Subgrid Scale Physics in Galaxy Simulations

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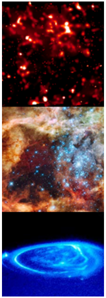
CRC 963 Astrophysical Turbulence and Flow Instabilities

Institut für Astrophysik Göttingen



with thanks to

Harald Braun, **Jan Frederic Engels**, **Jens Niemeyer**, IAG
Ann Almgren and **John Bell**, LBNL
Christoph Federrath, Monash University
yt-project.org

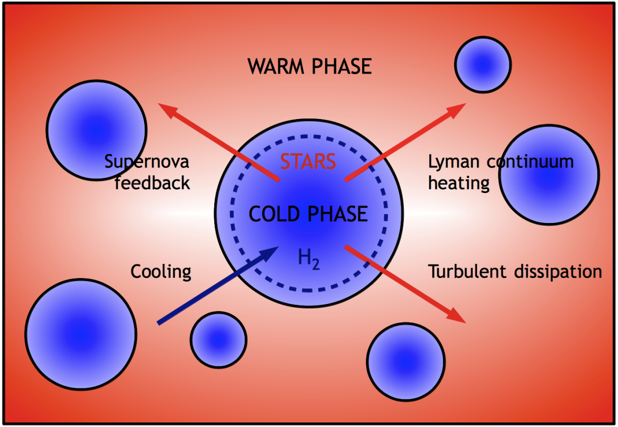
Galactic Scale Star Formation, Heidelberg, July/August 2012

Objective

- There are many star formation and feedback recipes for simulations (Robertson & Kravtsov 2008, Agertz et al. 2009, Tasker & Tan 2009, Bournaud et al. 2010, Dobbs & Pringle 2010, Governato et al. 2010, Greif et al. 2010, etc.)
- We do not aim at galaxy simulations in a static environment with resolution $\lesssim 1\,\mathrm{pc}$:
  - want to study galaxies in their fully dynamical cosmological environment, including galactic outflows
  - apply a subgrid scale model for the multiphase turbulent ISM (Braun & WS 2012)
  - simulations of isolated disk galaxies mainly serve as as a testing case for the model

A Simple Two-Phase Model

Split mass contents of grid cells into cold and warm phases with average densities $\rho_{\mathrm{c,pa}} = m_{\mathrm{c}}/V_{\mathrm{c}}$ and $\rho_{\mathrm{w,pa}} = m_{\mathrm{w}}/V_{\mathrm{w}}$ (Springel & Hernquist '03):



Effective Pressure Equilibrium

- Basic assumption: two-phase structure given by generalized virial theorem for ensemble of cold-gas clouds embedded in the warm medium:

$$\underbrace{3P_{\mathrm{c,eff}}}_{\text{int. + kin.}} - \underbrace{\frac{\pi}{10} G\rho_{\mathrm{c,pa}}^2 l_{\mathrm{c}}^2}_{\text{grav.}} - \underbrace{3P_{\mathrm{w,eff}}}_{\text{ext.}} \simeq 0$$

- Effective pressure $P_{\mathrm{c,eff}} = \rho_{\mathrm{c,pa}}\sigma_{\mathrm{c,eff}}^2$ (Chandrasekhar 1951):

$$\sigma_{\mathrm{c,eff}}^2 = \frac{c_{\mathrm{c}}^2}{\gamma} + \sigma_{\mathrm{c,turb}}^2 = \gamma(\gamma-1)e_{\mathrm{c}}\left(\frac{1}{\gamma} + \frac{1}{3}\mathcal{M}_{\mathrm{c,turb}}^2\right)$$

- If the bulk of the cold gas is not strongly self-gravitating, then $P_{\mathrm{c,eff}} \simeq P_{\mathrm{w,eff}}$ implies

$$\frac{\rho_{\mathrm{c,pa}}}{\rho_{\mathrm{w,pa}}} = \frac{\sigma_{\mathrm{w,eff}}^2}{\sigma_{\mathrm{c,eff}}^2}$$

Star Formation Model

- Cold gas is converted into star particles at a rate

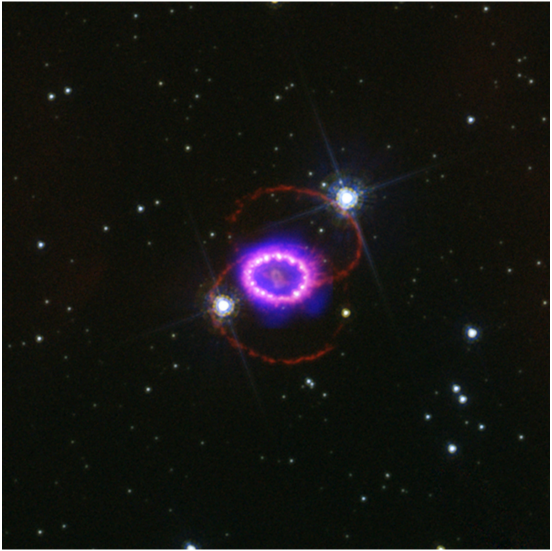
$$\dot{\rho}_{\mathrm{s}} = \epsilon_{\mathrm{core}} \frac{\mathrm{SFR}_{\mathrm{ff}} f_{\mathrm{H}_2} \rho_{\mathrm{c}}}{t_{\mathrm{c,ff}}}, \quad \text{where } t_{\mathrm{c,ff}} = \left(\frac{3\pi}{32 G \rho_{\mathrm{c,pa}}}\right)^{1/2}$$

- Molecular gas fraction $f_{\mathrm{H}_2} = m_{\mathrm{H}_2}/(\rho_{\mathrm{c}}\Delta^3)$ is determined by a Strömgren-like approach similar to Krumholz et al. 2009
- Dimensionless star formation rate per free fall time is given by (Padoan & Nordlund 2011)

$$\mathrm{SFR}_{\mathrm{ff}} = \int_{x_{\mathrm{crit}}}^{\infty} x p(x)\,\mathrm{d}x, \quad \text{where } x_{\mathrm{crit}} \approx 0.037\, \underbrace{\frac{15\sigma_{\mathrm{c,turb}}^2}{\pi G \rho_{\mathrm{c,pa}} l_{\mathrm{c}}^2}}_{\alpha_{\mathrm{vir}}} \mathcal{M}_{\mathrm{c,turb}}^2$$

- Turbulent density PDF $p(x)$ is assumed to be log-normal with variance (Federrath et al. 2010)

$$\sigma^2 \approx \ln\left(1 + b^2 \mathcal{M}_{\mathrm{c,turb}}^2\right), \quad \text{where } b = 1/3 \text{ (soln.) or } 1 \text{ (compr.)}$$



Composite optical HST and Chandra X-ray image of supernova 1987a

Supernova Feedback Model

- Supernova rate is determined by the star formation rate and the Chabrier (2001) fit to the IMF:

$$\dot{\rho}_{\mathrm{s,fb}}(t) = \int_{t_{\mathrm{b}}}^{t_{\mathrm{e}}} \dot{\rho}_{\mathrm{s}}(t - t')\,\mathrm{IMF}(m_*)\frac{\mathrm{d}m_*}{\mathrm{d}t'}\,\mathrm{d}t',$$

- Increase of warm-gas thermal energy due to heating and cold-gas evaporation (McKee & Ostriker 1977):

$$\left.\frac{\mathrm{d}(\rho_{\mathrm{w}} e_{\mathrm{w}})}{\mathrm{d}t}\right|_{\mathrm{SN}} = [(1-\epsilon_{\mathrm{SN}})e_{\mathrm{SN}} + Ae_{\mathrm{c}}]\dot{\rho}_{\mathrm{s,fb}}, \quad \text{where } e_{\mathrm{SN}} \approx 6\cdot 10^{49}\,\mathrm{erg}/M_{\odot}$$

- Production of turbulent pressure $P_{\mathrm{turb}} = \frac{2}{3}\rho K$:

$$\left.\frac{\mathrm{d}(\rho K)}{\mathrm{d}t}\right|_{\mathrm{SN}} = \epsilon_{\mathrm{SN}}\, e_{\mathrm{SN}}\, \dot{\rho}_{\mathrm{s,fb}}, \quad \text{where } \epsilon_{\mathrm{SN}} \approx 0.085$$

The Euler Equations with Subgrid-Scale Dynamics

Couple Euler equations for resolved flow variables to unresolved turbulence energy ρK such that $\rho(E+K)$ is conserved:

$$\frac{\partial}{\partial t}\rho + \nabla\cdot(\mathbf{u}\rho) = 0$$

$$\frac{\partial}{\partial t}(\rho\mathbf{u}) + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u}) = -\nabla\underbrace{\left(P+\frac{2}{3}\rho K\right)}_{\text{eff. pressure}} + \underbrace{\nabla\cdot\boldsymbol{\tau}^{*}_{\mathrm{sgs}}}_{\text{nondiag. stresses}} + \rho(\mathbf{g}+\mathbf{f}_{\mathrm{ext}})$$

$$\frac{\partial}{\partial t}\rho E + \nabla\cdot(\rho\mathbf{u}E) = -\nabla\cdot\left[\mathbf{u}\left(P+\frac{2}{3}\rho K\right)\right] + \nabla\cdot(\mathbf{u}\cdot\boldsymbol{\tau}^{*}_{\mathrm{sgs}}) + \rho\mathbf{u}\cdot(\mathbf{g}+\mathbf{f}_{\mathrm{ext}}) \underbrace{-\Lambda+\Gamma}_{\text{radiative}} \underbrace{-\Sigma+\rho\epsilon}_{\text{turbulent}}$$

$$\frac{\partial}{\partial t}\rho K + \nabla\cdot(\rho\mathbf{u}K) = \mathfrak{D} + \Sigma - \rho\epsilon$$

(Schmidt et al. 2006)

Closures for the Compressible Turbulent Cascade

- Production rate of SGS turbulence energy $\Sigma = \tau_{ij} S_{ij}$, where (Woodward et al. 2006, WS & Federrath 2011)

$$\tau_{ij} = \underbrace{C_1 \Delta \rho K^{1/2} S_{ij}^*}_{\text{linear eddy-viscosity part}} - \underbrace{2 C_2 \rho K \frac{2 u_{i,k} u_{j,k}}{|\nabla \otimes \mathbf{u}|^2}}_{\text{non-linear part}} - \frac{2}{3}(1 - C_2)\rho K \delta_{ij}$$

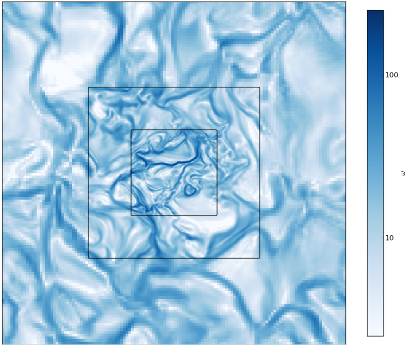
- $\forall C_1, C_2$: turbulent pressure given by

$$P_{\mathrm{turb}} = \frac{2}{3}\rho K = -\frac{1}{3}\tau_{ii}$$

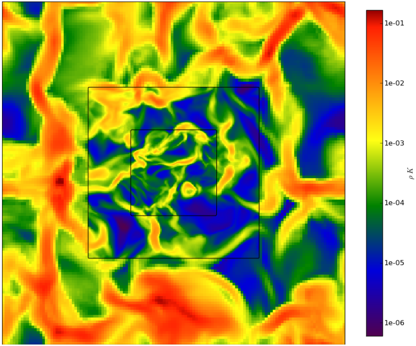
- Closure coefficients $C_1 \approx 0.02$ and $C_2 \approx 0.7$ depend only little on the Mach number in the supersonic regime
- Turbulent dissipation rate $\epsilon = C_\epsilon K^{3/2}/\Delta$

LES of Supersonic Turbulence on Nested Grids

Vorticity modulus ω



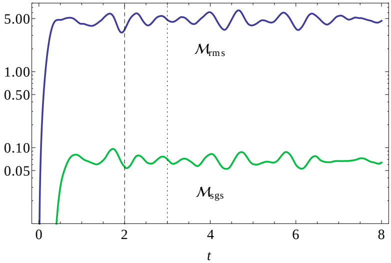
SGS turbulence energy ρK



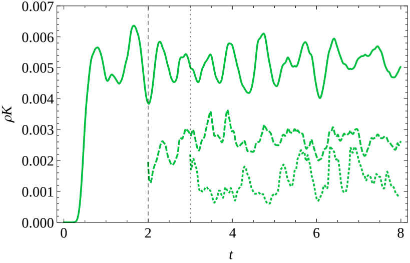
- Forced turbulence in a periodic box (128^3 root grid)
- Cooling $\mathcal{L} = \chi\rho(e - e_0)$ keeps int. energy roughly constant
- Statistically stationary state after ~ 2 integral time scales

LES of Supersonic Turbulence on Nested Grids

$\mathcal{M}_{\mathrm{rms}} \approx 5$



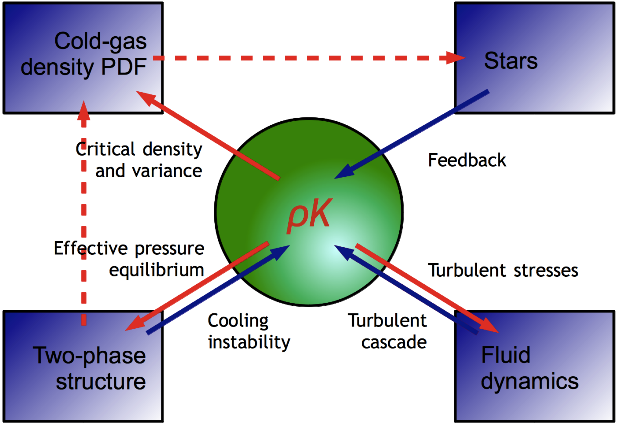
Mean SGS turbulence energy



- Turbulent Mach numbers: ratio of resolved/unresolved kinetic to thermal energies
- SGS turbulence energy scales down with refinement level
- Variables from higher levels are averaged down to coarser cells and energy correction is applied!

The Role of Subgrid Scale Turbulence

$$\mathcal{M}_{\mathrm{c,turb}} = \sqrt{3}\sigma_{\mathrm{c,turb}}/c_{\mathrm{c}}, \text{ where } 3\sigma_{\mathrm{c,turb}}^2 = 2K(\lambda_{\mathrm{J,c}}/\Delta)^{2\eta}$$



SGS Turbulence Energy Equation for Galaxy Simulations

$$\frac{\partial}{\partial t}\rho K + \nabla\cdot(\rho\mathbf{u}K) = \mathfrak{D} + \underbrace{\epsilon_{\rm SN} e_{\rm SN}\dot{\rho}_{\rm s,fb} + (1 - f_{\rm th})\epsilon_{\rm tt}\Lambda_{\rm eff}\rho_{\rm w}}_{\text{internal}} + \underbrace{(\tau^{*}_{ij})_{\rm sgs}S_{ij}}_{\text{external}} - \frac{2}{3}\rho K d - \rho C_{\epsilon}\frac{K^{3/2}}{\Delta}$$

Internal driving:

- Production by supernova feedback $\propto e_{\rm SN}\dot{\rho}_{\rm s,fb}$
- Production by thermal instability $\propto \rho_{\rm w}\Lambda_{\rm eff}$, where $\Lambda_{\rm eff} = \Lambda_{\rm rad} - \Gamma_{\rm PAH} - \Gamma_{\rm Lyc} - \epsilon$ is the effective cooling rate

External driving:

- Production through turbulent cascade from length sales $L \gg \ell$
- Coupling to resolved turbulence driven by gravity and shear of the disk

The Cosmological Fluid Dynamics Code Nyx

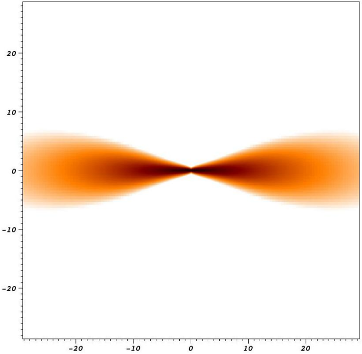
Initiators: Jens Niemeyer (IAG), Peter Nugent (LBNL)
Code paper: Almgren et al. (ApJ submitted)

BERKELEY LAB

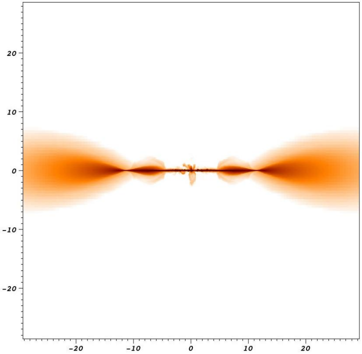
- Boxlib framework for block-structured AMR
- Hybrid OMP/MPI parallelization for up to $\sim$ 100000 cores
- Unsplit-PPM hydro solver
- Multi-grid Poisson solver for self-gravity
- PM treatment of dark matter/star particles
- CLOUDY cooling
- SGS model for turbulent multiphase ISM

Initialization: Stable Adiabatic disk

Inititalization:

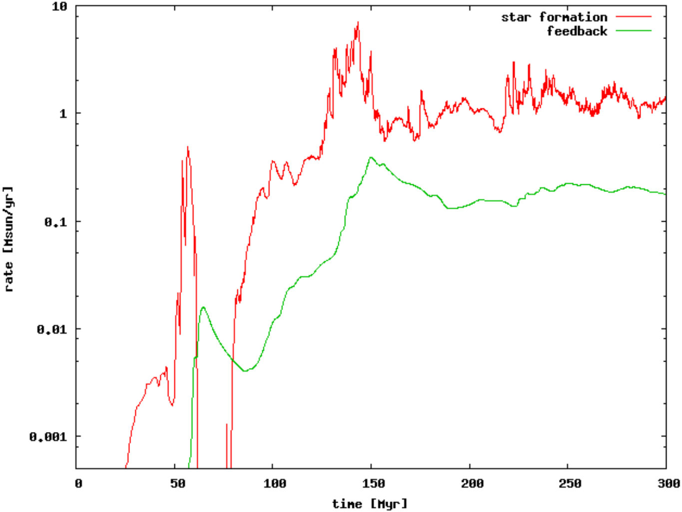


Collapse into a thin disk after 100 Myr:



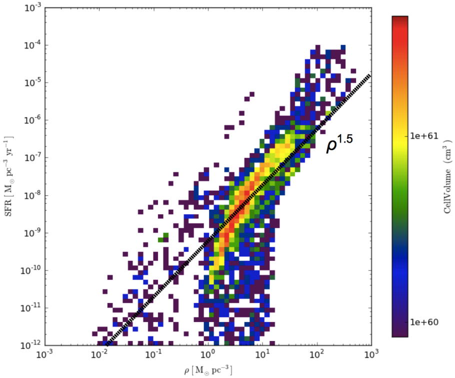
- IC: stable rotating disk with (r, θ)-profile of Wang et al. 2010 and initial temperature 4×10^4 K
- Static DM halo, $10^{10}\,\mathrm{M}_\odot$ baryons ($Z/Z_\odot = 0.1$) in a 1 Mpc^3 box
- Development runs: 128^3 root grid, 8 refinement levels (30 pc resolution)

Evolution of Star Formation and Feedback

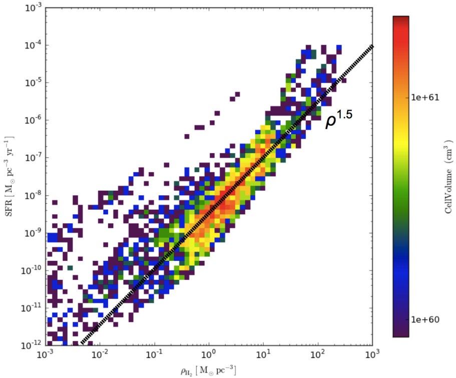


Star Formation (300 Myr)

Local star formation rate $\dot{\rho}_{\mathrm{s}}$ vs. ρ

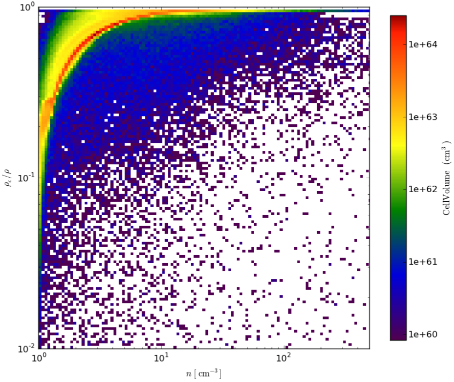


Local star formation rate $\dot{\rho}_{\mathrm{s}}$ vs. ρ_{H_2}

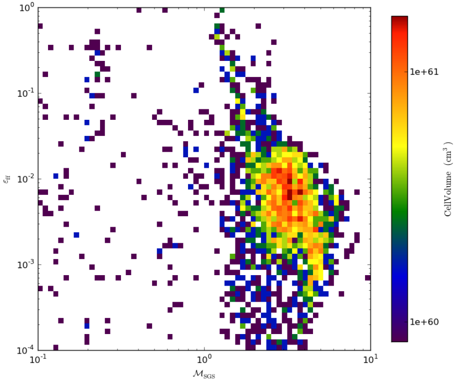


Cold gas and Turbulence (300 Myr)

Fraction of cold gas ρ_c/ρ vs. ρ

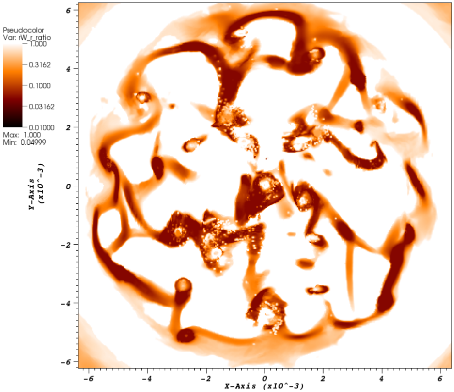


Typical star formation efficiency
$\epsilon_{\rm ff} \sim 0.01$ for $1 < \mathcal{M}_{\rm sgs} < 10$

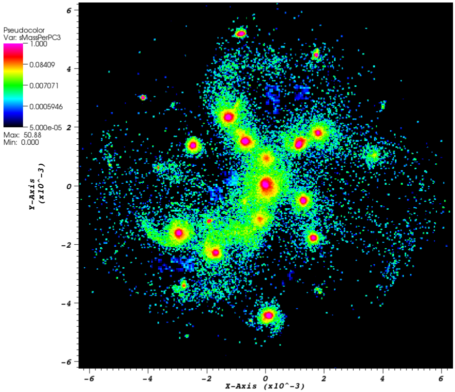


Warm-Gas Fraction and Stars (300 Myr)

Fraction of warm gas ρ_{w}/ρ



Stellar mass in $M_{\odot}/\mathrm{pc}^3$



Conclusions

- Cooling and gravity initially form large massive clumps in a gas-dominated adiabatically stable disk
  - cannot be prevented by feedback, although feedback subsequently strips off gas
  - clumps may merge and migrate toward the center to form a bulge or be torn apart and further fragment
- Turbulence-regulated star formation saturates quickly at a few solar masses per year
- Thermal and turbulent feedback pressurizes disk and drives galactic outflows