Cuscuton Dark Energy

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December 4th, 2008

Ref.:  
R. Durrer and R. Maartens  
Dark Energy and Modified Gravity  
N. Afshordi, D. J. H. Chung, M. Doran and G. Geshnizjani  
Cuscuton Cosmology: Dark Energy meets Modified Gravity
Overview

1. Introduction
2. Fundamental theories
3. Cuscuton and cosmology
4. Two examples
5. Summary
The need for dark energy

Why $\Lambda$

- CMB power spectrum
- Flat universe is clearly favoured → What is the rest?

Common realisations

- A cosmological constant
- A dynamical scalar field (quintessence, k-essence)
- Modified gravity (TeVeS, f(R), DGP, ...)
- Abandonment of FRW (Bubbles, voids, back-reaction...)

![Diagram showing the relationship between $\Omega_m$ and $\Omega_k$ with data points for BAO, SNe, and CMB.](Image)
A dark overview

Figure: Stolen from R. Durrer, she stole it from de Rham & Tolley (2008)
Fundamental physical theories

How do we want them to be?

- Allows for a mathematical description (Welcome to Physics)
- Lagrangian formulation
- Lorentz invariance
- No ghosts
- No tachyons
- Non-superluminal motion and causality
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Some examples

Scalar fields (spin 0)
\[ \mathcal{L} = -\frac{1}{2} \partial^\mu \phi \partial_\mu \phi + V(\phi) \]

QCD (spin 1/2 and spin 1)
\[ \mathcal{L} = i \bar{\psi}_i D_{ij} \psi_j - m_l \bar{\psi}_l \psi_l \]
\[ - \frac{1}{2} \text{TR}(F^{\mu\nu} F_{\mu\nu}) \]

Electrodynamics (spin 1)
\[ \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + J^\mu A_\mu \]

QED (spin 1/2 and spin 1)
\[ \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i \bar{\psi} \gamma^\mu \psi \\
- m \bar{\psi} \psi + e \bar{\psi} \gamma_\mu \gamma^\mu \psi A_\mu \]

General relativity (spin 2)
\[ \mathcal{L} = \frac{1}{16\pi G} R \sqrt{-g} \]
Different possibilities

After having constructed a "well-behaving" Lagrangian, one can construct:

Classical theories

- Variational principle delivers EOM’s
- Very elegant formalism
- No particle creation
- Examples: Maxwell’s theory, general relativity

Quantum theories

- Canonical quantisation or path-integral formalism delivers Feynman rules for the theory
- Mathematically difficult
- Allows particle creation (infinite degrees of freedom)
- Examples: The standard model, BCS theory

Full theory - straight line = quantum corrections
Different possibilities

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Mathematically difficult
Allows particle creation (infinite degrees of freedom)
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Dark energy and scalar fields

- Low energy effective field theories (QC protection)
- Described by Lagrangian:

\[ L_\phi = F(\phi, X) - V(\phi) \quad \text{where} \quad X \equiv -\frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi \]

- Field fluctuation propagate with the sound speed:

\[ c_s^2 = \frac{F_{,X}}{F_{,X} + 2XF_{,XX}} \]

- Quintessence, standard Lagrangian:

\[ F(\phi, X) = X \quad \text{with} \quad c_s^2 = 1 \]

- k-essence with non-standard kinetic term:

\[ \text{e.g. } F(\phi, X) = \phi^{-2} f(X) \quad \text{with an epoch during which} \quad c_s^2 > 1 \]
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These theories are meant to solve the coincidence problem but allow for superluminal motion and therefore violate causality.
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Defining an action

- One starts with a general k-essence Lagrangian

\[ S_\varphi = \int d^4x \sqrt{-g} \left[ \frac{1}{2} F(X, \varphi) - V(\varphi) \right] \]

- In the homogeneous limit \( F(X, \varphi(x)) \to F(\dot{\varphi}^2, \varphi(t)) \)
we want the field to lose its dynamics (kinetic term can be written as total derivative with \( \dot{\varphi} \neq 0 \))

\[ S_\varphi^{\text{homog.}} = - \int d^4x \sqrt{-g} V(\varphi) \]

- When coupled to another field we have not added an additional dynamical degree of freedom, but obtain only a constraint equation

\[ S_\chi = \int d^4x \left[ \mathcal{L}_\chi(\chi, \varphi) - V(\varphi) \right] \]
\[ \Rightarrow - \frac{\partial V}{\partial \varphi} + \frac{\mathcal{L}_\varphi(\chi, \varphi)}{\partial \varphi} = 0 \]
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  S_\phi = \int d^4x \sqrt{-g} \left[ \frac{1}{2} F(X, \varphi) - V(\varphi) \right]
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  \]
Cuscutoon action

\[(\text{Obviously})\]
\[S_\varphi = \int d^4x \sqrt{-g} \left[ \mu^2 \sqrt{|g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi|} - V(\varphi) \right]\]

does the job.

Properties (Afshordi, Chung and Geshnizjani 2006):

- \(c_s = \infty\)!
- Still, the Cuscutoon field is causal.
  (phase space volume of linear perturbations vanishes in the homogeneous limit)
- Hypersurfaces of constant \(\varphi\) have constant mean curvature
  (spacetime soap bubbles)
- Cuscutoon field is protected against quantum corrections at low energies.
Defining a name

- Cuscuta, dodder
- German: Seide, Teufelszwirn oder Kletterhur
- 100-170 species of parasitic plants
- wraps around host and connects to vascular system
- roots only exist in seed-phase but die when host is found
Why Cuscuton?

More "natural" than quintessence: Minimal model for evolving dark energy

- No additional, dynamical degree of freedom
- Just adds a constraint-equation, but still affects other fields
  ⇒ Might be observable

Could provide a viable low energy effective theory

- The cosmological constant for example is protected very bad against quantum corrections
  ⇒ fine-tuning problem
- Also many quintessence models suffer under this problem
Cuscuton cosmology

1. Constraints
   - Homogeneous Universe
   - Inhomogeneous Universe

2. A quadratic potential
   ($\sim \Lambda$CDM)

3. An exponential potential
   ($\sim$ DGP)

Figure: Stolen from N. Afshordi
We will use the reduced Planck mass $M_p = (8\pi G)^{-1/2}$

- Starting with a FRW metric and an homogeneous field configuration $ds^2 = dt^2 - a(t)^2 dx^i dx^i$ and $\varphi = \varphi(t)$,

  the Cuscuton action becomes

  $$ S = \int a^3 dt \left[ \mu^2 |\dot{\varphi}| - V(\varphi) \right] . $$

- Varying this with respect to $\varphi$ gives the field equation

  $$ (3\mu^2 H) \text{sgn}(\dot{\varphi}) + V'(\varphi) = 0 $$

- Remember the $^0_0$Einstein equation in an homogeneous fluid

  $$ H^2 = \frac{\rho_{\text{tot}}}{3M_p^2} $$

Julian Merten (ZAH/ITA)  Cuscuton DE  December 4th, 2008
Expansion history

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$$H^2 = \frac{\rho_{\text{tot}}}{3M_p^2}$$
What about $\rho$?

- Most general form of the energy momentum tensor for scalar fields $a$

$$T^\mu_\nu \equiv - \frac{\partial L(x)}{\partial (\partial_\mu \varphi_a(x))} \partial^\nu \varphi_a(x) + g^\mu\nu L(x)$$

- Fortunately we look at an homogeneous universe and only at the $0^0$ component of a single field

$$\rho = \frac{1}{2} \left( \frac{d\varphi}{dt} \right)^2 + V(\varphi) .$$

- And remember how Cuscuton was constructed

$$\rho = V(\varphi)$$

- This gives us

$$\left( \frac{M_p^2}{3 \mu^4} \right) V^2(\varphi) - V(\varphi) = \rho_m .$$
What about $\rho$?

- Most general form of the energy momentum tensor for scalar fields is given by:

$$T^\mu_\nu \equiv -\frac{\partial L(x)}{\partial(\partial_\mu \varphi_a(x))} \partial^\nu \varphi_a(x) + g^{\mu\nu} L(x)$$

- Fortunately, we look at an homogeneous universe and only at the $0^0$ component of a single field:

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Constraints on the potential

Now we can put constraints through $\rho_m$:

- **Weak energy condition**: $\rho_m > 0$

  $$V(\varphi) < \left( \frac{M_p^2}{3\mu^4} \right) V''(\varphi)$$

- **Null energy condition**:

  \[ \rho_m + p_m > 0 \]

  \[ V''(\varphi) = \frac{1}{2} \frac{dV'^2(\varphi)}{dV(\varphi)} > \frac{3\mu^4}{2M_p^2} \]
Scalar metric perturbations

We now look at a linearly perturbed FRW metric in the presence of dust.

1. The field equation becomes:

\[3\dot{\varphi}(\dot{\Phi} + H\Phi) + a^{-2}\nabla^2\delta\varphi - \mu^{-2}|\varphi|V''(\varphi)\delta\varphi = 0\]

giving for the field perturbations in Fourier space:

\[\delta\varphi = \frac{3\dot{\varphi}(\dot{\Phi} + H\Phi)}{k^2 a^2 - 3\dot{H}}\]

2. For the metric perturbations you find the Poisson equation:

\[\left(\frac{k^2}{a^2}\right)\Phi + \left[3H + \frac{9H(2\dot{H} + 3H^2\Omega_m)}{2\left(\frac{k^2}{a^2} - 3\dot{H}\right)}\right](\dot{\Phi} + H\Phi) + (2M_p^2)^{-1}\delta\rho_m = 0\]

3. \(\delta\varphi\) drops out and Cuscuton acts as a modification of gravity.
Another thing on modified gravity

Using the homogeneous field equations, we find:

$$H^2 = \frac{1}{3M_p^2} \left\{ \rho_m + V \left[ V'^{-1}(3\mu^2 H) \right] \right\},$$

where $V'^{-1}$ is the inverse function of $V'$.

This again shows, that Cuscuton can be interpreted as a modification of gravity.

Look at a quadratic potential:

$$V = \frac{1}{2}m^2\varphi^2.$$ 

Inserting this, we obtain again a Friedmann equation with a modified Planck mass:

$$M_p^2 \rightarrow M_p^2 - \frac{3\mu^4}{2m^2}.$$ 

In contrast to other models, we have not introduced an additional degree of freedom.
In addition to the change to the matter power spectrum, we have an induced CMB anisotropy due to the Fourier mode $\Phi_k$:

$$\Theta_{l,k} = \int_0^{\eta_0} d\eta \ g(\eta)(\Theta_0 + \Phi_k) \ j_l[k(\eta_0 - \eta)] + \frac{1}{ik} \int_0^{\eta_0} d\eta \ \nu_b(k) \ g(\eta) \frac{\partial}{\partial \eta} j_l[k(\eta_0 - \eta)] + 2 \int_0^{\eta_0} d\eta \ \frac{\partial \Phi_k}{\partial \eta} e^{-\tau(\eta)} j_l[k(\eta_0 - \eta)]$$

The ISW contribution to the CMB anisotropies can be computed perturbatively:

$$\frac{\partial \Phi_{(1),k}}{\partial \eta} = a_i \left( \frac{t(\eta)}{t_i} \right)^{-2} \int_{t_i}^{t(\eta)} dt' \left( \frac{t'}{t_i} \right)^{8/3} \ S_k(t') \quad S_k(t) \equiv - \left\{ 3V + \frac{3t \dot{V}}{4} + \frac{3}{k^2 a^2 + \frac{2}{t^2}} \left[ \frac{\dot{V}}{t} + \frac{\ddot{V}}{2t} + \left( \frac{\dot{V}}{2t} \right)^2 \frac{4k^2}{a^2} + \frac{4}{t^2} \right] \right\} \frac{\Phi_{(0),k}}{3M_p^3}$$
A quadratic potential I

\[ V(\varphi) = V_0 + \frac{1}{2}m^2\varphi^2 \]

- \(V_0\) is nothing else but a cosmological constant, but for the quadratic term we have:

\[ \Omega_Q = \frac{1}{2} \frac{m^2 \varphi^2}{\rho_{tot}} = \frac{3\mu^4}{2M_p^2 m^2} = \text{const.} \]

- This kind of tracking behaviour is identical to quintessence (EDE).
- Since the number of rel. neutrinos during radiation domination also does not change \(H(z)\) there is a degeneracy.
  \[ \Rightarrow \Omega_Q \lesssim 10\% \]
- Large scale structure and CMB are putting much tighter constraints:
  \[ \Rightarrow \Omega_Q < 1.6\% \] (best constraint on EDE to the authors’ knowledge)
A quadratic potential II

Figure: $\Omega_Q = 0, 0.05, 0.1$; solid, dotted, dashed
An exponential potential I

\[ V(\varphi) = V_0 \exp \left[ - \left( \frac{\mu^2 r_c}{M_P^2} \right) \varphi \right] \]

- Substituting into the homogeneous equations yields:

\[ H = \frac{1}{2r_c} + \sqrt{\frac{1}{4r_c^2} + \frac{\rho_m}{3M_P^2}} \]

which is exactly the DGP expansion history.

- As a result it is not possible to distinguish exponential Cuscuton from DGP with geometrical tests.

- Distinction is possible through e.g. ISW
An exponential potential II

\[ 1\left(1 + 1\right)\frac{C_l(\mu K^2)}{(2\pi)} \]

\[ 10^4 \]

\[ 1000 \]

\[ 100 \]

\[ l \]

\[ 5 \quad 10 \quad 15 \quad 20 \]
Successful theories seem to and should have a common foundation and justification.

Cuscuton DE is a minimal extension to a cosmological constant.

The effect of the Cuscuton field can be tested with cosmological experiments.

Cuscuton blurs the line between dark energy and modified gravity models. (argument stolen from N. Afshordi)

A quadratic potential mimics the tracking behaviour of quintessence models.

An exponential potential gives the expansion history of DGP, but a distinction is possible.