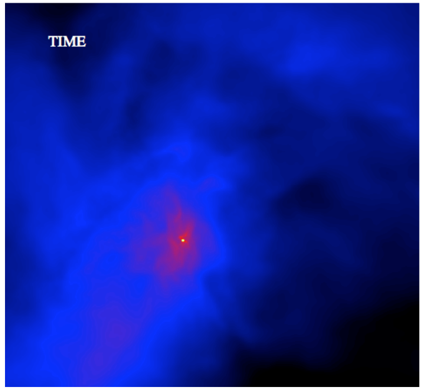
$t = t_{SF}$ - 67 yr



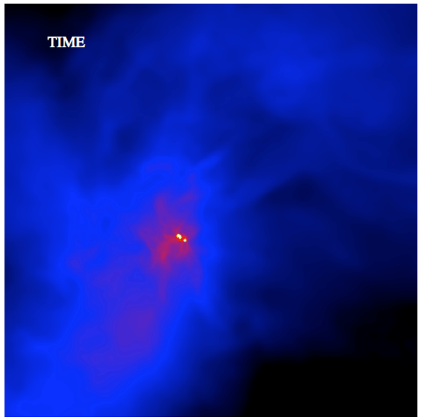

$t = t_{SF}$ - 20 yr



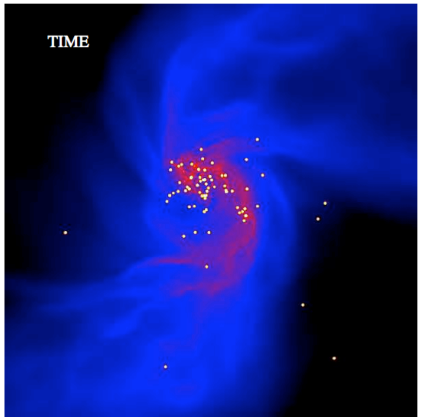

$t = t_{SF}$



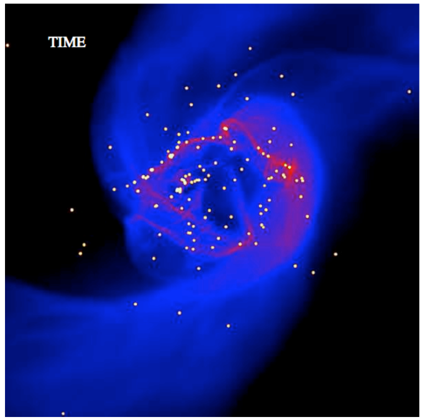

$t = t_{SF}$ + 53 yr




$t = t_{SF}$ + 233 yr

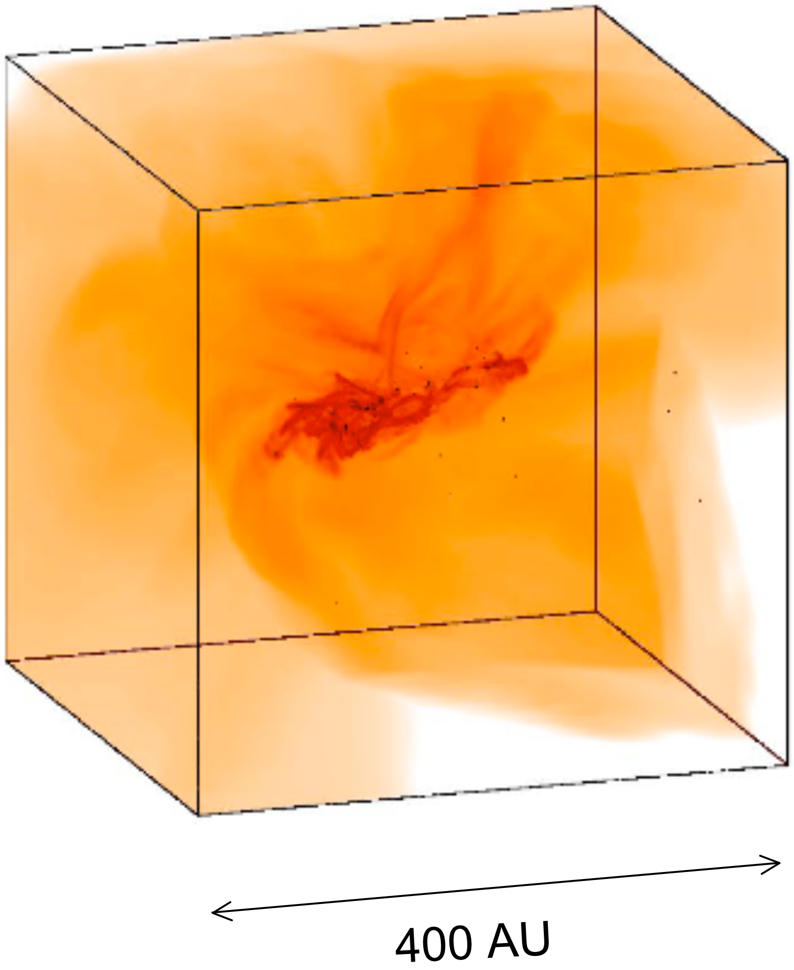



$t = t_{SF}$ + 420 yr




(Clark et al. 2007)

# dust induced fragmentation at $Z=10^{-5}$




dense cluster of low-mass protostars builds up:

- mass spectrum peaks below 1 $M_{sun}$
- cluster VERY dense $n_{stars} = 2.5 \times 10^9 \, pc^{-3}$
- fragmentation at density $n_{gas} = 10^{12} - 10^{13} \, cm^{-3}$

(Clark et al. 2007)

# cluster build-up

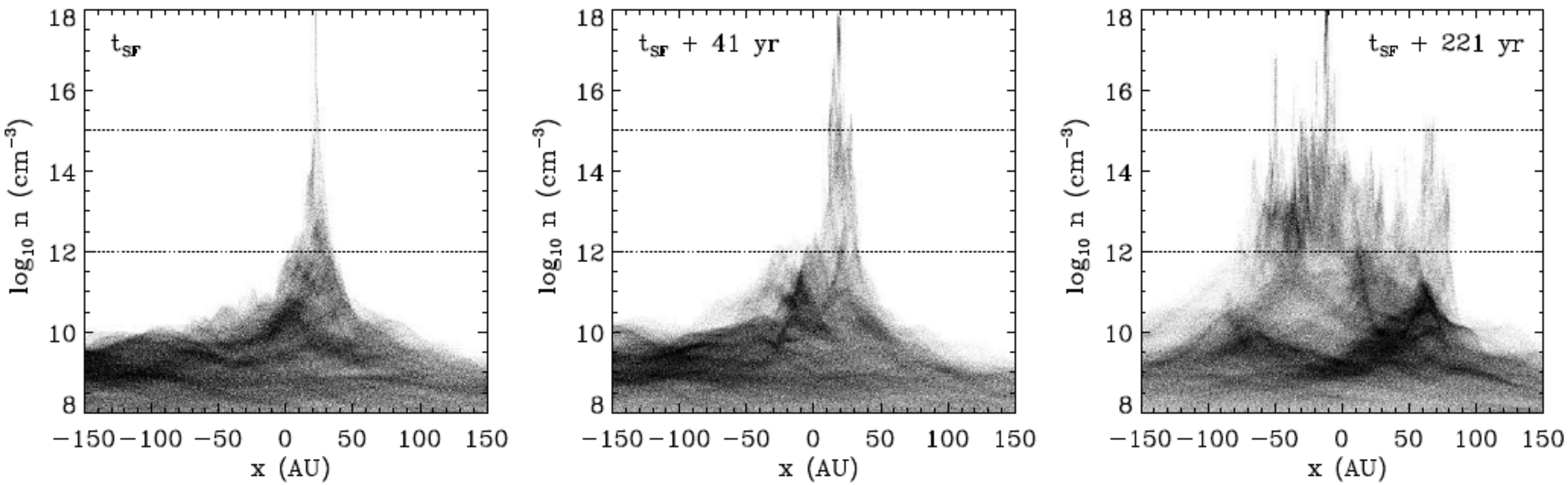
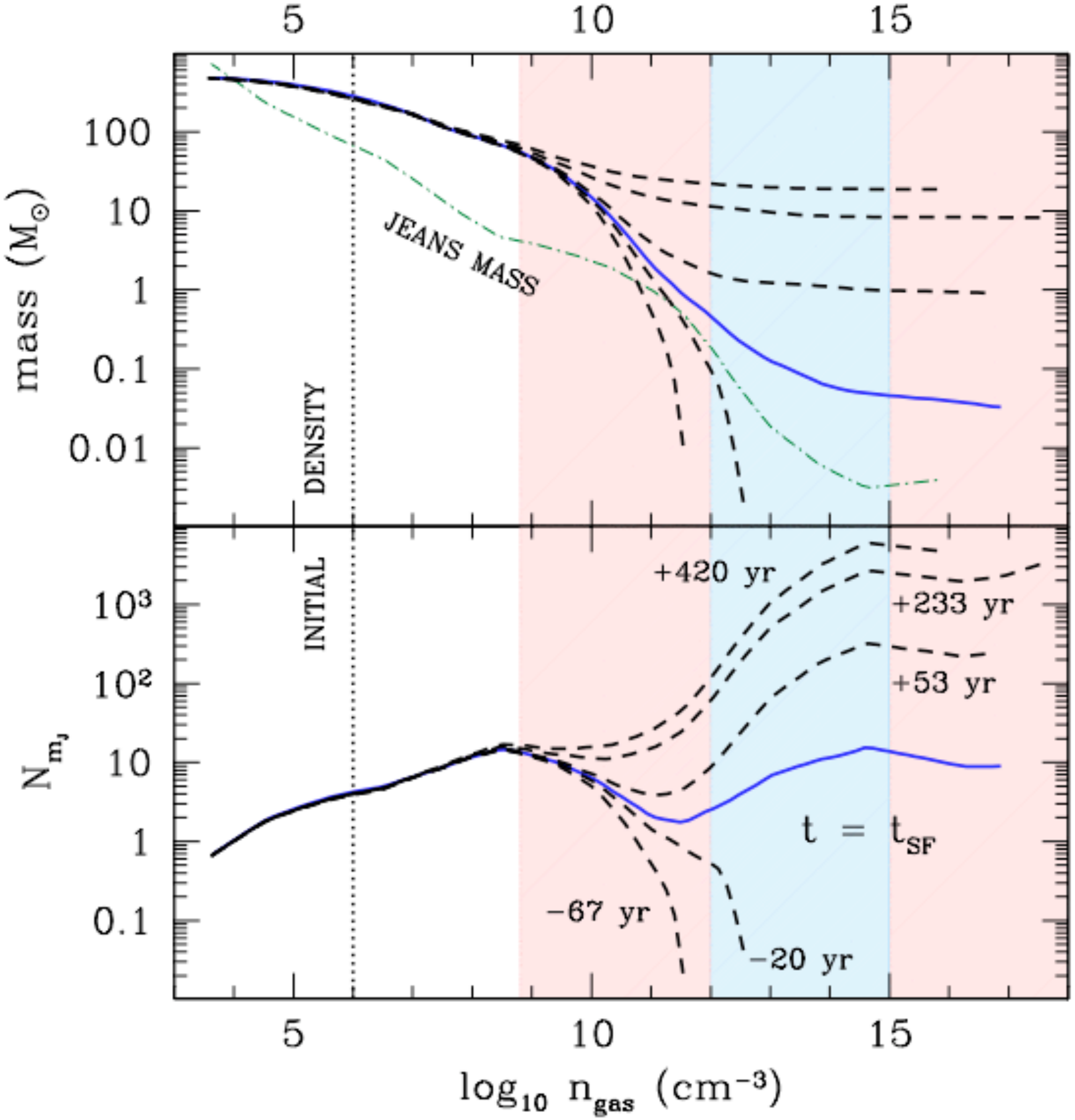



FIG. 3.— We illustrate the onset of the fragmentation process in the high resolution $Z = 10^{-5} Z_{\odot}$ simulation. The graphs show the densities of the particles, plotted as a function of their x-position. Note that for each plot, the particle data has been centered on the region of interest. We show here results at three different output times, ranging from the time that the first star forms ($t_{sf}$) to 221 years afterwards. The densities lying between the two horizontal dashed lines denote the range over which dust cooling lowers the gas temperature.

(Clark et al. 2007)

# cluster build-up

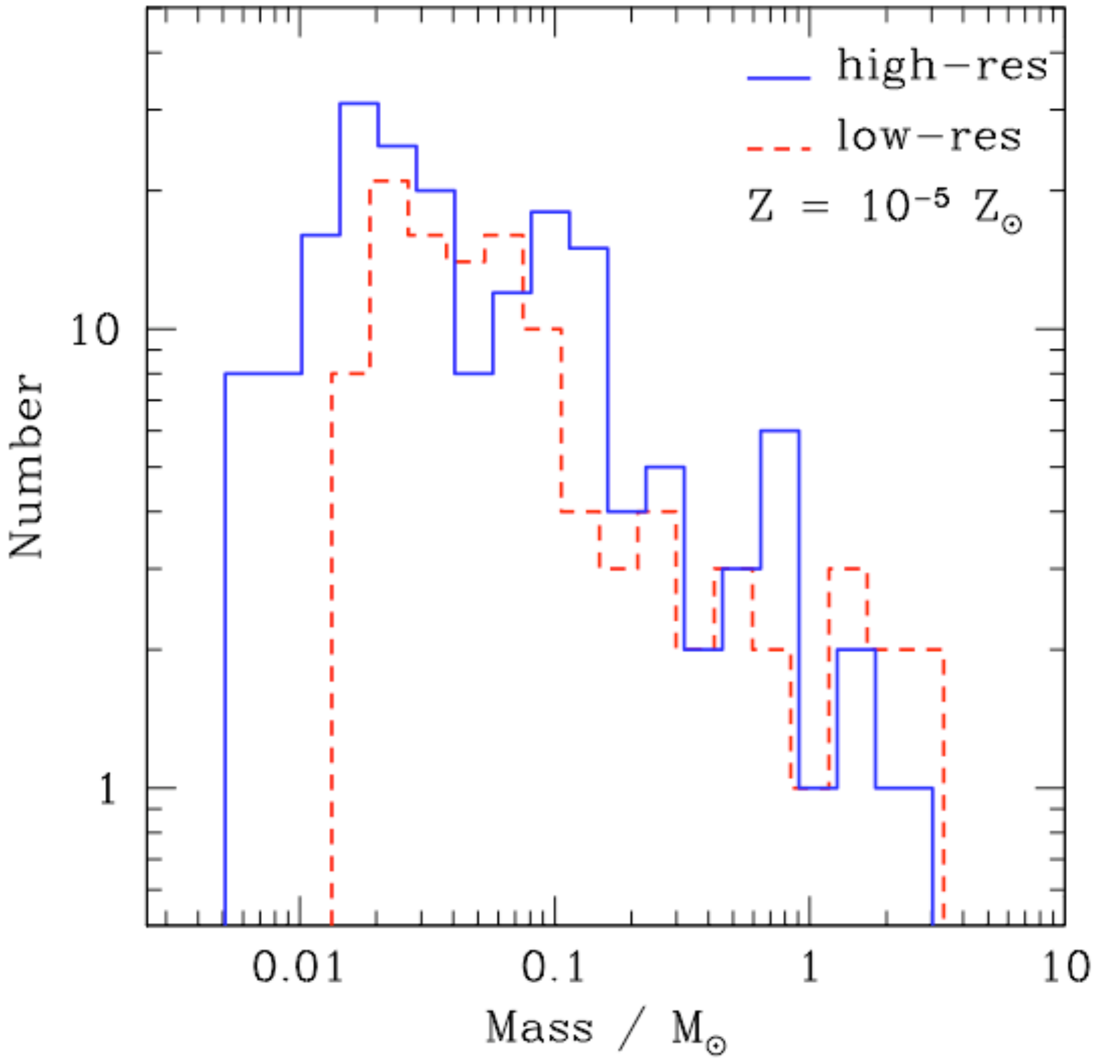



$\gamma > 1$
(heating)

$\gamma < 1$
(cooling)

(Clark et al. 20

# dust induced fragmentation at $Z=10^{-5}$

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(Clark et al. 2007)

# comparison for different Z

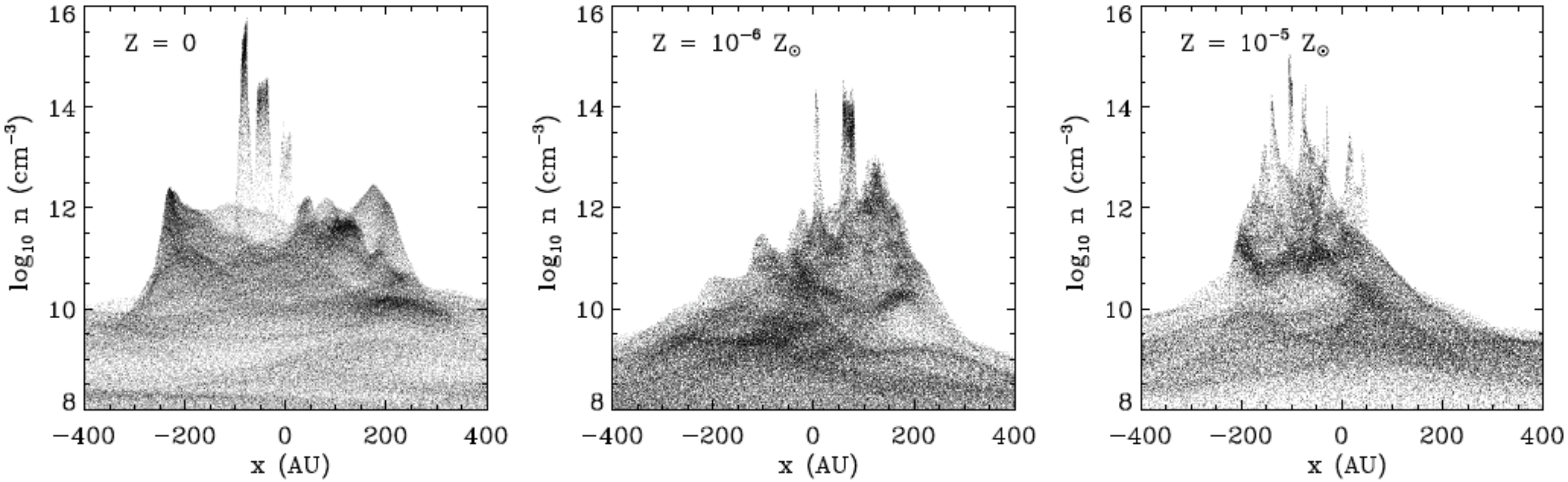



FIG. 6.— Particle densities as a function of position in the low-resolution simulations, for the primordial (left), $Z = 10^{-6}\,Z_\odot$ (middle) and $Z = 10^{-5}\,Z_\odot$ simulations (right). The particles are plotted once the protostars in each simulation have accreted 19 $M_\odot$ of gas.

even zero-metallicity case fragments
(although much more weakly)

(Clark et al. 2007)

# comparison for different Z

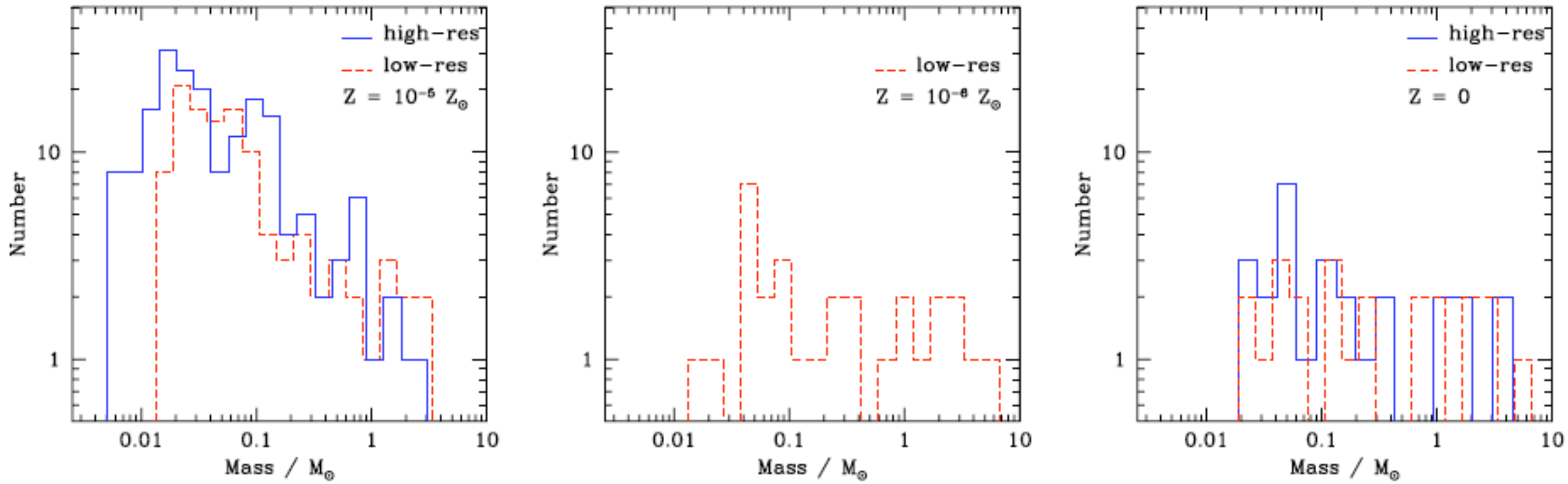


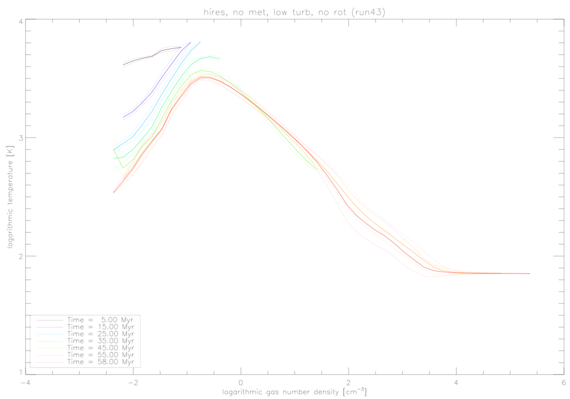

FIG. 4.— Mass functions resulting from simulations with metallicities $Z = 10^{-5}\ Z_\odot$ (left-hand panel), $Z = 10^{-6}\ Z_\odot$ (center panel), and $Z = 0$ (right-hand panel). The plots refer to the point in each simulation at which 19 $M_\odot$ of material has been accreted (which occurs at a slightly different time in each simulation). The mass resolutions are 0.002 $M_\odot$ and 0.025 $M_\odot$ for the high and low resolution simulations, respectively. Note the similarity between the results of the low-resolution and high-resolution simulations. The onset of dust-cooling in the $Z = 10^{-5}\ Z_\odot$ cloud results in a stellar cluster which has a mass function similar to that for present day stars, in that the majority of the mass resides in the lower-mass objects. This contrasts with the $Z = 10^{-6}\ Z_\odot$ and primordial clouds, in which the bulk of the cluster mass is in high-mass stars.

even zero-metallicity case fragments
(although much more weakly)

(Clark et al. 2007)

# Simple EOS vs. radiation transfer

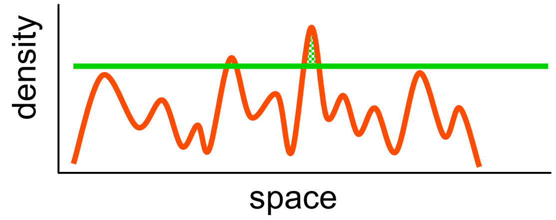
- how good is EOS approach?
  - time to reach chemical and thermal equilibrium shorter than dynamical time?
  - how does EOS depend on dynamics? (e.g. 1D collapse with large-gradient approx. versus complex 3D turbulent flows)




- how important is heating from stars?
  - accretion luminosity may heat gas and reduce degree of cloud fragmentation (cluster formation vs. high-mass SF)
- how can we model that best?
  - full radiation transfer vs. approximate schemes

# summary

# Summary I




- interstellar gas is highly *inhomogeneous*
  - *thermal instability*
  - *gravitational instability*
  - *turbulent compression* (in shocks $\delta\rho/\rho \propto \mathcal{M}^2$; in atomic gas: $\mathcal{M} \approx 1...3$)
- cold *molecular clouds* can form rapidly in high-density regions at *stagnation points* of *convergent large-scale flows*
  - chemical *phase transition*: atomic → molecular
  - process is *modulated* by large-scale *dynamics* in the galaxy
- inside *cold clouds:* turbulence is highly supersonic ($\mathcal{M} \approx 1...20$)
  → *turbulence* creates density contrast, *gravity* selects for collapse

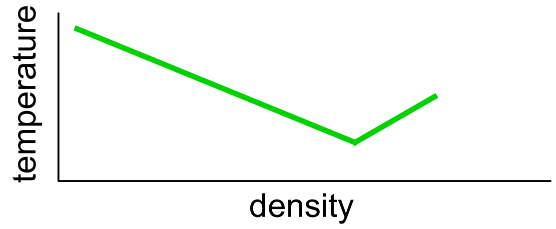
→ **GRAVOTUBULENT FRAGMENTATION**

- *turbulent cascade:* local compression *within* a cloud provokes collapse → formation of individual *stars* and *star clusters*
- *star cluster:* gravity dominates in large region (--> competitive accretion)

(e.g. Mac Low & Klessen, 2004, Ballesteros-Paredes et al. 2006, McKee & Ostriker 2007)

# Summary II

- *thermodynamic response* (EOS) determines fragmentation behavior
    - characteristic stellar mass from fundamental atomic and molecular parameters
      --> explanation for quasi-universal IMF?




- *stellar feedback* is important
    - accretion heating may reduce degree of fragmentation
    - ionizing radiation will set efficiency of star formation
- *CAVEATS:*
    - star formation is *multi-scale, multi-physics* problem --> VERY difficult to model
    - in simulations: very small turbulent inertial range (Re < 1000)
    - can we use EOS to describe thermodynamics of gas, or do we need time-dependent chemical network and radiative transport?
    - stellar feedback requires (at least approximative) radiative transport, most numerical calculations so far have neglected that aspect

(e.g. Mac Low & Klessen, 2004, Ballesteros-Paredes et al. 2006, McKee & Ostriker 2007)

# Thanks!