We perform a set of ray-tracing simulations, using numerical cluster models, aiming at evaluating how the galaxy cluster efficiency for producing strong lensing events changes in different cosmological models with dark energy. The sample of investigated clusters for which we present our results here is composed by 7 dark matter halos. Each of them was simulated in 8 different cosmological models with constant and time-variable equation of state of dark energy. For all the clusters in the sample, we have measured the lensing cross sections for producing giant arcs, i.e. arcs having a minimum length-to-width ratio. We find that the lensing cross section for giant arcs is sensitive to the equation of state of quintessence. Indeed, the optical depth, which can be translated into a number of arcs by multiplying by the correct density of source galaxy on the sky, spans more than one order of magnitude among different cosmological models.

Keywords: Cosmology; Galaxy Clusters; Gravitational Lensing

PACS Nos.: include PACS Nos.

1. Introduction
This research project aims at evaluating how dark energy affects the ability of galaxy clusters for producing strong lensing events. Previous analytic computations
and subsequent numerical N-body simulations showed that clusters are characterized by different concentration parameters in different dark energy cosmologies with constant and time-variable equation of state \(^1\).\(^2\).

In particular, numerical simulations show that the cluster concentration depends on the dark energy equation of state at the cluster formation redshift \(z_{\text{coll}}\) through the linear growth factor \(D_\Delta(z_{\text{coll}})\): density perturbations grow in a different way in cosmological models with different equation of state of dark energy, leading to different cluster formation epochs. The concentration reflects the mean value of the density parameter of the Universe when the halo formed. Therefore, halos in dark energy models with equation of state \(w > -1\) are found to have concentrations which interpolate between those of corresponding halos in standard ΛCDM and the OCDM models.

Since the strong lensing cross section of galaxy clusters is known to be very sensitive to the lens concentration, we expect large changes in the number of gravitational arcs which clusters in different cosmologies with dark energy are able to produce \(^2\). Therefore, quantifying the cluster optical depth for strong lensing as a function of the dark energy equation of state through numerical simulations could offer a new opportunity for constraining dark energy by comparing to the observed number of gravitational arcs in the sky.

2. Numerical models

The results we present here were obtained by making ray-tracing simulations with a sample of seven dark matter halos. Each of them was simulated in an open Cold Dark Matter (OCDM) and four flat dark-energy cosmologies. The latter are a cosmological constant (ΛCDM) model \((w = -1)\), a dark-energy model with constant equation of state (DECDM, \(w = -0.6\)), and two quintessence models, one with inverse power-law Ratra-Peebles potential \(^3\) (RP) and one with SUGRA potential \(^4\) (S). The present time value of the equation of state parameter \(w\), describing the ratio between the dark energy pressure and energy density, has been set equal to \(-0.83\) for the RP and S models. For the DECDM, RP, and S models, two sets of simulations were performed by normalizing the power spectrum of the primordial density perturbations on large scales, with the observed Cosmic Microwave Background (CMB) anisotropies (e.g. \(^5\)), or on small scales, using the observed cluster abundance, are shown. In the second case, we choose \(\sigma_8 = 0.9\) in all the models. Full descriptions of the numerical models and of the techniques used in the lensing simulations can be found in \(^1\) and \(^6\), \(^7\).

3. Lensing cross sections and optical depth

For each cluster we have measured the lensing cross sections for producing giant arcs, i.e. arcs having a minimum length-to-width ratio \(L/W\). The lensing cross section for any arc property is defined as the area on the source plane where a source must be placed in order to be imaged as an arc characterized by the requested property.
Our results are shown in Fig. 1. On the left panel we show the mean cross section of our cluster sample for arcs with length-to-width ratio larger than 7.5 as a function of the cluster redshift. Different curves refer to different cosmological models. Eight curves are shown here: the black and the blue curves refer to the OCDM and to the ΛCDM models, respectively. The red, the green and the cyan curves refer to the previously defined dark energy models, instead.

As expected the lensing cross sections reflect the differences in the concentration of dark matter halos in different cosmological models. Assuming the same normalization of the power spectrum, the lensing cross sections for the OCDM and the ΛCDM models differ by roughly a factor of six and the cross sections for the other cosmological models with dark energy interpolate between them. Despite the equation of state of dark energy today is the same for the RP and S models, their lensing cross sections differ significantly at higher redshift.

For models where the CMB normalization of the power spectrum is used, we find mean lensing cross sections smaller by more than one order of magnitude compared to the OCDM model. In fact, the normalization of the power spectrum derived from the CMB is generally slightly smaller than that obtained using the observed cluster abundance, because of the Integrated Sachs Wolfe (ISW) effect affecting the large scale CMB anisotropies in the cosmologies we consider. When adopting this normalization, the formation epoch of our simulated clusters is delayed and consequently they exhibit a smaller concentration.

The cluster sample is still too small for the mean cross section to be a smooth function of redshift. In fact, the curves exhibit strong peaks which are connected to merger events arising in single clusters. It has been recently shown that during such events, which occur on timescales of some Gyr, the cluster efficiency for strong
lensing is strongly enhanced, due to the combined effect of the increasing shear and convergence. It is interesting to note that, by comparing the cross sections as function of redshift for different cosmological models, there is a strict correspondence between the number of the peaks in the curves. This is obvious, since we are comparing the same clusters in all cosmological models. However the position and the amplitude of the peaks is strongly dependent on the cosmological model. Indeed the impact of a merger events appear to be larger in cosmologies where halos are less concentrated. Moreover, the different position of the peaks is produced by the different formation epoch and evolution of clusters, depending of the equation of state of dark energy. The shift of the peaks is more evident at higher redshift, since our sample is build such to obtain similar objects at redshift $z = 0$.

On the right panel of Fig. 1 we show the lensing optical depth for sources at redshift $z_s = 1$, i.e. the integral of the lensing cross section, weighted by the cluster mass function, between redshift $z = 0$ and $z = 1$, as a function of the normalization of the primordial power spectrum of the density fluctuations used for obtaining our numerical clusters. Reflecting the results shown for the lensing cross sections, our results confirm that the cluster strong lensing efficiency is very sensitive to the equation of state of dark energy. Indeed, the optical depth, which can be translated into a number of arcs by multiplying by the correct density of source galaxy on the sky, spans more than one order of magnitude among different cosmological models.

4. Conclusions

We used the ray-tracing technique for investigating how the strong lensing efficiency of a sample of 7 numerically simulated galaxy clusters change in several cosmological models with dark energy. Our main results are

- the lensing cross section for arc with a minimal length-to-width ratio is sensitive to the equation of state of dark energy. In particular, cosmological models with dark energy are found to interpolate between the standard $\Lambda$CDM and the OCDM models, reflecting the differences between the typical concentrations of dark matter halos in these cosmological models;
- the mean lensing cross sections of our sample exhibit strong peaks, corresponding to merger events occurring in single clusters. The position and the amplitude of these peaks is strongly dependent on the cosmological model.
- the optical-depths of our sample in different cosmologies span over more than one order of magnitude.

Based on these results, we conclude that quantifying the cluster optical depth for strong lensing as a function of the dark energy equation of state through numerical simulations could offer a new opportunity for constraining dark energy by comparing to the observed number of gravitational arcs in the sky. Moreover, since the lensing cross section is very sensitive to merger events arising in clusters and therefore can be used for tracing their time evolution, it should be possible to discriminate be-
between different cosmologies with dark energy by looking at the frequency of strong gravitational lenses at high redshift, where evolutionary differences between cosmological models are mostly evident.

Acknowledgments
This work has been partially supported by Italian MIUR (Grant 2001, prot. 2001028932, “Clusters and groups of galaxies: the interplay of dark and baryonic matter”), CNR and ASI.