Exercises for Radiative Transfer in Astrophysics (WS2017)

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Exercise sheet 6

Spherical circumstellar dusty envelope model (part II)

We continue with the model of an optically thick spherically symmetric dusty cloud around a star of exercise sheet 5.

1. Optical depth of the envelope

In this exercise we are going to calculate the radial optical depth of the envelope (from the star out to infinity) for the case of rhodust0=1d-18 (i.e. $\rho_{\rm dust,0}=10^{-18}$ gram/cm³, the normal case). Let us first do this at a wavelength of around $\lambda \simeq 0.5$ μ m. Do this with the following steps:

- (a) Look in the table in the file dustkappa_silicate.inp and find the wavelength point closest to $\lambda \simeq 0.5 \ \mu m$. Take the corresponding κ_{abs} as the opacity. If you want to do an even better job (voluntary): take the two closest wavelength points and do a linear interpolation of κ_{abs} .
- (b) Write down the analytic form of the integral of the optical depth at $\lambda \simeq 0.5$ μm for this powerlaw density distribution, and solve it. You will obtain an expression for the optical depth at this wavelength, $\tau_{0.5}$, as a function of $\rho_{\rm dust,0}$. By inserting the value of $\rho_{\rm dust,0} = 10^{-18}$ you get the optical depth. This should, for $\rho_{\rm dust,0} = 10^{-18}$ gram/cm³, give a value around $\tau_{0.5} \simeq 580$.

If this were a grey envelope, then the optical depth of 580 would mean that radiation would have a very hard time to escape. A radiation package would get absorbed and re-emitted on average 580^2 times (note that an absorption + re-emission event is in some sense like a scattering event, so we can use what we've learned about the number of scatterings at some optical depth). However, the opacity table is definitely non-grey. Let us take this into account now.

- (c) The dust in the inner edge of the envelope has a temperature of about 70 K. The peak of the function $\nu B_{\nu}(T)$ is located at a frequency $\nu = 4kT/h$. What is the opacity at that frequency? If you take the opacity at that frequency, show that the optical depth is roughly around 6. How many absorption + re-emission events do you expect?
- (d) RADMC-3D, however, writes that on average the photons experience even fewer events (roughly 11). Any idea why? Tip: The temperatures at larger radii than the inner edge are lower than 70 K.

2. The SED

- (a) Use RADMC-3D to create the SED of this envelope covering all wavelength of interest (from the UV to the millimeter) and make a plot of this. The observer is at a distance of 100 parsec. Note: since RADMC-3D is optimized for 3-D transfer, it is not particularly fast for 1-D problems, so you may need to wait a few minutes before it is finished. Tip: Find in the manual of RADMC-3D how to make an SED. The file spectrum.out that RADMC-3D produces contains some header lines and then two columns. The first one is the wavelength, the second one is the flux (in erg/cm²/s/Hz) of the object when the observer is at a distance of 1 parsec.
- (b) Do the same, but now for an envelope with $\rho_{\rm dust,0}=10^{-21}~{\rm gram/cm^3}$ (i.e. a 1000x lower optical depth). Explain the difference with the previous one.