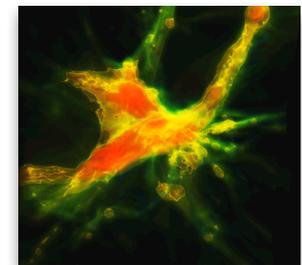
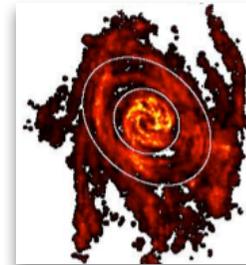
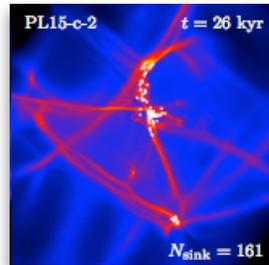
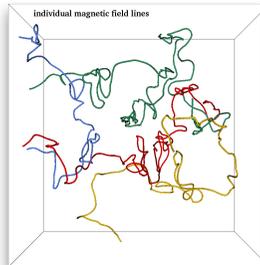


# Modeling ISM Dynamics and Star Formation



**Ralf Klessen**



Universität Heidelberg, Zentrum für Astronomie  
Institut für Theoretische Astrophysik



disclaimer

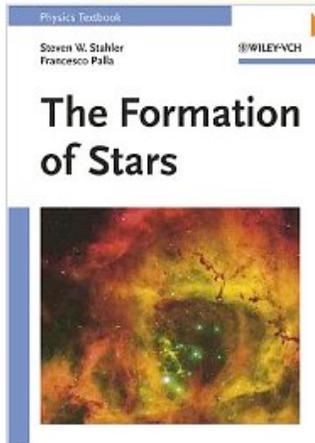
# Disclaimer

- I try to cover the field as broadly as possible, however, there will clearly be a bias towards my personal interests and many examples will be from my own work.

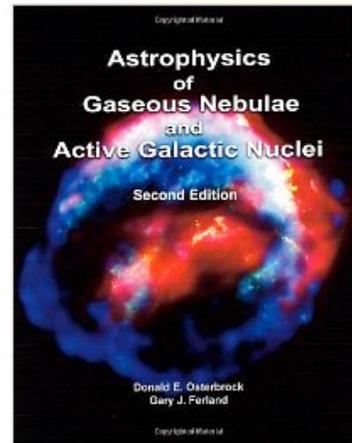
literature

# Literature

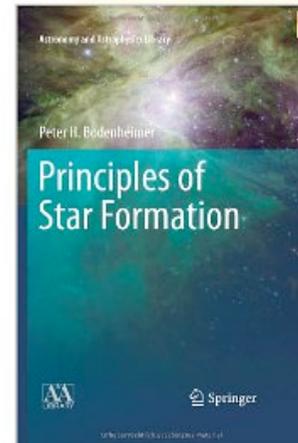
Click to **LOOK INSIDE!**



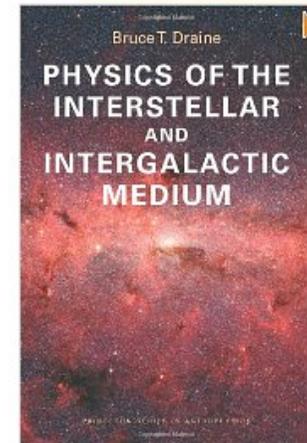
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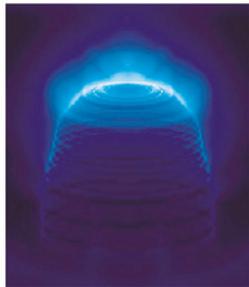


PHYSICS TEXTBOOK

George B. Rybicki  
Alan P. Lightman

WILEY-VCH

**Radiative Processes  
in Astrophysics**

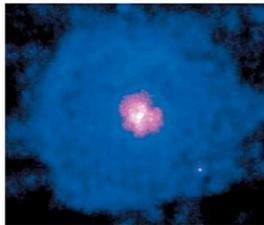


PHYSICS TEXTBOOK

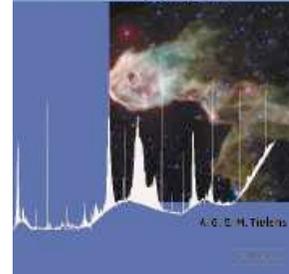
Lyman Spitzer, Jr.

WILEY-VCH

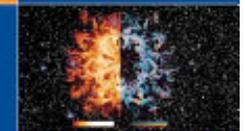
**Physical Processes in the  
Interstellar Medium**



The Physics and Chemistry of the  
**Interstellar  
Medium**



Stars in Atmospheres and Atmospheres



**NUMERICAL METHODS  
IN ASTROPHYSICS**

An Introduction

Peter Bodenheimer  
Gerson P. Laganak  
Mohit Rastogi  
Ramon N. Toral

Taylor & Francis

## 🌟 Books

- 🌟 Spitzer, L., 1978/2004, Physical Processes in the Interstellar Medium (Wiley-VCH)
- 🌟 Rybicki, G.B., & Lightman, A.P., 1979/2004, Radiative Processes in Astrophysics (Wiley-VCH)
- 🌟 Stahler, S., & Palla, F., 2004, "The Formation of Stars" (Weinheim: Wiley-VCH)
- 🌟 Tielens, A.G.G.M., 2005, The Physics and Chemistry of the Interstellar Medium (Cambridge University Press)
- 🌟 Osterbrock, D., & Ferland, G., 2006, "Astrophysics of Gaseous Nebulae & Active Galactic Nuclei, 2<sup>nd</sup> ed. (Sausalito: Univ. Science Books)
- 🌟 Bodenheimer, P., et al., 2007, Numerical Methods in Astrophysics (Taylor & Francis)
- 🌟 Draine, B. 2011, "Physics of the Interstellar and Intergalactic Medium" (Princeton Series in Astrophysics)
- 🌟 Bodenheimer, P. 2012, "Principles of Star Formation" (Springer Verlag)

# Literature

## ● Review Articles

- Mac Low, M.-M., Klessen, R.S., 2004, "The control of star formation by supersonic turbulence", Rev. Mod. Phys., 76, 125
- Elmegreen, B.G., Scalo, J., 2004, "Interstellar Turbulence 1", ARA&A, 42, 211
- Scalo, J., Elmegreen, B.G., 2004, "Interstellar Turbulence 2", ARA&A, 42, 275
- Bromm, V., Larson, R.B., 2004, "The first stars", ARA&A, 42, 79
- Zinnecker, H., Yorke, McKee, C.F., Ostriker, E.C., 2008, "Toward Understanding Massive Star Formation", ARA&A, 45, 481 - 563
- McKee, C.F., Ostriker, E.C., 2008, "Theory of Star Formation", ARA&A, 45, 565
- Kennicutt, R.C., Evans, N.J., 2012, "Star Formation in the Milky Way and Nearby Galaxies", ARA&A, 50, 531

# Further resources

## Internet resources

-  Cornelis Dullemond: *Radiative Transfer in Astrophysics*  
[http://www.ita.uni-heidelberg.de/~dullemond/lectures/radtrans\\_2012/index.shtml](http://www.ita.uni-heidelberg.de/~dullemond/lectures/radtrans_2012/index.shtml)
-  Cornelis Dullemond: *RADMC-3D: A new multi-purpose radiative transfer tool*  
<http://www.ita.uni-heidelberg.de/~dullemond/software/radmc-3d/index.shtml>

# Part 2: Dynamics of the ISM

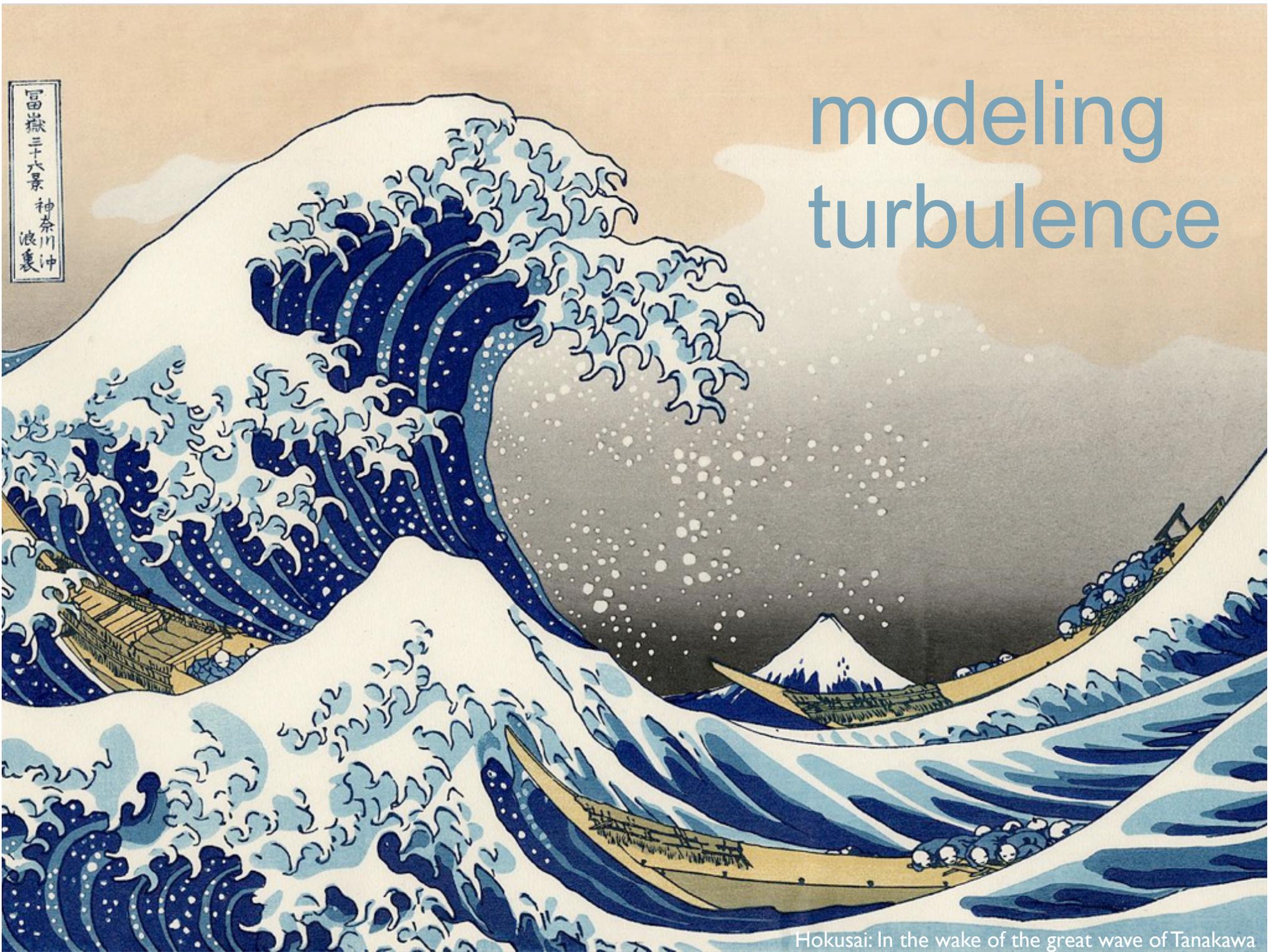


**Ralf Klessen**

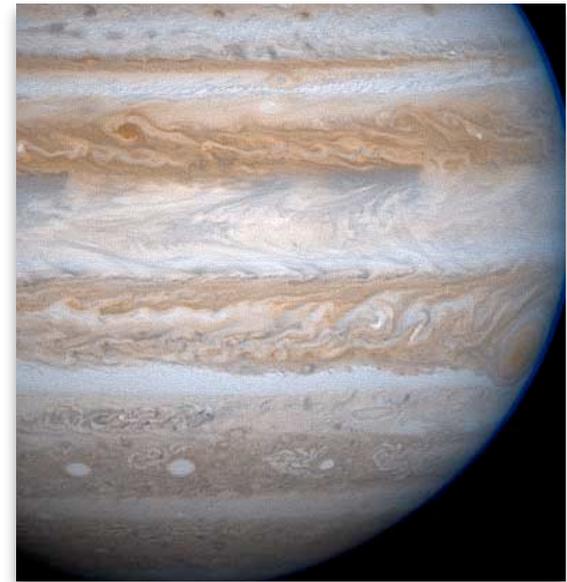
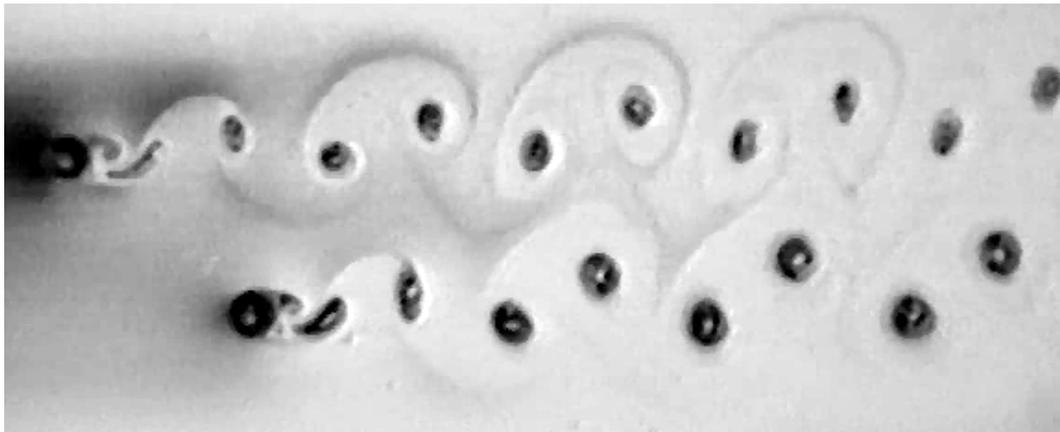
Universität Heidelberg, Zentrum für Astronomie  
Institut für Theoretische Astrophysik



# modeling turbulence



Hokusai: In the wake of the great wave of Tanakawa



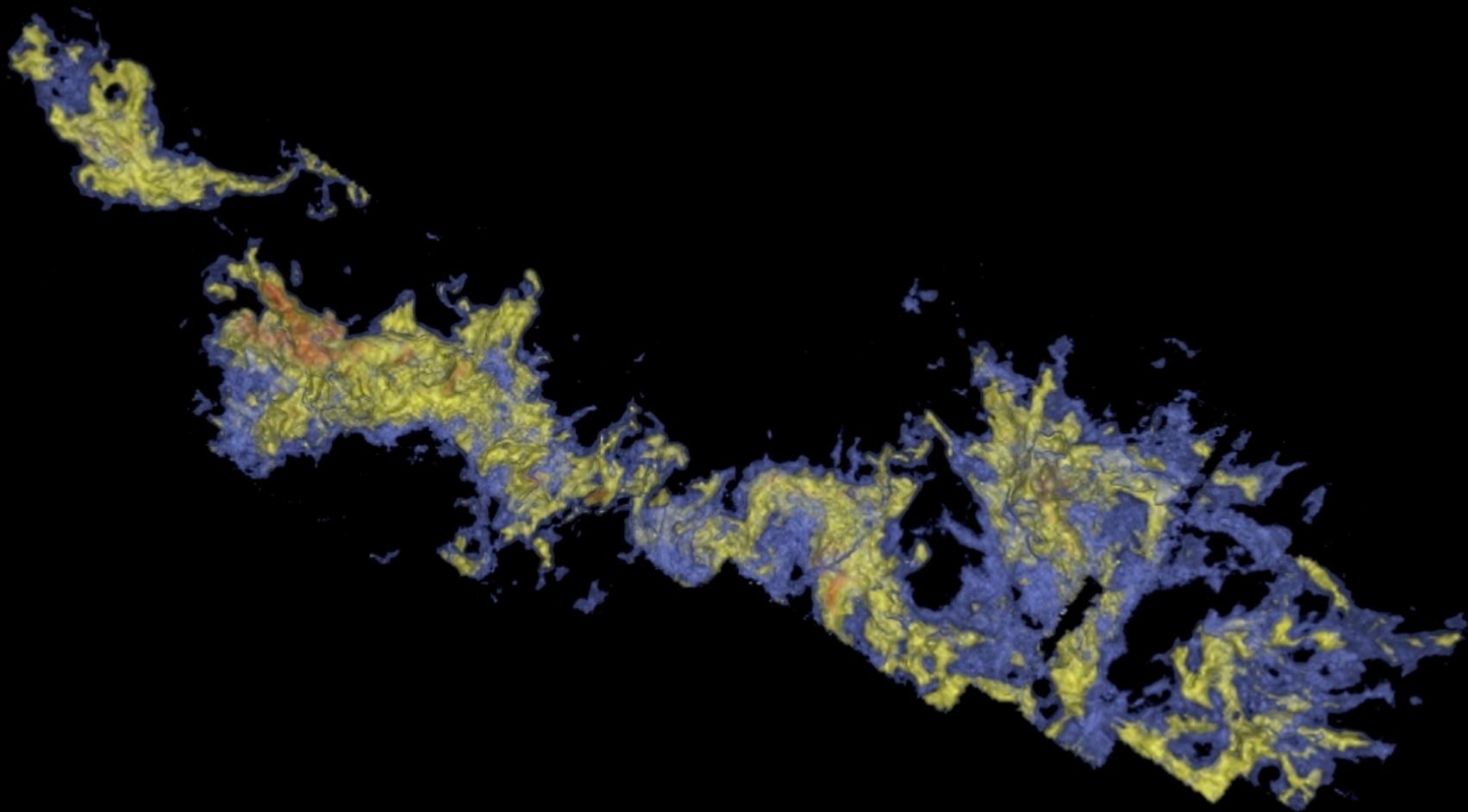
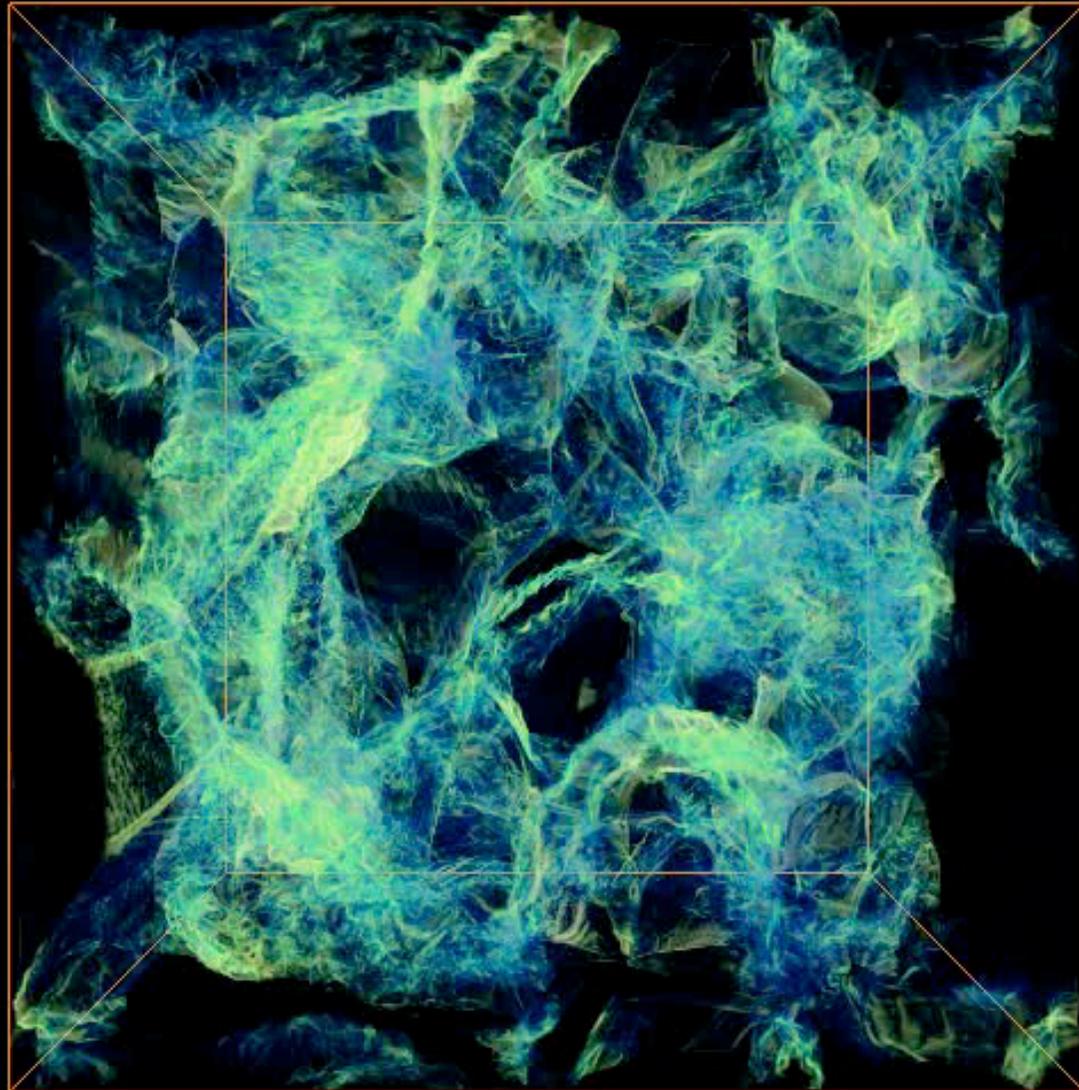


image from Alyssa Goodman: COMPLETE survey



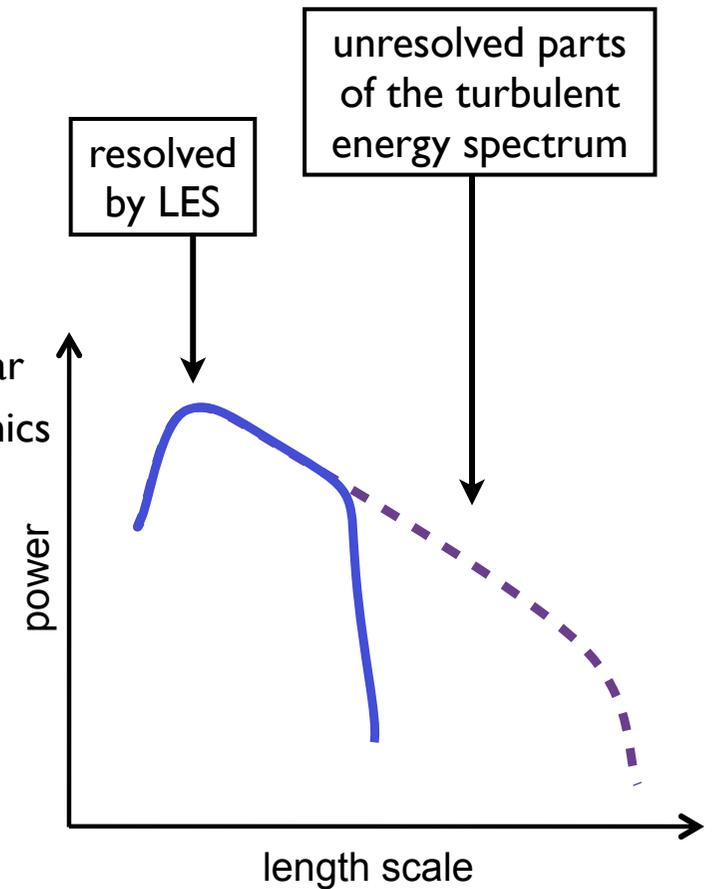
Schmidt et al. (2009, A&A, 494, 127)

# large eddie simulations

- large eddie simulations (LES) attempt to resolve at least parts of the turbulent cascade
  - principal problem: only large scale flow properties
  - Reynolds number:  $Re = LV/\nu$  ( $Re_{\text{nature}} \gg Re_{\text{model}}$ )
  - dynamic range much smaller than true physical one
- need subgrid model
  - (in our case simple: only dissipation)
  - more complex when processes (chemical reactions, nuclear burning, etc) on subgrid scale determine large-scale dynamics
- stochasticity → unpredictable when and where “interesting things” happen
  - occurrence of localized collapse
  - location and strength of shock fronts
  - etc.

# large eddy simulations

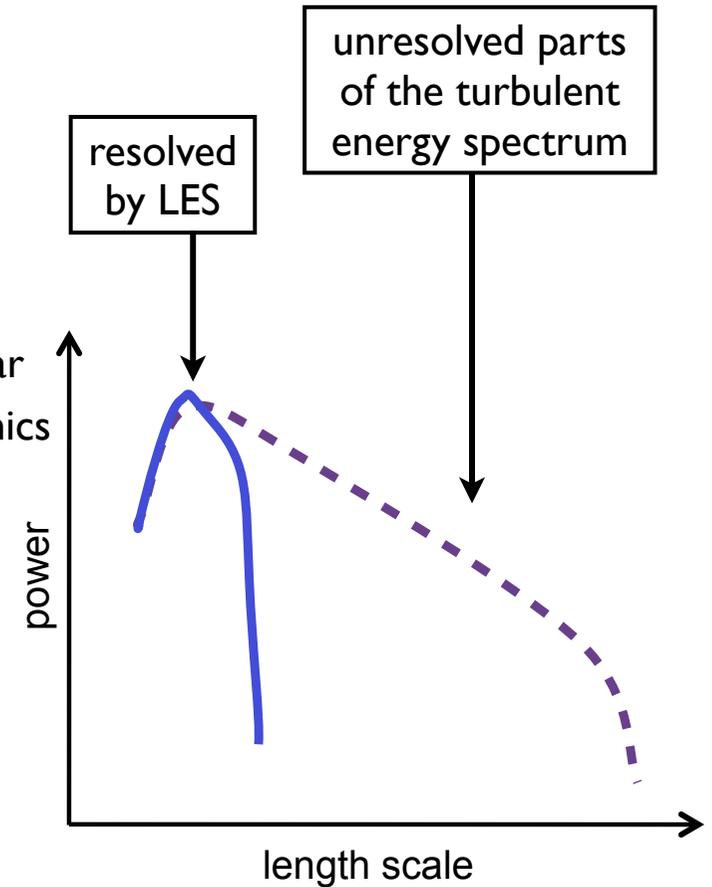
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We model honey instead of the ISM!!!



# driving turbulence 0

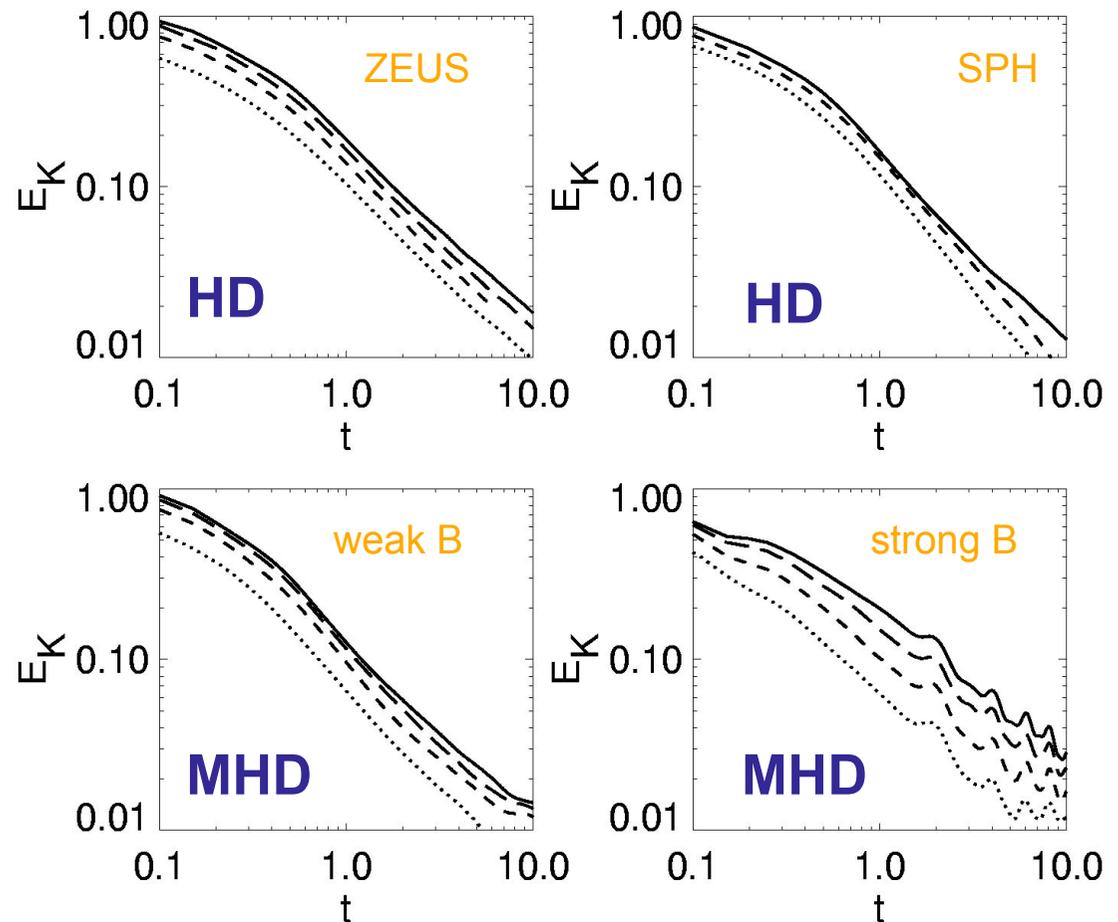
turbulence decays rapidly:

$E_{\text{kin}}$  decays as  $t^{-\eta}$ , with  $0.85 \approx \eta \approx 1.2$ .

turbulence *decays* on timescales *comparable* to the free-fall time  $\tau_{\text{ff}}$

(e.g. Mac Low et al. 1998, Stone et al. 1998, Padoan & Nordlund 1999)

steady state turbulence needs to be continuously driven!



(Mac Low, Klessen, Burkert, & Smith, 1998, PRL)

# driving turbulence 1

turbulent energy decays --> steady state turbulence needs to be driven --> insert energy at each timestep (or at least frequently)

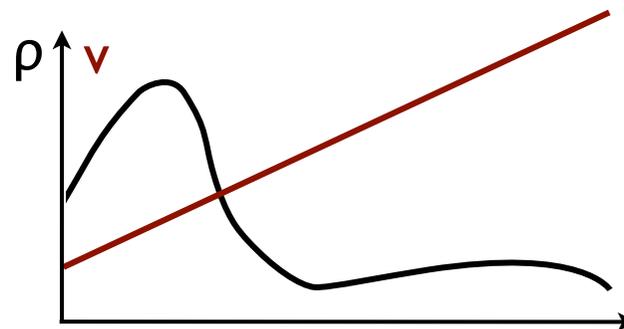
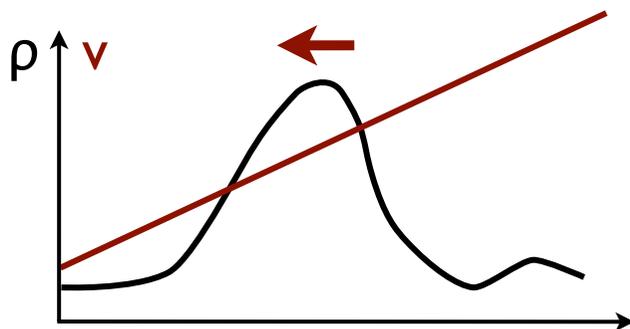
two possibilities:

-- include stochastic force term

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = \sum_i \vec{F}_i + \vec{f}_t$$

-- add  $\delta \vec{v}_t$  to the velocity  $\vec{v} \rightarrow \vec{v} + \delta \vec{v}_t$

for supersonic turbulence, keeping constant velocity dispersion requires some thoughts (because of compressibility of the medium)



# driving turbulence 2

turbulent energy decays --> steady state turbulence needs to be driven  
--> insert energy at each timestep (or at least frequently)

goal: keep rms velocity dispersion constant  
--> adjust the amount of energy added

$$\Delta E = \sum_i \frac{m_i}{2} (\vec{v} + \delta\vec{v})^2 - \sum_i \frac{m_i}{2} \vec{v}^2$$

resulting in

$$\Delta E = \sum_i \frac{m_i}{2} (\vec{v} + \delta\vec{v}) \delta\vec{v}$$

because  $m_i$  changes at each timestep,  $\Delta E$  needs to be adjusted.

write  $\delta\vec{v} = A\delta\tilde{v}$  with fixed  $\delta\tilde{v}$  and adjustable  $A$ .

solve quadratic equation to get  $A$ : 
$$\Delta E = \sum_i \frac{m_i}{2} (A\vec{v}\delta\tilde{v} + A^2\delta\tilde{v}^2)$$

# driving turbulence 3

Solve equations of (magneto)hydrodynamics on a computer.

Logarithmic density:  $s \equiv \ln \frac{\rho}{\langle \rho \rangle}$

HD equations are then:

$$\frac{\partial s}{\partial t} + (\mathbf{v} \cdot \nabla)s = -\nabla \cdot \mathbf{v} \quad \text{continuity}$$
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -c_s^2 \nabla s + \mathbf{f}, \quad \text{Euler}$$



Leonard Salomon Ornstein (1880-1941)



George Eugene Uhlenbeck (1900 -1988)

stochastic force term that follows  
an Ornstein-Uhlenbeck process

named after [Leonard Ornstein](#) and  
[George Eugene Uhlenbeck](#)

# driving turbulence 4

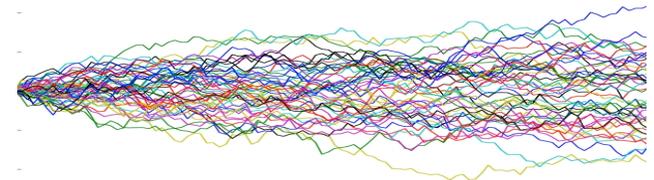
The OU process is a stochastic differential equation describing the evolution of the forcing term in Fourier space (k-space):

$$d\widehat{f}(\mathbf{k}, t) = f_0(\mathbf{k}) \underline{\mathcal{P}}^\zeta(\mathbf{k}) dW(t) - \widehat{f}(\mathbf{k}, t) \frac{dt}{T}$$

the first term on RHS is a diffusion term modeled as a Wiener process  $W(t)$  which adds a Gaussian random increment to the vector field given in the previous time step  $dt$ .

$$W(t) - W(t - dt) = N(0, dt)$$

where  $N(0, dt)$  denotes a Gaussian distribution with zero mean and standard deviation  $dt$



# driving turbulence 5

The OU process is a stochastic differential equation describing the evolution of the forcing term in Fourier space (k-space):

$$d\widehat{f}(\mathbf{k}, t) = f_0(\mathbf{k}) \underline{\mathcal{P}}^\zeta(\mathbf{k}) dW(t) - \widehat{f}(\mathbf{k}, t) \frac{dt}{T}$$

with projection tensor in Fourier space  $\underline{\mathcal{P}}^\zeta(\mathbf{k})$  (Helmholtz decomposition)

in index notation:  $\mathcal{P}_{ij}^\zeta(\mathbf{k}) = \zeta \mathcal{P}_{ij}^\perp(\mathbf{k}) + (1 - \zeta) \mathcal{P}_{ij}^\parallel(\mathbf{k}) = \zeta \delta_{ij} + (1 - 2\zeta) \frac{k_i k_j}{|\mathbf{k}|^2}$

$\mathcal{P}_{ij}^\perp = \delta_{ij} - k_i k_j / k^2$  fully solenoidal projection operator

$\mathcal{P}_{ij}^\parallel = k_i k_j / k^2$  fully compressive projection operator

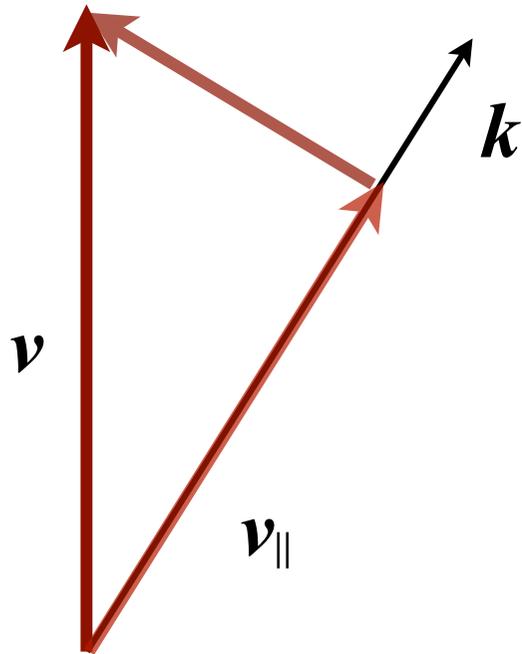


Hermann Ludwig Ferdinand von Helmholtz (1821-1894)

# driving turbulence 6

$\mathcal{P}_{ij}^{\perp} = \delta_{ij} - k_i k_j / k^2$  fully solenoidal projection operator

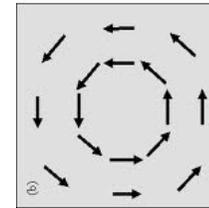
$\mathcal{P}_{ij}^{\parallel} = k_i k_j / k^2$  fully compressive projection operator



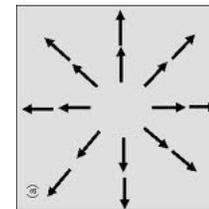
# driving turbulence 7

in index notation:  $\mathcal{P}_{ij}^{\zeta}(\mathbf{k}) = \zeta \mathcal{P}_{ij}^{\perp}(\mathbf{k}) + (1 - \zeta) \mathcal{P}_{ij}^{\parallel}(\mathbf{k}) = \zeta \delta_{ij} + (1 - 2\zeta) \frac{k_i k_j}{|\mathbf{k}|^2}$

$\zeta = 1$  for purely solenoidal force field



$\zeta = 0$  for purely compressive force field



by adjusting  $\zeta$  any combination of solenoidal and compressive force fields are possible

# driving turbulence 8

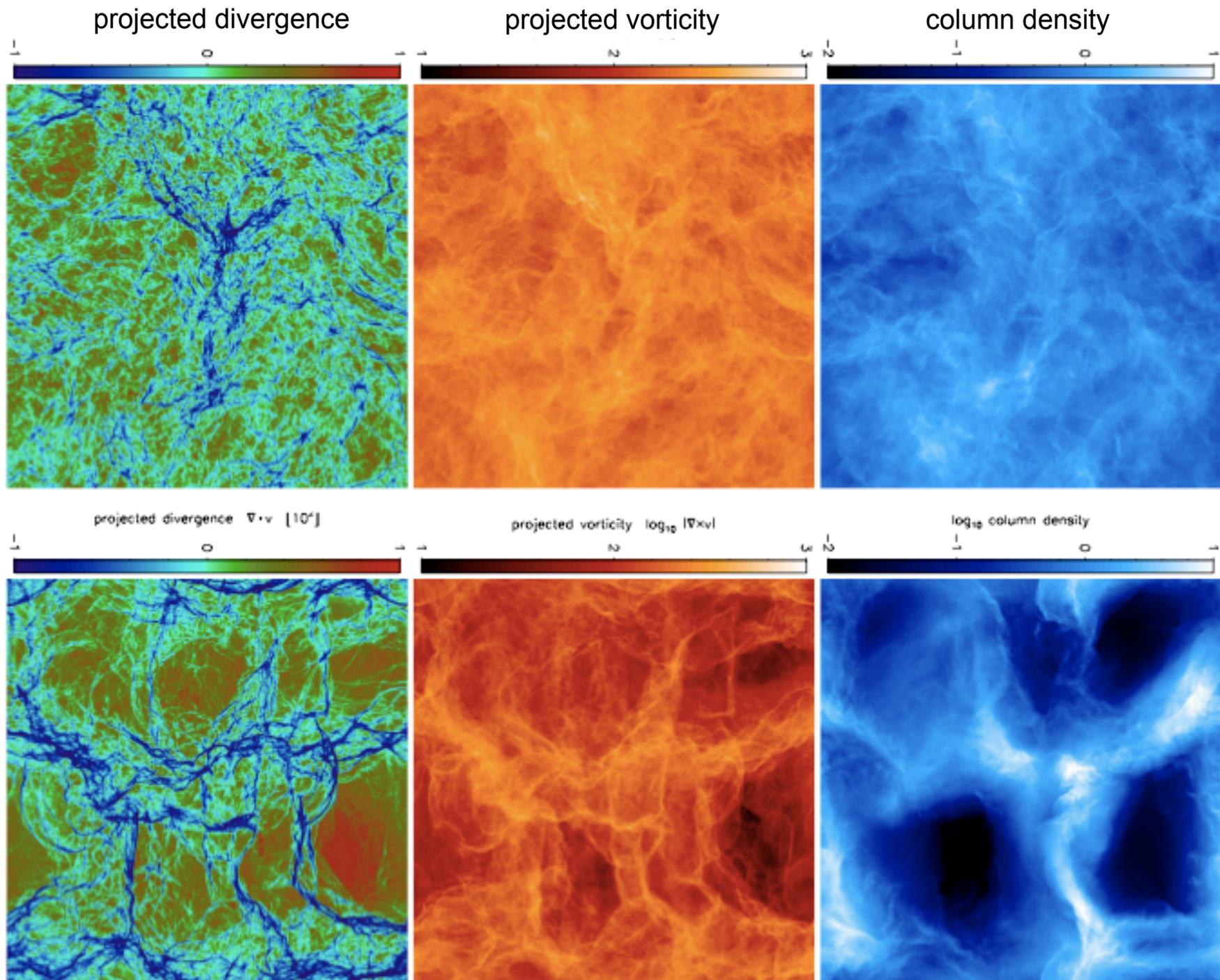
density as function of time / projected density of low-res.  $128^3$  cube simulation (FLASH)



compressive  
*larger structures, higher  $\rho$ -contrast*



rotational  
*smaller structures, narrow  $\rho$ -pdf*



# driving turbulence 8

The analytical ratio of compressive power to total power can be derived by evaluating the norm of the compressive component of the projection tensor

$$\left| (1 - \zeta) \mathcal{P}_{ij}^{\parallel} \right|^2 = (1 - \zeta)^2,$$

and by evaluating the norm of the full projection tensor

$$\left| \mathcal{P}_{ij}^{\zeta} \right|^2 = 1 - 2\zeta + D\zeta^2,$$

where  $D$  denotes the dimensionality of the problem ( $D = 1, 2, 3$ )

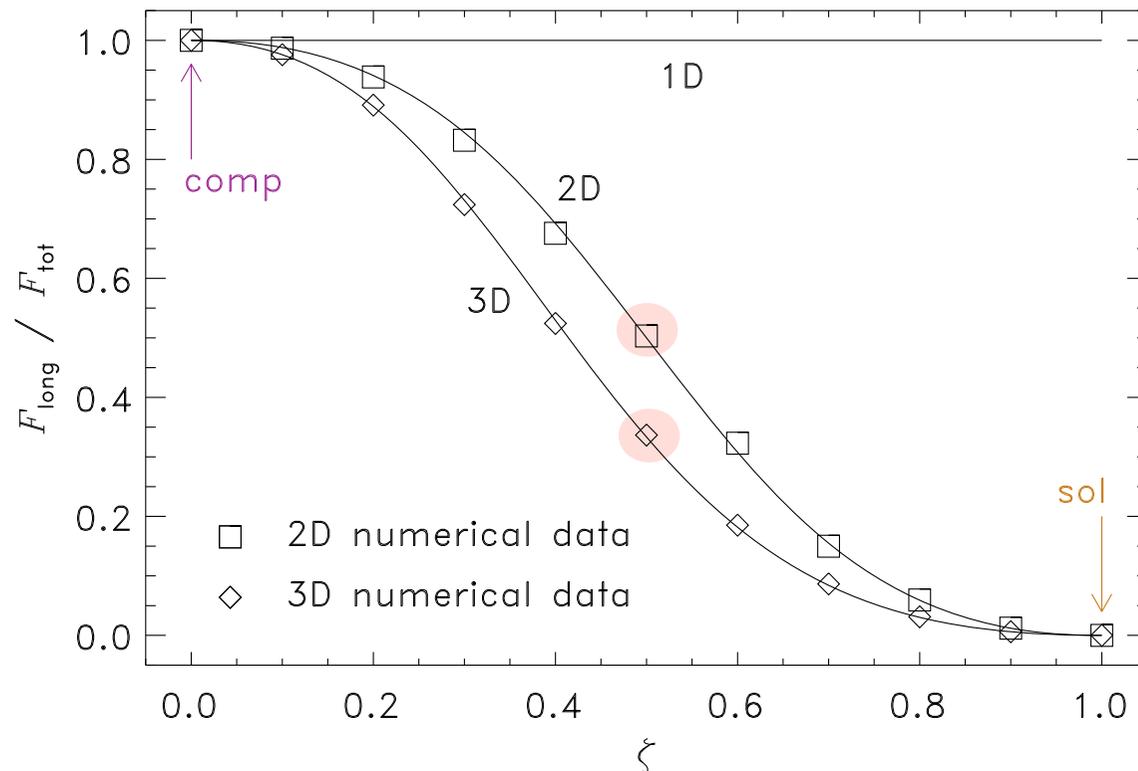
ratio of compressive forcing power to total forcing power is then:

$$\frac{F_{\text{long}}}{F_{\text{tot}}} = \frac{(1 - \zeta)^2}{1 - 2\zeta + D\zeta^2}$$

# driving turbulence 9

ratio of compressive forcing power to total forcing power is then:

$$\frac{F_{\text{long}}}{F_{\text{tot}}} = \frac{(1 - \zeta)^2}{1 - 2\zeta + D\zeta^2}$$



natural ratio of solenoidal and compressive forcing for

$$\zeta = 0.5.$$

then we get (for  $D=3$ )

$$F_{\text{long}} / F_{\text{tot}} = 1/3$$

and ( $D=2$ )

$$F_{\text{long}} / F_{\text{tot}} = 1/2$$

# driving turbulence 10

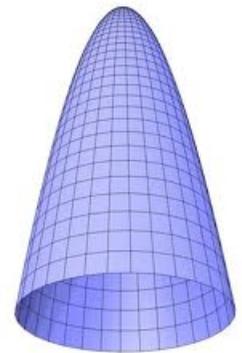
The OU process is a stochastic differential equation describing the evolution of the forcing term in Fourier space (k-space):

$$d\widehat{f}(\mathbf{k}, t) = f_0(\mathbf{k}) \underline{\mathcal{P}}^\zeta(\mathbf{k}) dW(t) - \widehat{f}(\mathbf{k}, t) \frac{dt}{T}$$

second term is drift term, and models the exponentially decaying timescale of the force field with itself --> autocorrelation timescale  $T$  of the forcing field

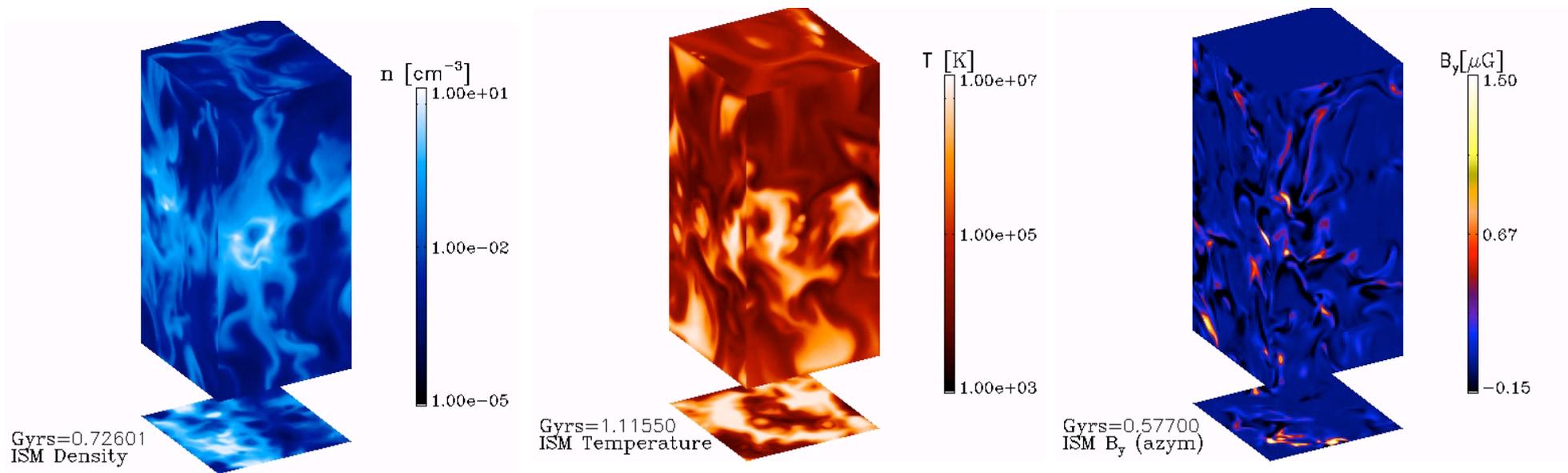
often  $T = L/(2V)$  with  $L$  being the size of the computational domain and  $V$  being the typical crossing time  $V = c_s \mathcal{M}$  (with Mach number  $\mathcal{M}$  sound speed  $c_s$ )

forcing amplitude  $f_0(\mathbf{k})$  is a paraboloid in 3D Fourier space, only containing power on the largest scales in a small interval of wavenumbers  $k_{\min} < |\mathbf{k}| < k_{\max}$

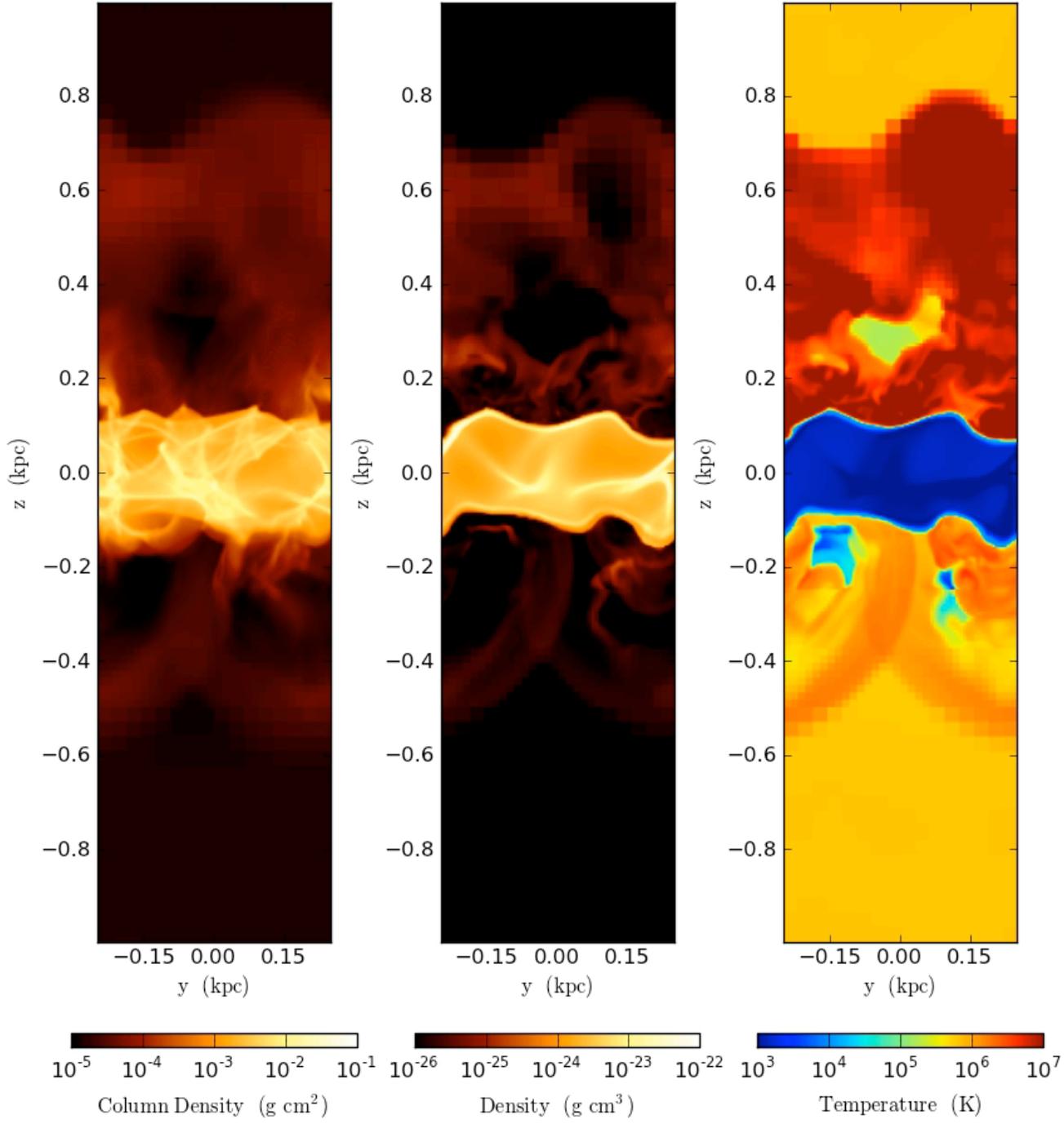


# driving turbulence 11

Supernova explosions as drivers of ISM turbulence



movies from Philipp Girichidis (University of Sheffield)



SILCC project

# what drives ISM turbulence?

- seems to be driven on large scales, little difference between star-forming and non-SF clouds  
→ rules out internal sources
- proposals in the literature
  - supernovae
  - expanding HII regions / stellar winds / outflows
  - spiral density waves
  - magneto-rotational instability
  - more recent idea: accretion onto disk

# what drives ISM turbulence?

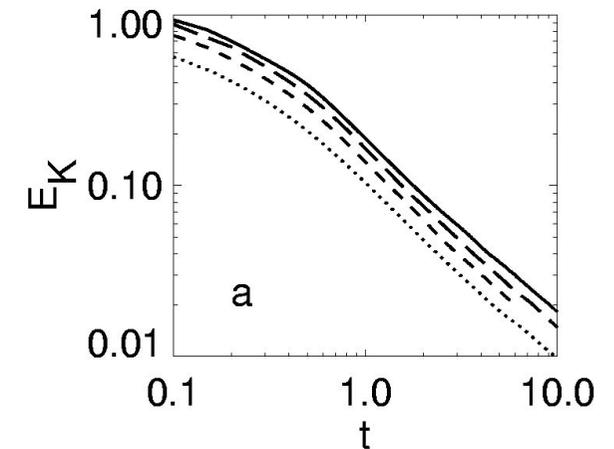
some energetic arguments...

energy decay by turbulent dissipation:

$$\begin{aligned}\dot{e} &\simeq -(1/2)\rho v_{\text{rms}}^3/L_d \\ &= -(3 \times 10^{-27}) \text{ erg cm}^{-3} \text{ s}^{-1} \left( \frac{n}{1 \text{ cm}^{-3}} \right) \\ &\quad \times \left( \frac{v_{\text{rms}}}{10 \text{ km s}^{-1}} \right)^3 \left( \frac{L_d}{100 \text{ pc}} \right)^{-1},\end{aligned}$$

decay timescale:

$$\begin{aligned}\tau_d = e/\dot{e} &\simeq L_d/v_{\text{rms}} \\ &= (9.8 \text{ Myr}) \left( \frac{L_d}{100 \text{ pc}} \right) \left( \frac{v_{\text{rms}}}{10 \text{ km s}^{-1}} \right)^{-1},\end{aligned}$$



(Mac Low et al. 1999)

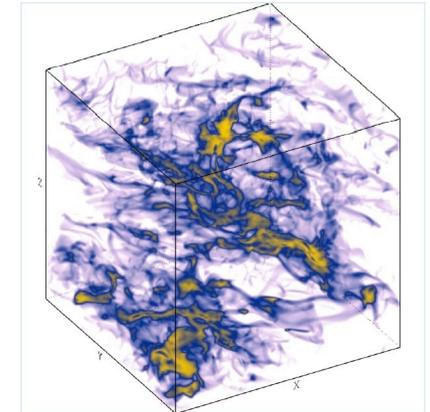
# what drives ISM turbulence?

magneto-rotational instability:

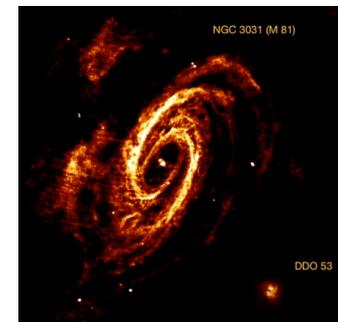
$$\dot{e} = (3 \times 10^{-29} \text{ erg cm}^{-3} \text{ s}^{-1}) \left( \frac{B}{3 \mu\text{G}} \right)^2 \left( \frac{\Omega}{(220 \text{ Myr})^{-1}} \right).$$

gravitational instability (spiral waves)

$$\begin{aligned} \dot{e} &\simeq G(\Sigma_g/H)^2 \lambda^2 \Omega \\ &\simeq (4 \times 10^{-29} \text{ erg cm}^{-3} \text{ s}^{-1}) \\ &\quad \times \left( \frac{\Sigma_g}{10 M_\odot \text{ pc}^{-2}} \right)^2 \left( \frac{H}{100 \text{ pc}} \right)^{-2} \\ &\quad \times \left( \frac{\lambda}{100 \text{ pc}} \right)^2 \left( \frac{\Omega}{(220 \text{ Myr})^{-1}} \right), \end{aligned}$$



(from Pietek & Ostriker 2005)



(from Walter et al. 2008)

# what drives ISM turbulence?

protostellar outflows

$$\begin{aligned} \dot{e} &= \frac{1}{2} f_w \eta_w \frac{\dot{\Sigma}_*}{H} v_w^2 \\ &\simeq (2 \times 10^{-28}) \text{ erg cm}^{-3} \text{ s}^{-1} \left( \frac{H}{200 \text{ pc}} \right)^{-1} \left( \frac{f_w}{0.4} \right) \\ &\quad \times \left( \frac{v_w}{200 \text{ km s}^{-1}} \right) \left( \frac{v_{\text{rms}}}{10 \text{ km s}^{-1}} \right) \\ &\quad \times \left( \frac{\dot{\Sigma}_*}{4.5 \times 10^{-9} M_\odot \text{ pc}^{-2} \text{ yr}^{-1}} \right), \end{aligned}$$

(Li & Nakamura 2006, Wang et al. 2010 vs. Banerjee et al. 2008)

expanding HII regions

$$\begin{aligned} \dot{e} &= \frac{\langle \delta p \rangle \mathcal{N}(>1) v_i}{V t_i} \\ &= (3 \times 10^{-30}) \text{ erg s}^{-3} \\ &\quad \times \left( \frac{N_H}{1.5 \times 10^{22} \text{ cm}^{-2}} \right)^{-3/14} \left( \frac{M_{cl}}{10^6 M_\odot} \right)^{1/14} \\ &\quad \times \left( \frac{\langle M_* \rangle}{440 M_\odot} \right) \left( \frac{\mathcal{N}(>1)}{650} \right) \left( \frac{v_i}{10 \text{ km s}^{-1}} \right) \\ &\quad \times \left( \frac{H_c}{100 \text{ pc}} \right)^{-1} \left( \frac{R_{sf}}{15 \text{ kpc}} \right)^{-2} \left( \frac{t_i}{18.5 \text{ Myr}} \right)^{-1} \end{aligned}$$

(note: different numbers by Matzner 2002)

(from Mac Low & Klessen, 2004)

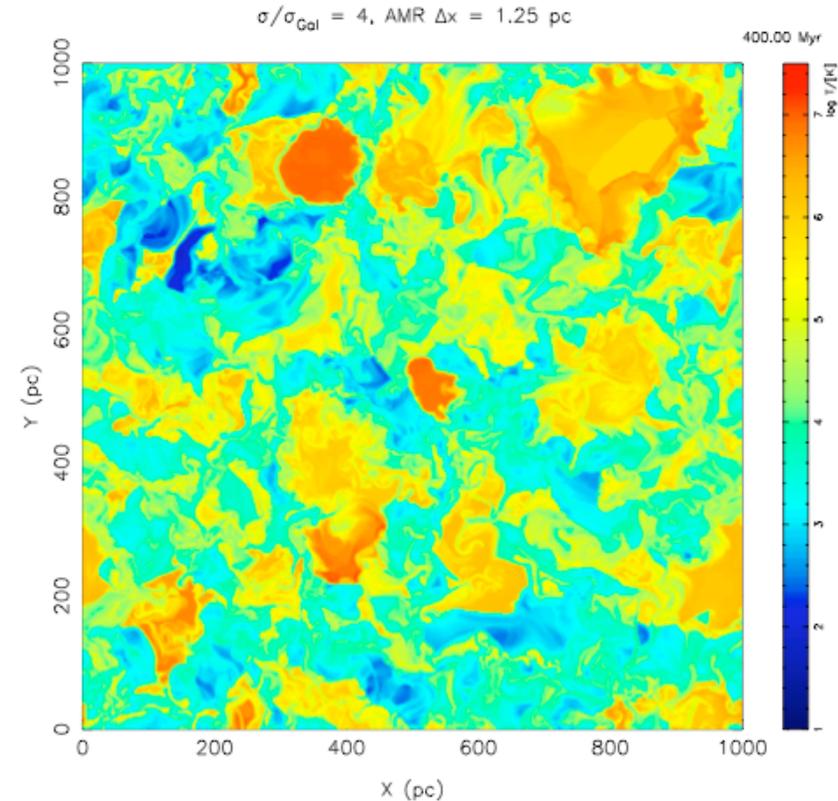
# what drives ISM turbulence?

supernovae

$$\begin{aligned}\dot{e} &= \frac{\sigma_{SN} \eta_{SN} E_{SN}}{\pi R_{sf}^2 H_c} \\ &= (3 \times 10^{-26} \text{ erg s}^{-1} \text{ cm}^{-3}) \left( \frac{\eta_{SN}}{0.1} \right) \left( \frac{\sigma_{SN}}{1 \text{ SNU}} \right) \\ &\quad \times \left( \frac{H_c}{100 \text{ pc}} \right)^{-1} \left( \frac{R_{sf}}{15 \text{ kpc}} \right)^{-2} \left( \frac{E_{SN}}{10^{51} \text{ erg}} \right).\end{aligned}$$

in star-forming parts of the disk,  
clearly SN provide enough energy  
to compensate for the decay of  
ISM turbulence.

**BUT:** what is outside the disk?



(distribution of temperature in SN driven disk turbulence, by  
de Avillez & Breitschwerdt 2004)

# accretion driven turbulence

- yet another thought:
  - astrophysical objects *form* by *accretion* of ambient material
  - the *kinetic energy* associated with this process is a key agent *driving internal turbulence*.
  - this works on *ALL* scales:
    - galaxies
    - molecular clouds
    - protostellar accretion disks

# concept

- turbulence decays on a crossing time

$$\tau_d \approx \frac{L_d}{\sigma}$$

- energy decay rate

$$\dot{E}_{\text{decay}} \approx \frac{E}{\tau_d} = -\frac{1}{2} \frac{M\sigma^3}{L_d}$$

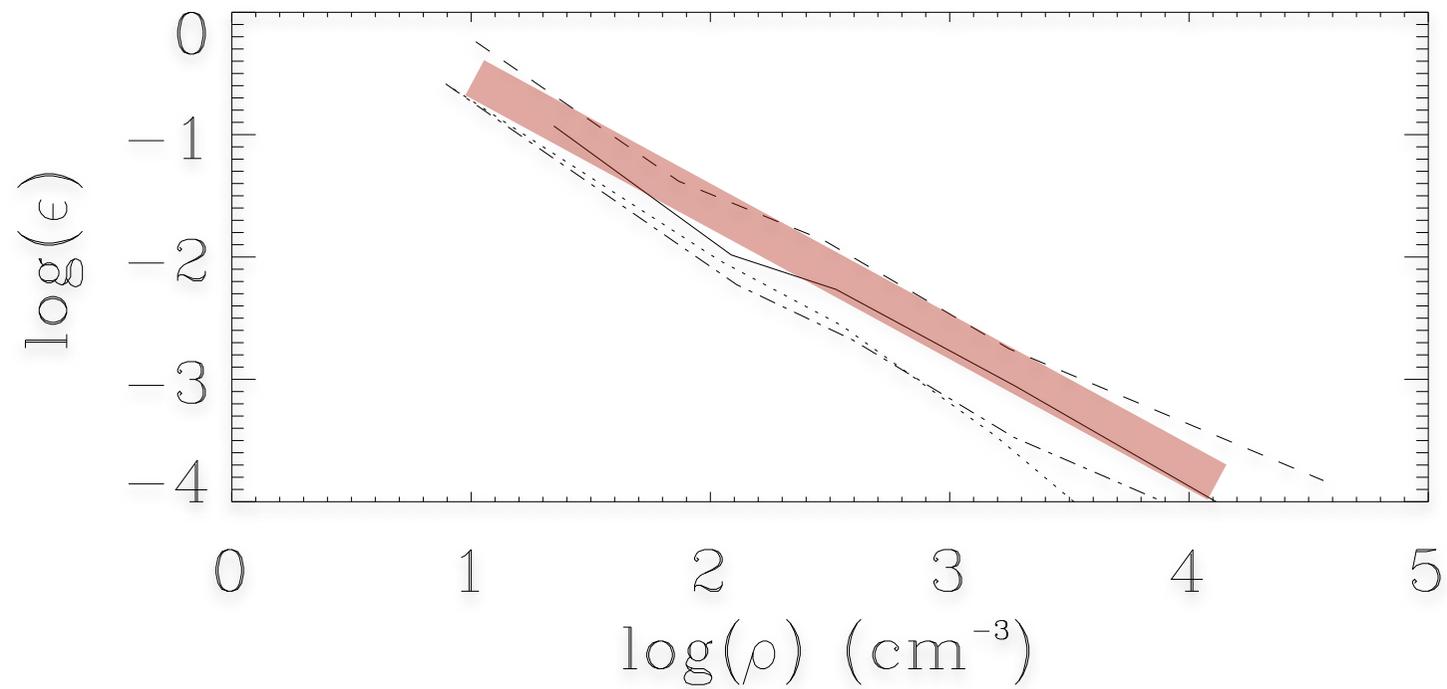
- kinetic energy of infalling material

$$\dot{E}_{\text{in}} = \frac{1}{2} \dot{M}_{\text{in}} v_{\text{in}}^2$$

- can both values match, modulo some efficiency?

$$\epsilon = \left| \frac{\dot{E}_{\text{decay}}}{\dot{E}_{\text{in}}} \right|$$

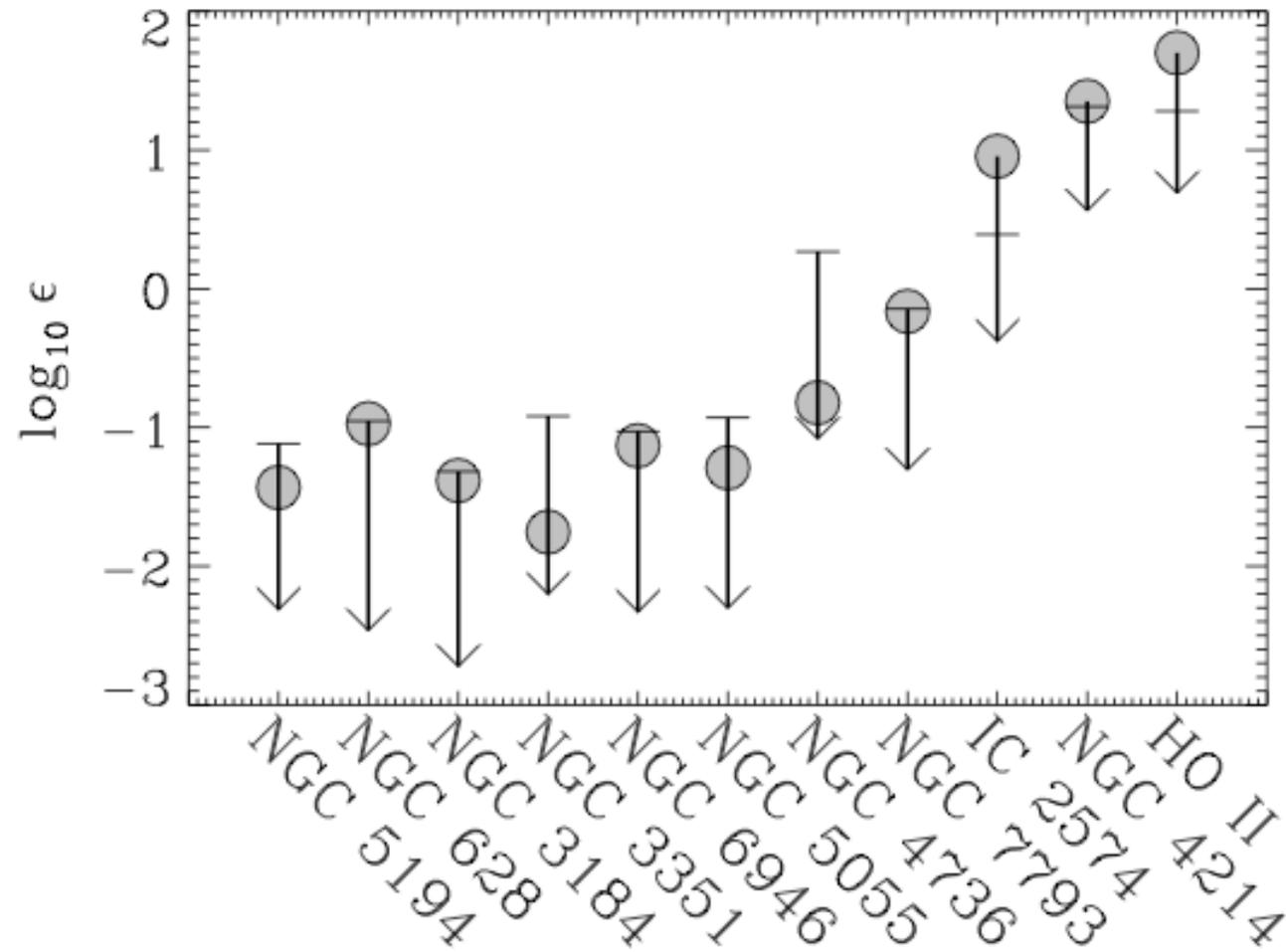
some estimates from convergent flow studies



# application to galaxies

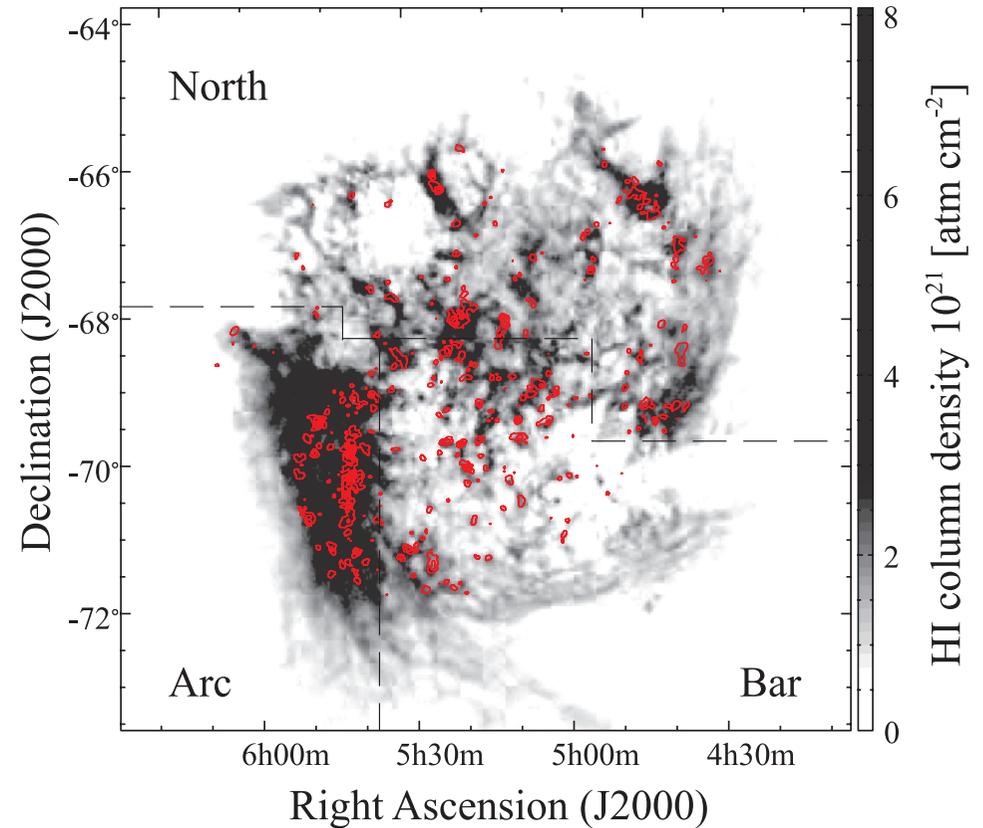
- underlying assumption
  - galaxy is in steady state
    - > accretion rate equals star formation rate
  - what is the required efficiency for the method to work?
- study Milky Way and 11 THINGS
  - excellent observational data in HI:  
velocity dispersion, column density, rotation curve

# 11 THINGS galaxies



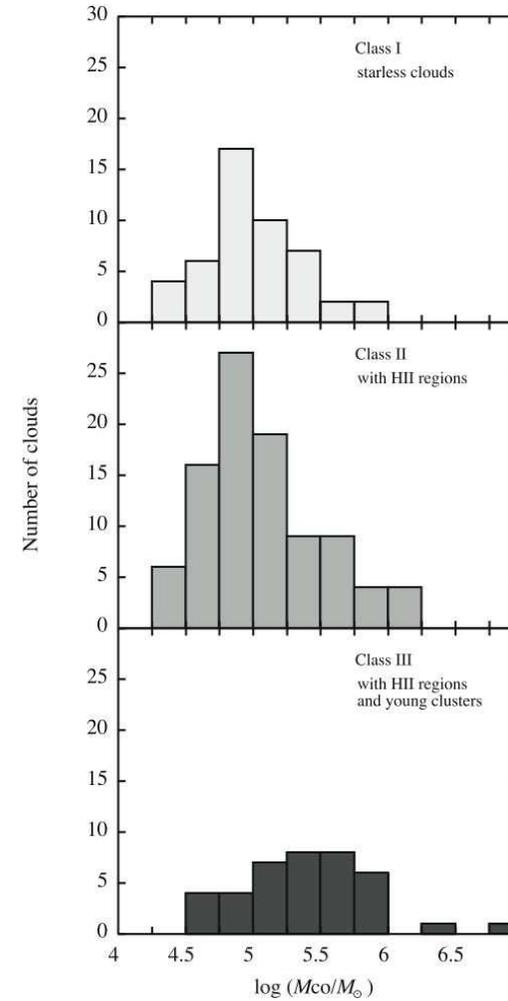
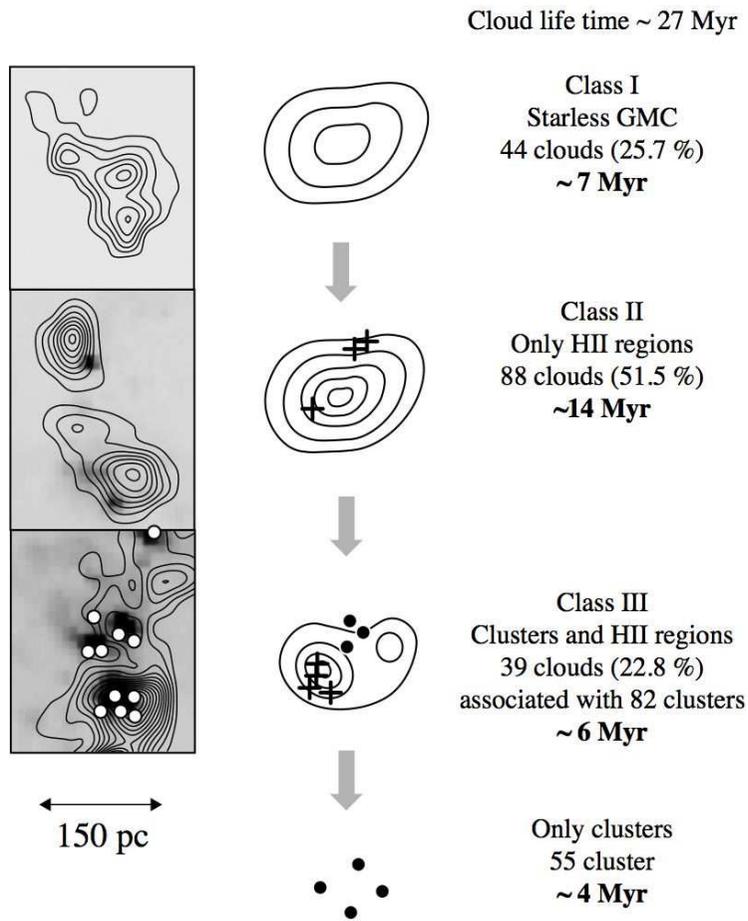
# molecular cloud scales

- molecular clouds grow in mass
- this is inferred by looking at molecular clouds in different evolutionary phases in the LMC (Fukui et al. 2008, 2009)



Fukui et al. (2009)

# molecular cloud scales



Blitz et al. (2007, PPV)

## some further thoughts

- method works for Milky Way type galaxies:
  - required efficiencies are  $\sim 1\%$  only!
- relevant for outer disks (extended HI disks)
  - there are not other sources of turbulence (certainly not stellar sources, maybe MRI)
- works well for molecular clouds
  - example clouds in the LMC (Fukui et al.)
- potentially interesting for TTS
  - model reproduces  $dM/dt - M$  relation (e.g Natta et al. 2006, Muzerolle et al. 2005, Muhanty et al. 2005, Calvet et al. 2004, etc.)

end

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Hokusai: In the wake of the great wave of Tanakawa