Subgrid Scale Physics in Galaxy Simulations

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CRC 963 Astrophysical Turbulence and Flow Instabilities

with thanks to

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yt-project.org

Galactic Scale Star Formation, Heidelberg, July/August 2012
Objective

- There are many star formation and feedback recipes for simulations (Robertson & Kravtsov 2008, Agertz et al. 2009, Tasker & Tan 2009, Bournaud et al. 2010, Dobbs & Pringle 2010, Governato et al. 2010, Greif et al. 2010, etc.)

- We do not aim at galaxy simulations in a static environment with resolution $\lesssim 1\, \text{pc}$:
  - want to study galaxies in their fully dynamical cosmological environment, including galactic outflows
  - apply a subgrid scale model for the multiphase turbulent ISM (Braun & WS 2012)
  - simulations of isolated disk galaxies mainly serve as a testing case for the model
A Simple Two-Phase Model

Split mass contents of grid cells into cold and warm phases with average densities $\rho_{c,pa} = m_c/V_c$ and $\rho_{w,pa} = m_w/V_w$ (Springel & Hernquist '03):
Effective Pressure Equilibrium

- Basic assumption: two-phase structure given by generalized virial theorem for ensemble of cold-gas clouds embedded in the warm medium:

\[
3P_{c,\text{eff}} = \frac{\pi}{10} G \rho_{c,\text{pa}}^2 l_c^2 - 3P_{w,\text{eff}} \approx 0
\]

- Effective pressure \( P_{c,\text{eff}} = \rho_{c,\text{pa}}\sigma_{c,\text{eff}}^2 \) (Chandrasekhar 1951):

\[
\sigma_{c,\text{eff}}^2 = \frac{c_s^2}{\gamma} + \sigma_{c,\text{turb}}^2 = \gamma(\gamma - 1)e_c \left( \frac{1}{\gamma} + \frac{1}{3} \mathcal{M}_{c,\text{turb}}^2 \right)
\]

- If the bulk of the cold gas is not strongly self-gravitating, then \( P_{c,\text{eff}} \approx P_{w,\text{eff}} \) implies

\[
\frac{\rho_{c,\text{pa}}}{\rho_{w,\text{pa}}} = \frac{\sigma_{w,\text{eff}}^2}{\sigma_{c,\text{eff}}^2}
\]
Star Formation Model

- Cold gas is converted into star particles at a rate

\[ \dot{\rho}_s = \epsilon_{\text{core}} \frac{\text{SFR}_{\text{ff}} f_{\text{H}_2} \rho_c}{t_{\text{c,ff}}} , \quad \text{where } t_{\text{c,ff}} = \left( \frac{3\pi}{32 \, G \rho_{c,\text{pa}}} \right)^{1/2} \]

- Molecular gas fraction \( f_{\text{H}_2} = m_{\text{H}_2}/(\rho_c \Delta^3) \) is determined by a Strömgren-like approach similar to Krumholz et al. 2009

- Dimensionless star formation rate per free fall time is given by (Padoan & Nordlund 2011)

\[ \text{SFR}_{\text{ff}} = \int_{x_{\text{crit}}}^{\infty} x p(x) \, dx , \quad \text{where } x_{\text{crit}} \approx 0.037 \frac{15 \sigma^2_{c,\text{turb}}}{\pi G \rho_{c,\pa} l^2_{c,\text{turb}}} \mathcal{M}^2_{c,\text{turb}} \]

- Turbulent density PDF \( p(x) \) is assumed to be log-normal with variance (Federrath et al. 2010)

\[ \sigma^2 \approx \ln \left(1 + b^2 \mathcal{M}^2_{c,\text{turb}}\right) , \quad \text{where } b = 1/3 \, (\text{soln.}) \text{ or } 1 \, (\text{compr.}) \]
Composite optical HST and Chandra X-ray image of supernova 1987a
Supernova Feedback Model

- Supernova rate is determined by the star formation rate and the Chabrier (2001) fit to the IMF:

\[
\dot{\rho}_{s,fb}(t) = \int_{t_b}^{t_e} \dot{\rho}_s(t - t') \text{IMF}(m_*) \frac{dm_*}{dt'} dt',
\]

- Increase of warm-gas thermal energy due to heating and cold-gas evaporation (McKee & Ostriker 1977):

\[
\left. \frac{d(\rho_w e_w)}{dt} \right|_{SN} = [(1 - \epsilon_{SN}) e_{SN} + A e_c] \dot{\rho}_{s,fb}, \quad \text{where} \ e_{SN} \approx 6 \cdot 10^{49} \text{erg}/M_\odot
\]

- Production of turbulent pressure \( P_{turb} = \frac{2}{3} \rho K \):

\[
\left. \frac{d(\rho K)}{dt} \right|_{SN} = \epsilon_{SN} e_{SN} \dot{\rho}_{s,fb}, \quad \text{where} \ \epsilon_{SN} \approx 0.085
\]
The Euler Equations with Subgrid-Scale Dynamics

Couple Euler equations for resolved flow variables to unresolved turbulence energy $\rho K$ such that $\rho(E + K)$ is conserved:

$$
\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho \mathbf{u}) = 0
$$

$$
\frac{\partial}{\partial t} (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = - \nabla \left( P + \frac{2}{3} \rho K \right) + \nabla \cdot \tau_{\text{sgs}}^* + \rho (\mathbf{g} + f_{\text{ext}})
$$

$$
\frac{\partial}{\partial t} \rho E + \nabla \cdot (\rho \mathbf{u} E) = - \nabla \cdot \left[ \mathbf{u} \left( P + \frac{2}{3} \rho K \right) \right] + \nabla \cdot (\mathbf{u} \cdot \tau_{\text{sgs}}^*) + \rho \mathbf{u} \cdot (\mathbf{g} + f_{\text{ext}}) - \Lambda + \Gamma - \Sigma + \rho \epsilon
$$

$$
\frac{\partial}{\partial t} \rho K + \nabla \cdot (\rho \mathbf{u} K) = \mathcal{D} + \Sigma - \rho \epsilon
$$

(Schmidt et al. 2006)
Closures for the Compressible Turbulent Cascade

- Production rate of SGS turbulence energy $\Sigma = \tau_{ij} S_{ij}$, where (Woodward et al. 2006, WS & Federrath 2011)

$$\tau_{ij} = C_1 \Delta \rho K^{1/2} S_{ij}^* - 2C_2 \rho K \frac{2u_{i,k} u_{j,k}}{\nabla \otimes u^2} - \frac{2}{3} (1 - C_2) \rho K \delta_{ij}$$

- $\forall C_1, C_2$: turbulent pressure given by

$$P_{\text{turb}} = \frac{2}{3} \rho K = -\frac{1}{3} \tau_{ii}$$

- Closure coefficients $C_1 \approx 0.02$ and $C_2 \approx 0.7$ depend only little on the Mach number in the supersonic regime

- Turbulent dissipation rate $\epsilon = C_\epsilon K^{3/2} / \Delta$
LES of Supersonic Turbulence on Nested Grids

Vorticity modulus $\omega$

SGS turbulence energy $\rho K$

- Forced turbulence in a periodic box ($128^3$ root grid)
- Cooling $\mathcal{L} = \chi \rho (e - e_0)$ keeps int. energy roughly constant
- Statistically stationary state after $\sim 2$ integral time scales
LES of Supersonic Turbulence on Nested Grids

\( \mathcal{M}_{\text{rms}} \approx 5 \)

- Turbulent Mach numbers: ratio of resolved/unresolved kinetic to thermal energies
- SGS turbulence energy scales down with refinement level
- Variables from higher levels are averaged down to coarser cells and energy correction is applied!
The Role of Subgrid Scale Turbulence

\[ M_{c, \text{turb}} = \sqrt{3} \sigma_{c, \text{turb}} / c_c, \text{ where } 3\sigma_{c, \text{turb}}^2 = 2K(\lambda_{J,c}/\Delta)^{2\eta} \]
SGS Turbulence Energy Equation for Galaxy Simulations

\[
\frac{\partial}{\partial t} \rho K + \nabla \cdot (\rho u K) = \mathcal{Q} + \epsilon_{SN} e_{SN} \dot{\rho}_{s,fb} + (1 - f_{th}) \epsilon_{tt} \Lambda_{\text{eff}} \rho_w \\
+ (\tau^{*}_{ij})_{\text{sgs}} S_{ij} - \frac{2}{3} \rho Kd - \rho C_{\epsilon} \frac{K^{3/2}}{\Delta}
\]

Internal driving:

- Production by supernova feedback \( \propto e_{SN} \dot{\rho}_{s,fb} \)
- Production by thermal instability \( \propto \rho_w \Lambda_{\text{eff}} \), where \( \Lambda_{\text{eff}} = \Lambda_{\text{rad}} - \Gamma_{\text{PAH}} - \Gamma_{\text{Lyc}} - \epsilon \) is the effective cooling rate

External driving:

- Production through turbulent cascade from length scales \( L \gg \ell \)
- Coupling to resolved turbulence driven by gravity and shear of the disk
The Cosmological Fluid Dynamics Code Nyx

Initiators: Jens Niemeyer (IAG), Peter Nugent (LBNL)

- Boxlib framework for block-structured AMR
- Hybrid OMP/MPI parallelization for up to \( \sim 100000 \) cores
- Unsplit-PPM hydro solver
- Multi-grid Poisson solver for self-gravity
- PM treatment of dark matter/star particles
- CLOUDY cooling
- SGS model for turbulent multiphase ISM
Initialization: Stable Adiabatic disk

IC: stable rotating disk with \((r, \theta)\)-profile of Wang et al. 2010 and initial temperature \(4 \times 10^4\) K

Static DM halo, \(10^{10}\ M_\odot\) baryons \((Z/Z_\odot = 0.1)\) in a 1 Mpc\(^3\) box

Development runs: \(128^3\) root grid, 8 refinement levels (30 pc resolution)
Evolution of Star Formation and Feedback

![Graph showing the evolution of star formation and feedback rate over time in Myr. The x-axis represents time in Myr, ranging from 0 to 300, and the y-axis represents the rate in $\text{Msun/yr}$. The graph includes two curves: red for star formation and green for feedback. There are peaks and troughs in both curves, indicating fluctuations in the rates. The graph illustrates the interplay between star formation and feedback processes in galaxy simulations.]
Star Formation (300 Myr)

Local star formation rate $\dot{\rho}_s$ vs. $\rho$

Local star formation rate $\dot{\rho}_s$ vs. $\rho_{H_2}$
Cold gas and Turbulence (300 Myr)

Fraction of cold gas $\rho_c/\rho$ vs. $\rho$

Typical star formation efficiency $\epsilon_{\text{ff}} \sim 0.01$ for $1 < M_{\text{sgs}} < 10$
Warm-Gas Fraction and Stars (300 Myr)

Fraction of warm gas $\rho_w / \rho$

Stellar mass in $M_\odot / pc^3$
Conclusions

- Cooling and gravity initially form large massive clumps in a gas-dominated adiabatically stable disk
  - cannot be prevented by feedback, although feedback subsequently strips off gas
  - clumps may merge and migrate toward the center to form a bulge or be torn apart and further fragment
- Turbulence-regulated star formation saturates quickly at a few solar masses per year
- Thermal and turbulent feedback pressurizes disk and drives galactic outflows