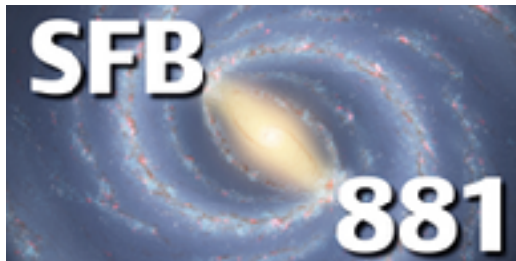


# Periodicity makes galactic shocks unstable

(mnras submitted)

**Mattia C. Sormani**

Zentrum für Astronomie der Universität Heidelberg  
Institut für Theoretische Astrophysik



# In collaboration with

**E. Sobacchi**



**Robin G. Treß**



UNIVERSITÄT  
HEIDELBERG  
ZUKUNFT  
SEIT 1385



**Steven N. Shore**



UNIVERSITÀ DI PISA



**Ralf S. Klessen**

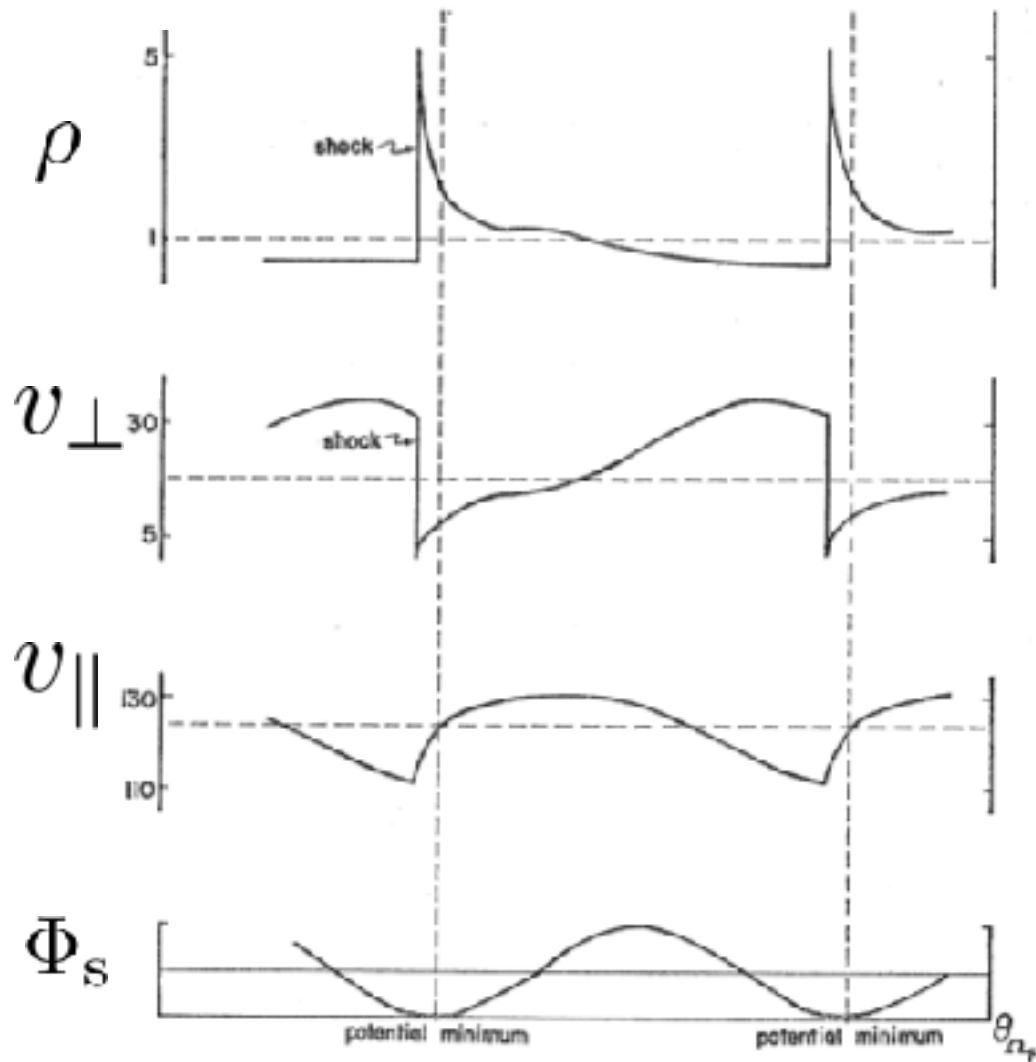


UNIVERSITÄT  
HEIDELBERG  
ZUKUNFT  
SEIT 1385



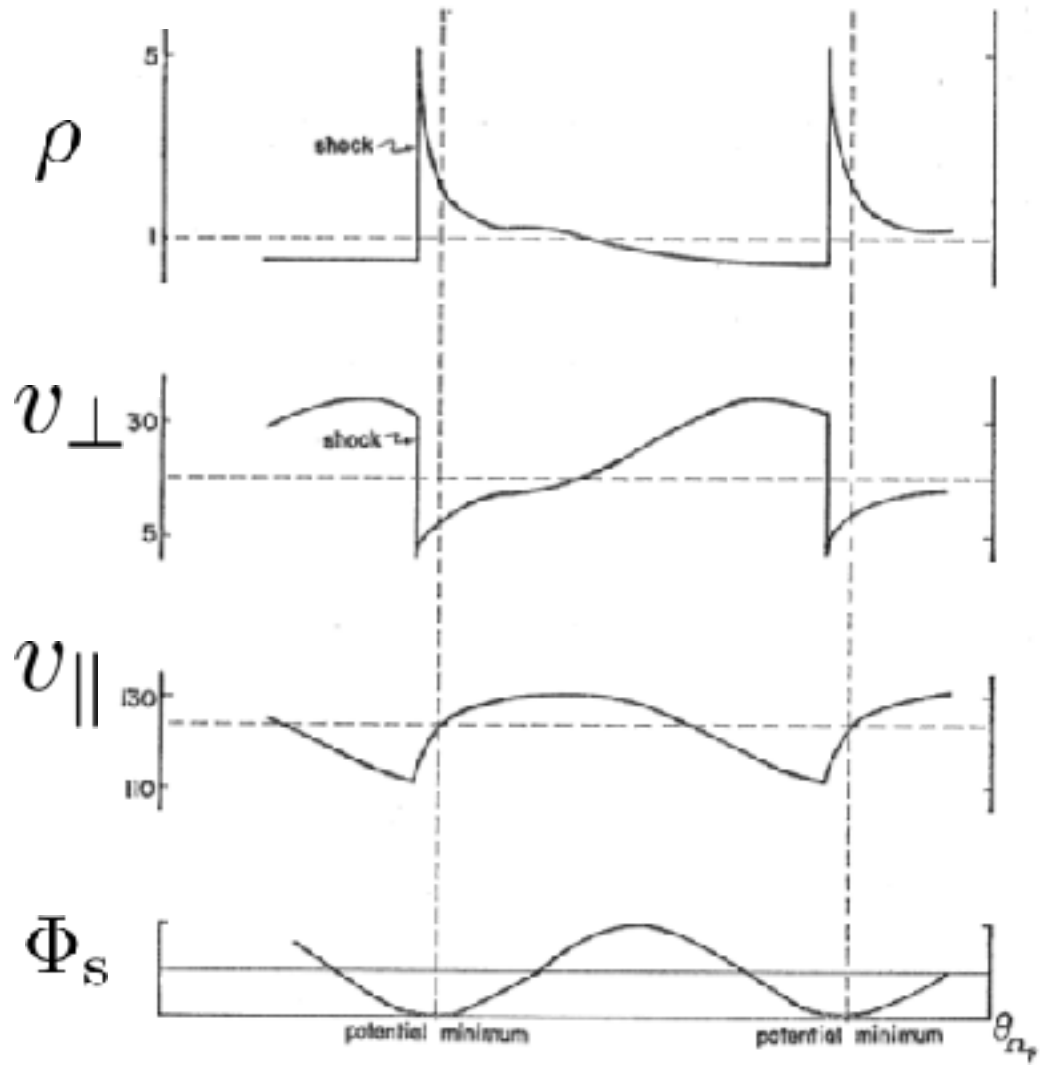
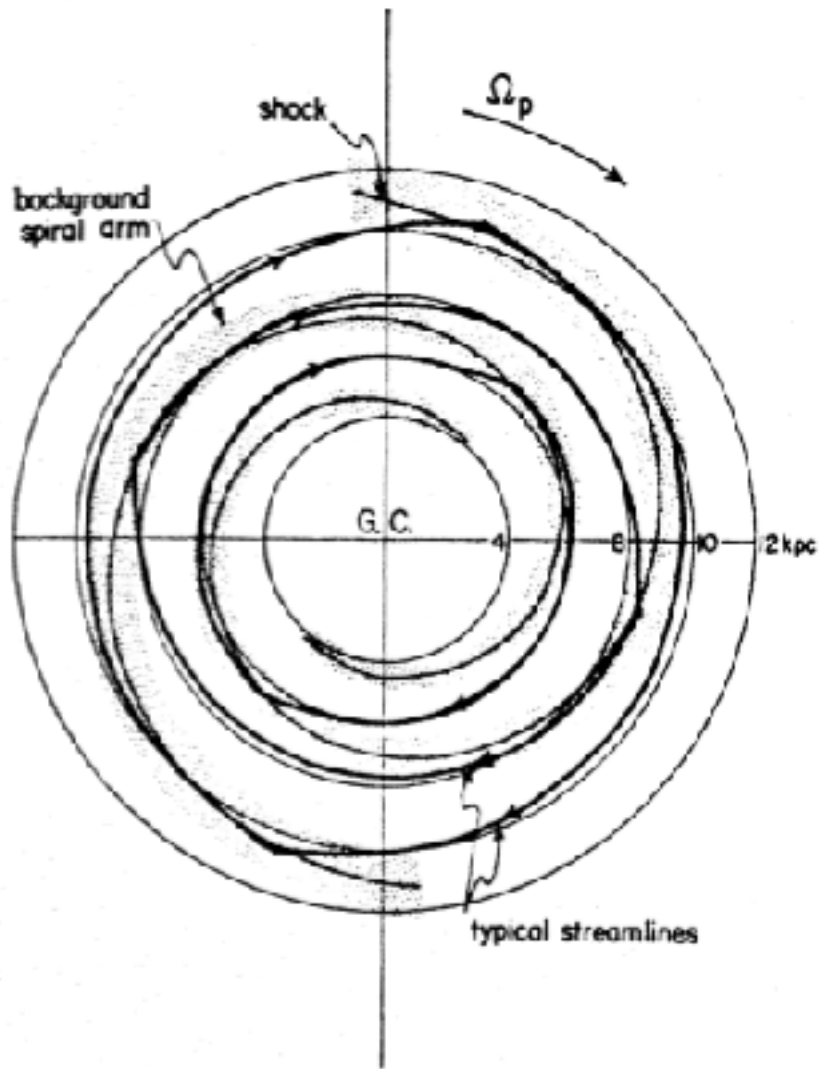
# Roberts 1969

- **Stationary spiral shocks** can result as gas response to externally imposed spiral potential
- **No self-gravity**
- **Isothermal gas**



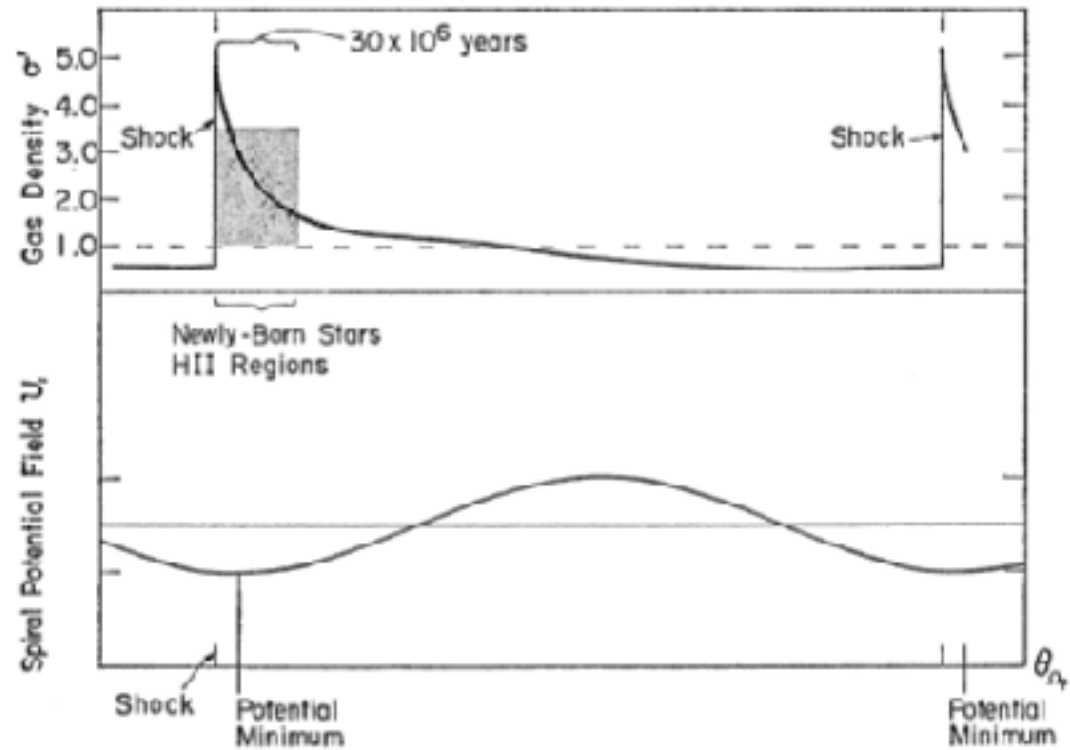
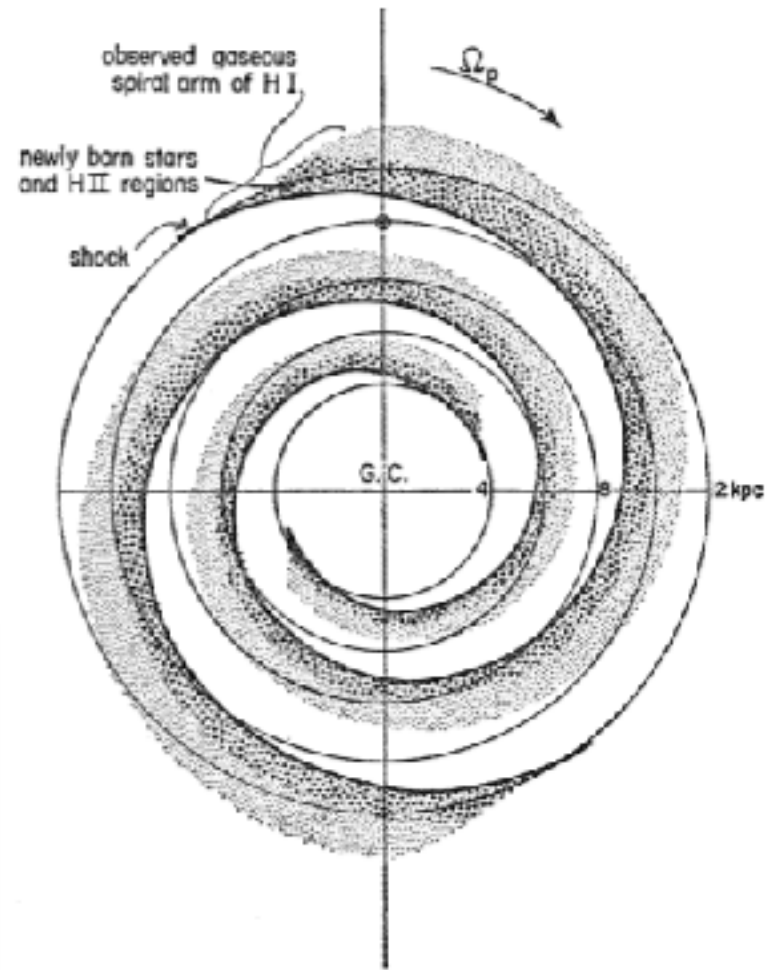
Coordinate perpendicular to spiral arm

# Roberts 1969



Coordinate perpendicular to spiral arm

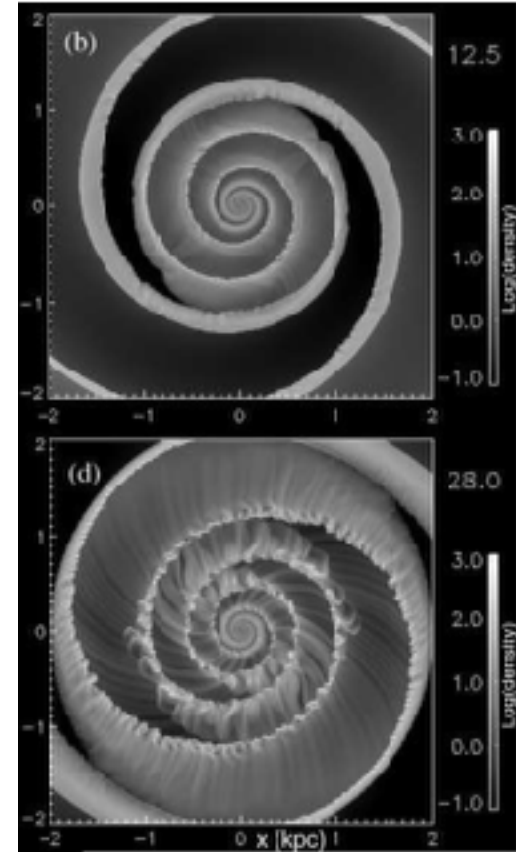
# Implications for star formation



Coordinate perpendicular to spiral arm

# Are these shocks stable?

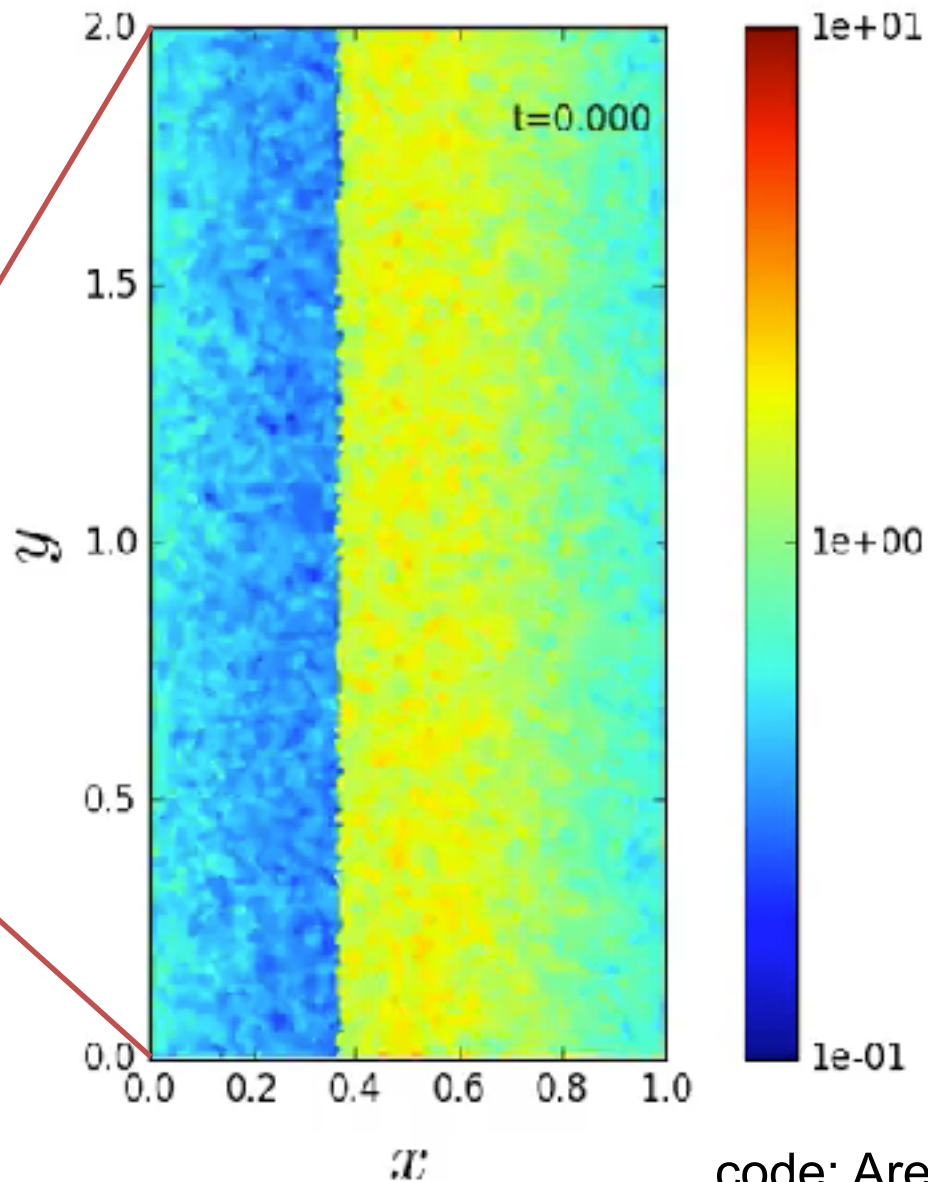
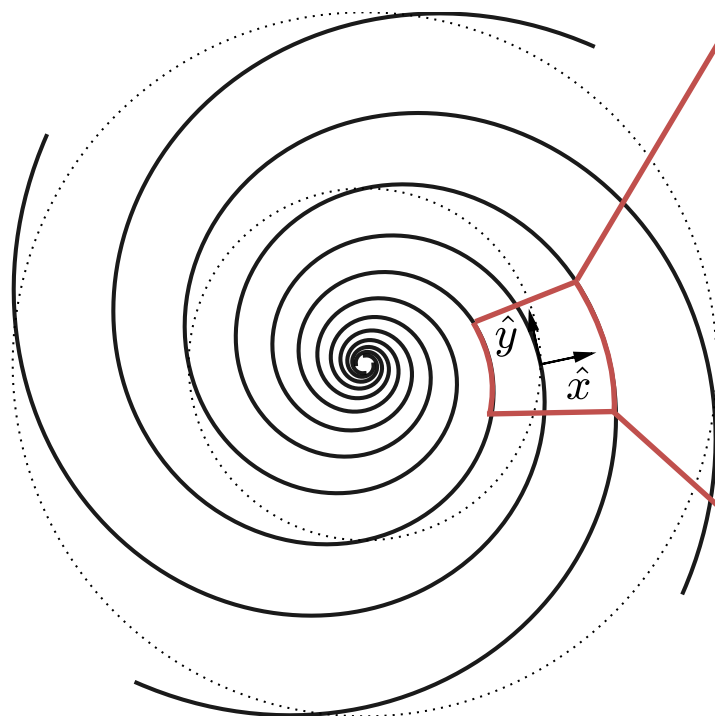
- **Shocks are usually stable** against corrugation of surface (D'yakov&Kontorovich classic result)
- Hence, people argued if instabilities are present is because:
  - **self-gravity** (because of high compression at shocks)
  - **shear** in the post-shock region.
- **70s, 80s, 90s:** several papers find **no evidence of instability**. Topic was thought to be dead...until
- **2000s:** Wada&Koda2004 revitalise the question. They run isothermal, 2D, non-self gravitating simulations and find “**wiggle instability**”. Interpret as KH. Some say is numerical artefact.
- **2010s:** New studies appear and **this time they find shocks to be unstable**.  
(e.g. Lee&Shu2012, KimKimKim2014)



1. **Contradictory results:** new&old papers study the same problem but obtain different results! Why?? Who is right?
2. **Is it just Kelvin-Helmholtz as some say (e.g. review by Shu2016) or is there more?**

# Zoom

Local patch  
around a spiral arm



# Confirmed by linear analysis

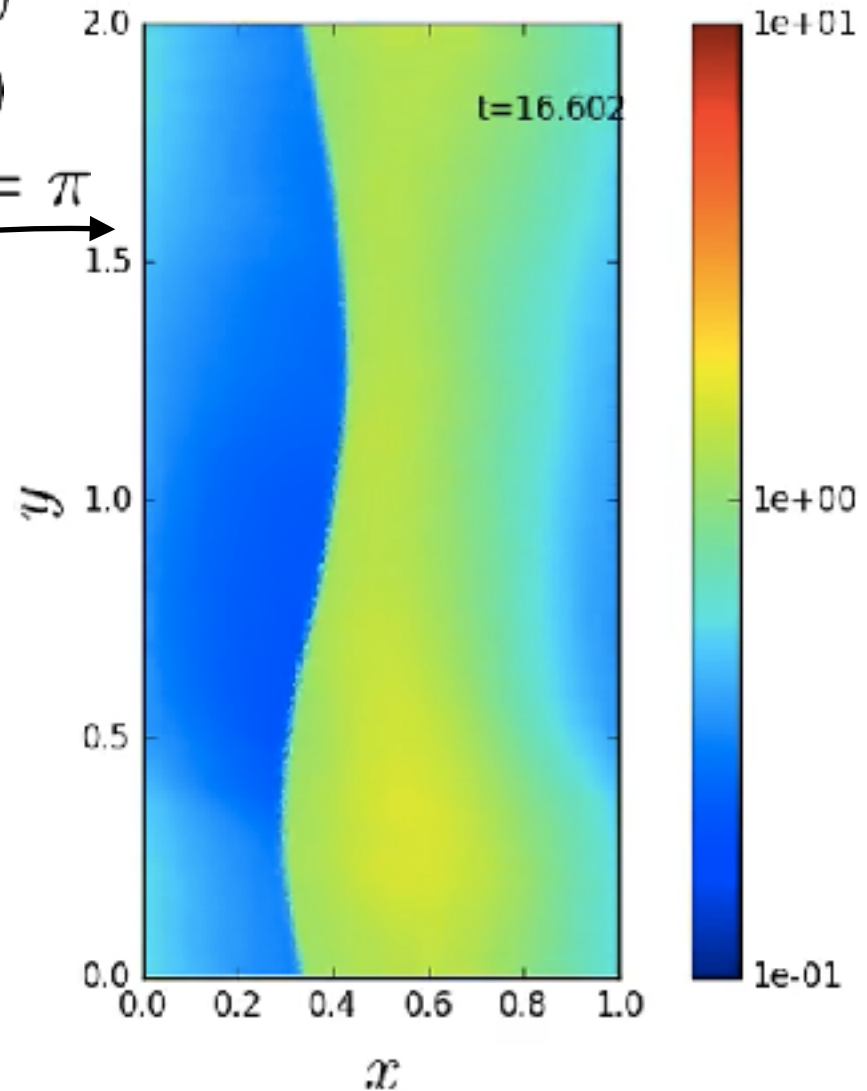
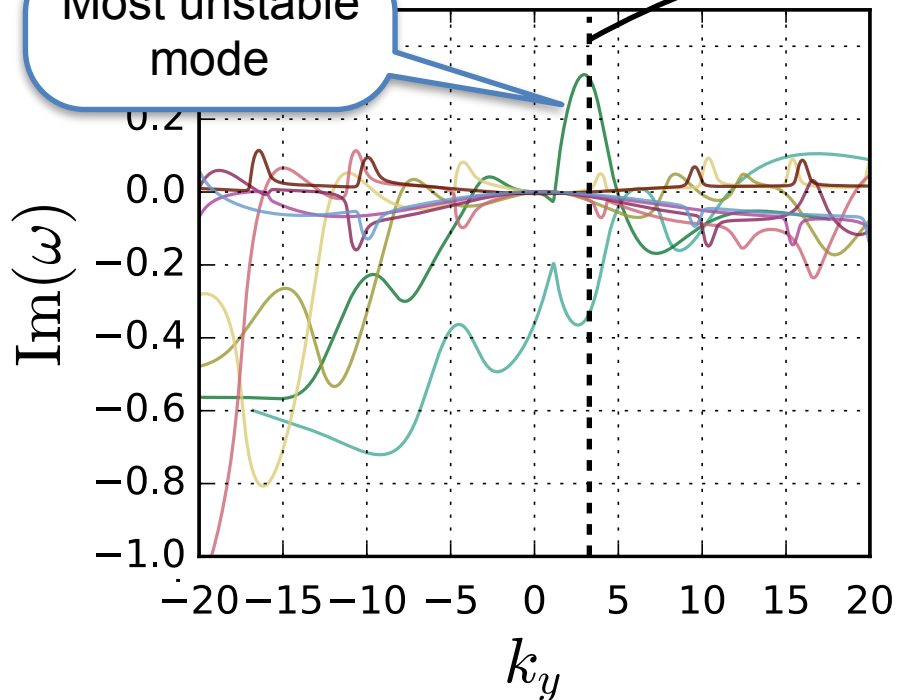
$$\rho = \rho_0(x) + \rho_1(x) \exp(ik_y y - i\omega t)$$

$$\mathbf{v} = \mathbf{v}_0(x) + \mathbf{v}_1(x) \exp(ik_y y - i\omega t)$$

Dispersion relation

$$k_y = \pi$$

Most unstable mode



Assumptions: 2D, isothermal



# Physical interpretation

- In D'yakov-Kontorovich analysis in which the upstream flow is left unperturbed, the shock is stable but can oscillate and emit small waves at some **characteristic frequencies**.
- However, if in the DK problem one sends **incident waves from upstream towards the shock**, these can be greatly amplified or even blow everything up if sent with the proper frequencies of the system.
- What happens if spontaneously emitted waves are somehow allowed to re-enter the shock from upstream? This is what happens with periodic boundary conditions. The shock can **“resonate with itself”**

# Conclusions

- **Stability depends on boundary conditions.** This explains apparently contradictory results
- **Galactic shocks are always unstable** because they are essentially periodic
- **The periodic shock instability is distinct from KH** otherwise it would not disappear by switching boundary conditions
- Relevant for **feathering/spurs of spiral arms.** (e.g. M51) and **Galactic centre bar shocks**
- For strong spiral potentials a **parasitic KH** can also be present on top of the periodic shock instability

**Thank You!**

# Extra

# Consider the following problem

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla P}{\rho} - \nabla \Phi - 2\Omega \times \mathbf{v} + \mathbf{F}$$

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$P = c_s^2 \rho$$

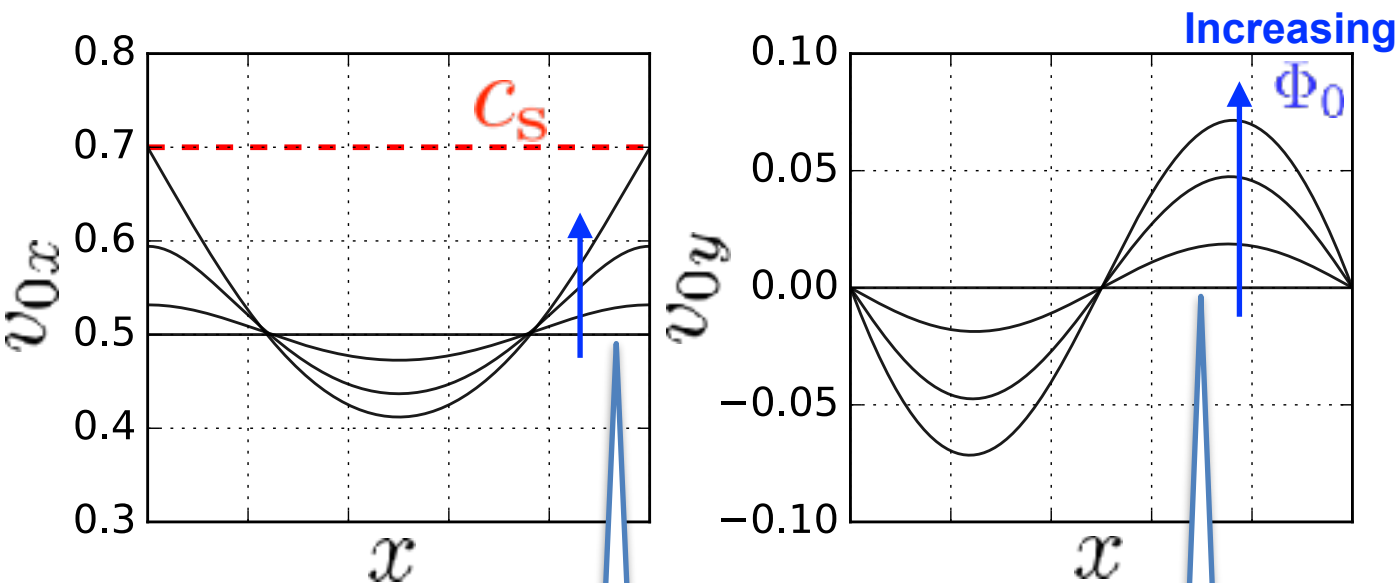
$$\Phi(x) = \Phi_0 \cos\left(\frac{2\pi x}{L}\right)$$

external pot.

Coriolis

constant force

## Steady states



- **Periodic in x**
- **Do not depend on y**

$$\Phi_0 = 0 \Rightarrow \mathbf{F} = 2\Omega \times \mathbf{v}$$

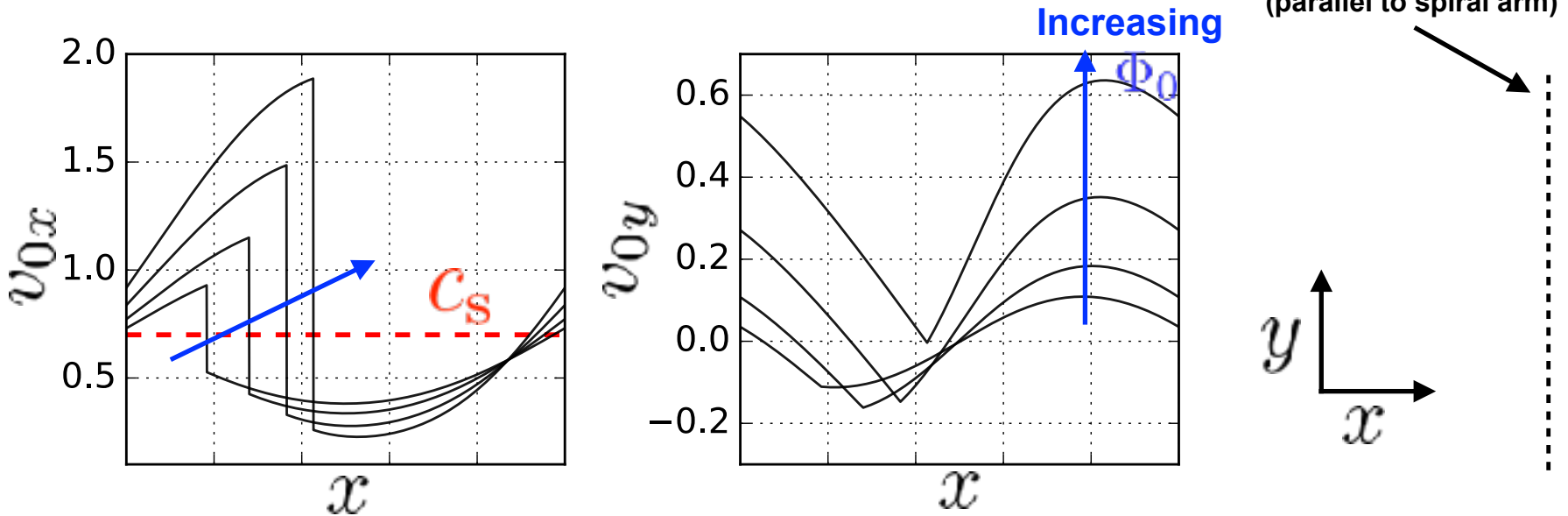
# Consider the following problem

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla P}{\rho} - \nabla \Phi - 2\Omega \times \mathbf{v} + \mathbf{F}$$
$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$P = c_s^2 \rho$$

$$\Phi(x) = \Phi_0 \cos\left(\frac{2\pi x}{L}\right)$$

## Steady states



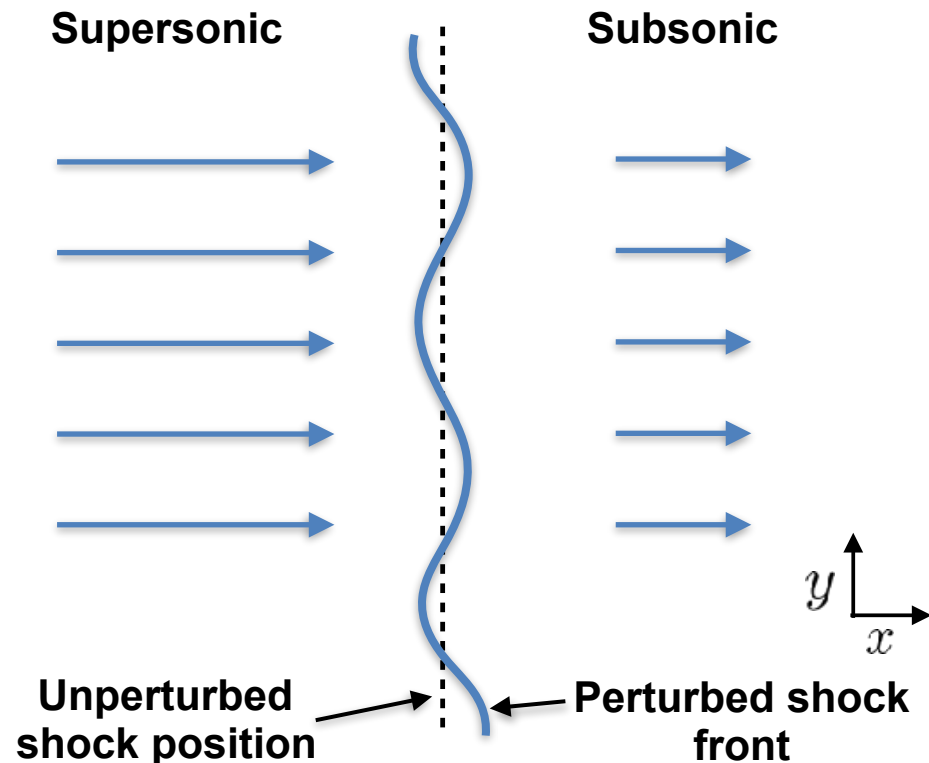
# Linearise around steady state and find eigenmodes

$$\rho = \rho_0(x) + \rho_1(x) \exp(ik_y y - i\omega t)$$

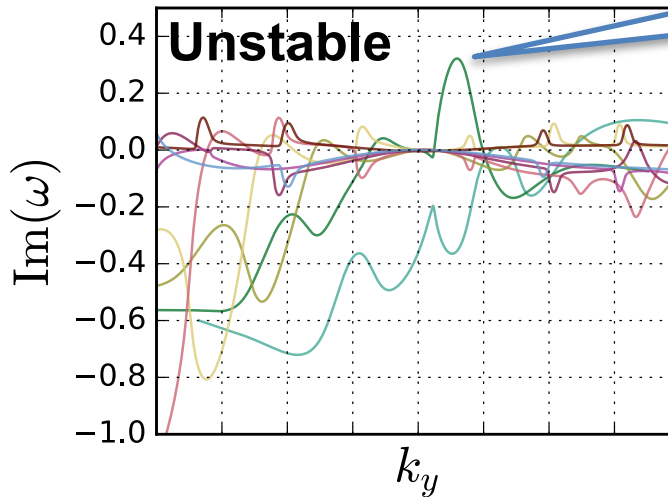
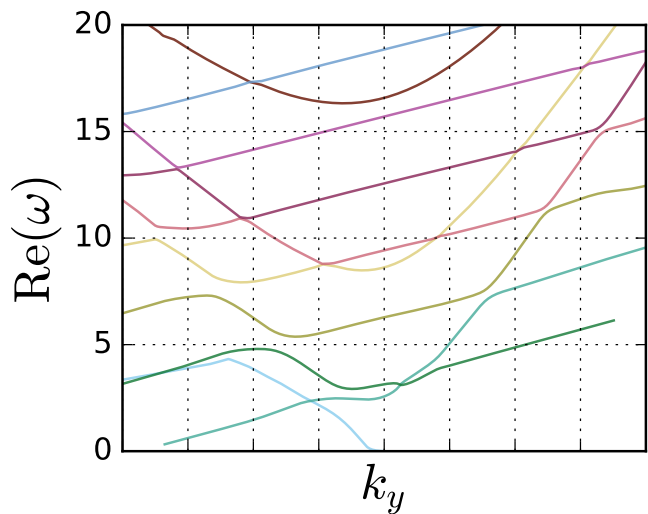
$$\mathbf{v} = \mathbf{v}_0(x) + \mathbf{v}_1(x) \exp(ik_y y - i\omega t)$$

## Two types of boundary conditions

1. **Periodic**
2. **D'yakov-Kontorovich**  
(upstream flow unperturbed because supersonic)

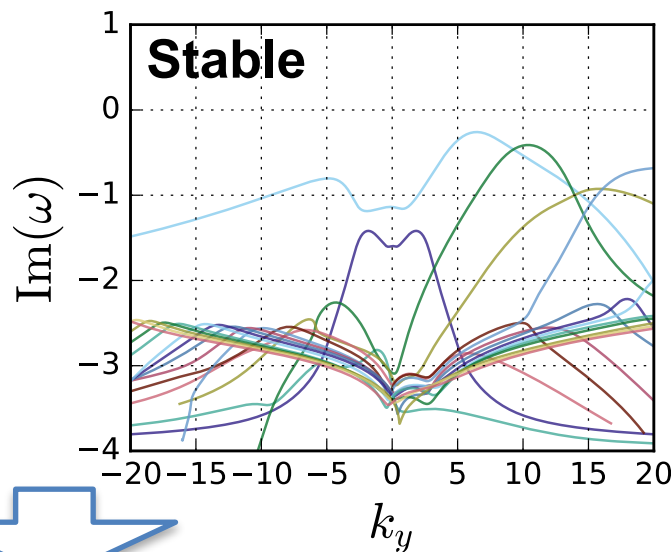
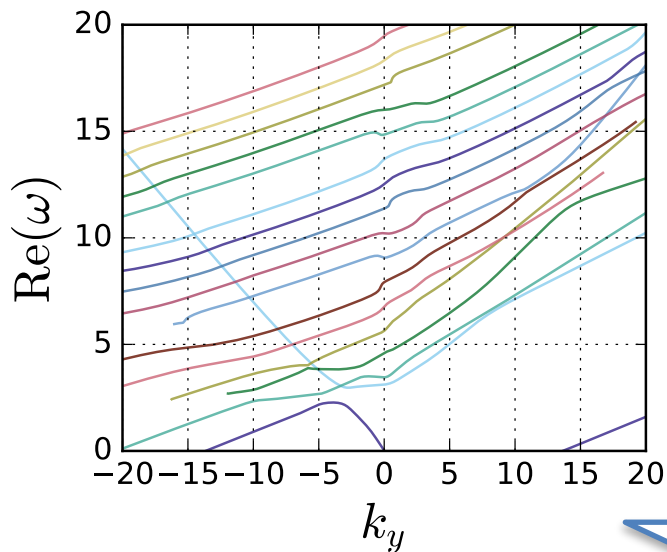


# Dispersion relation



Most unstable mode

$\Phi_0 = 0.25$   
periodic b.c.



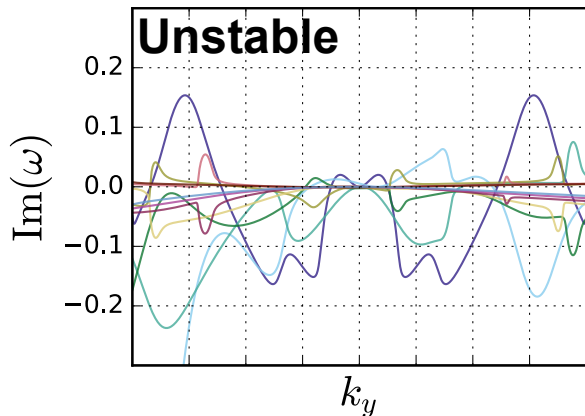
$\Phi_0 = 0.25$   
DK b.c.



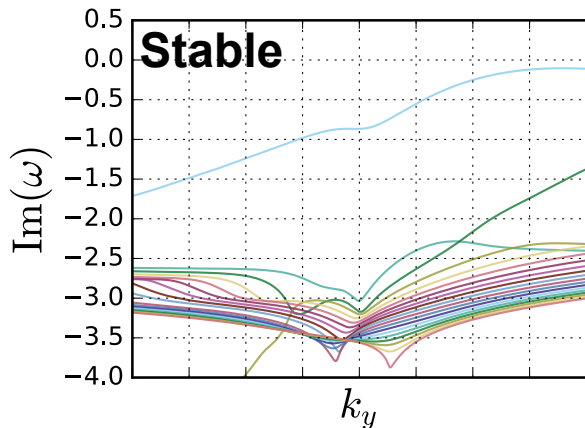
Changing boundary conditions can make the instability disappear!

# Dispersion relation

## Weak/Moderate spiral potential

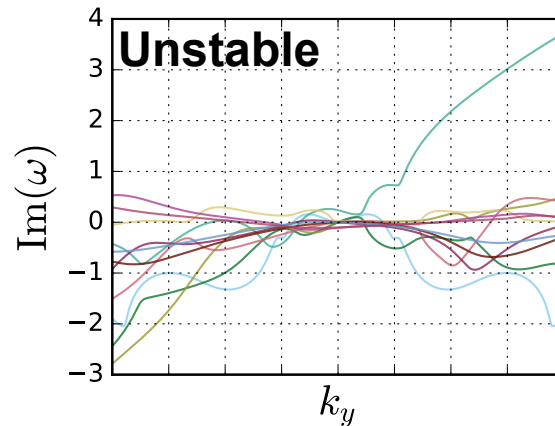


$\Phi_0 = 0.025$   
periodic b.c.

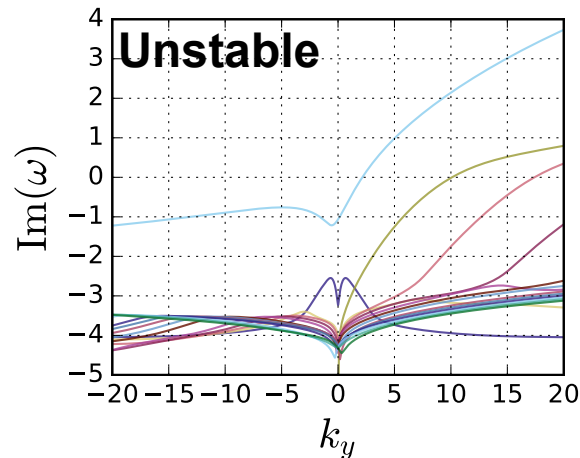


$\Phi_0 = 0.025$   
DK b.c.

## Strong spiral potential



$\Phi_0 = 0.25$   
periodic b.c.



$\Phi_0 = 0.25$   
DK b.c.

- instability disappears by switching b.c.
- **Not KH!**

- parasitic KH instability appears on top of periodic shock instability (shear)

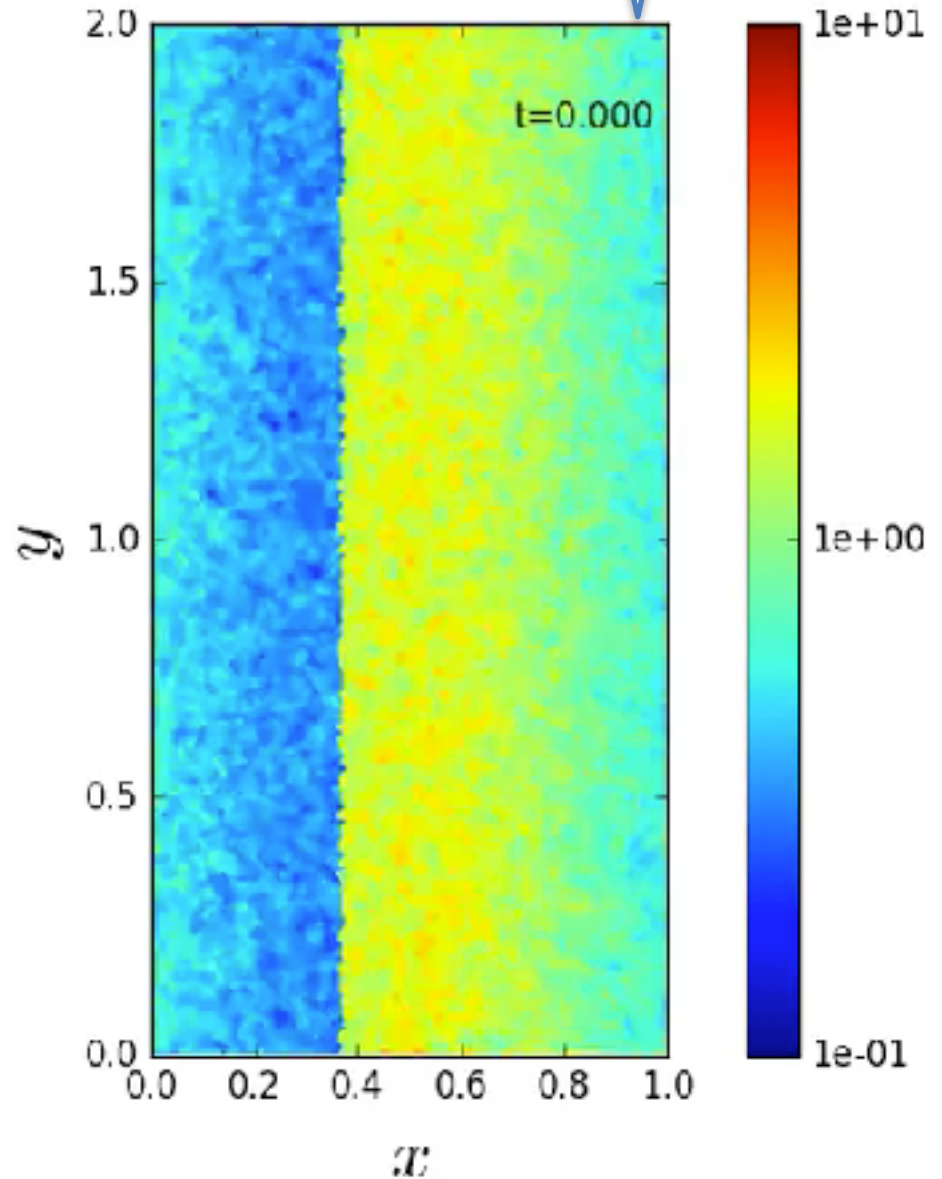
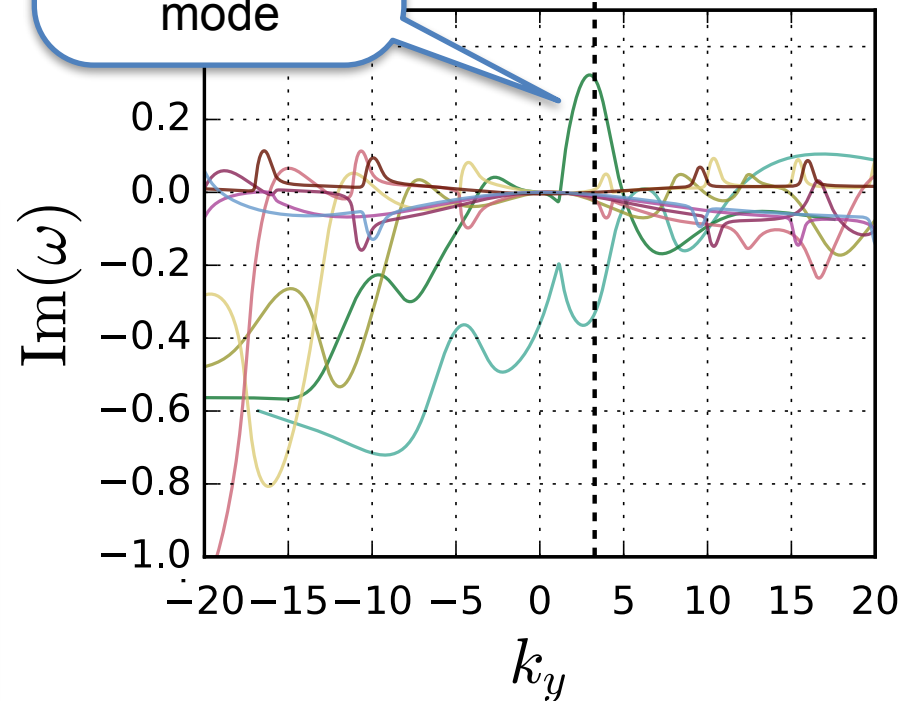


# Arepo simulation

- 2D, Isothermal
- Periodic b.c.

Same simulation with  
**inflow-outflow** on left and  
right boundaries: **is stable!**

Most unstable  
mode



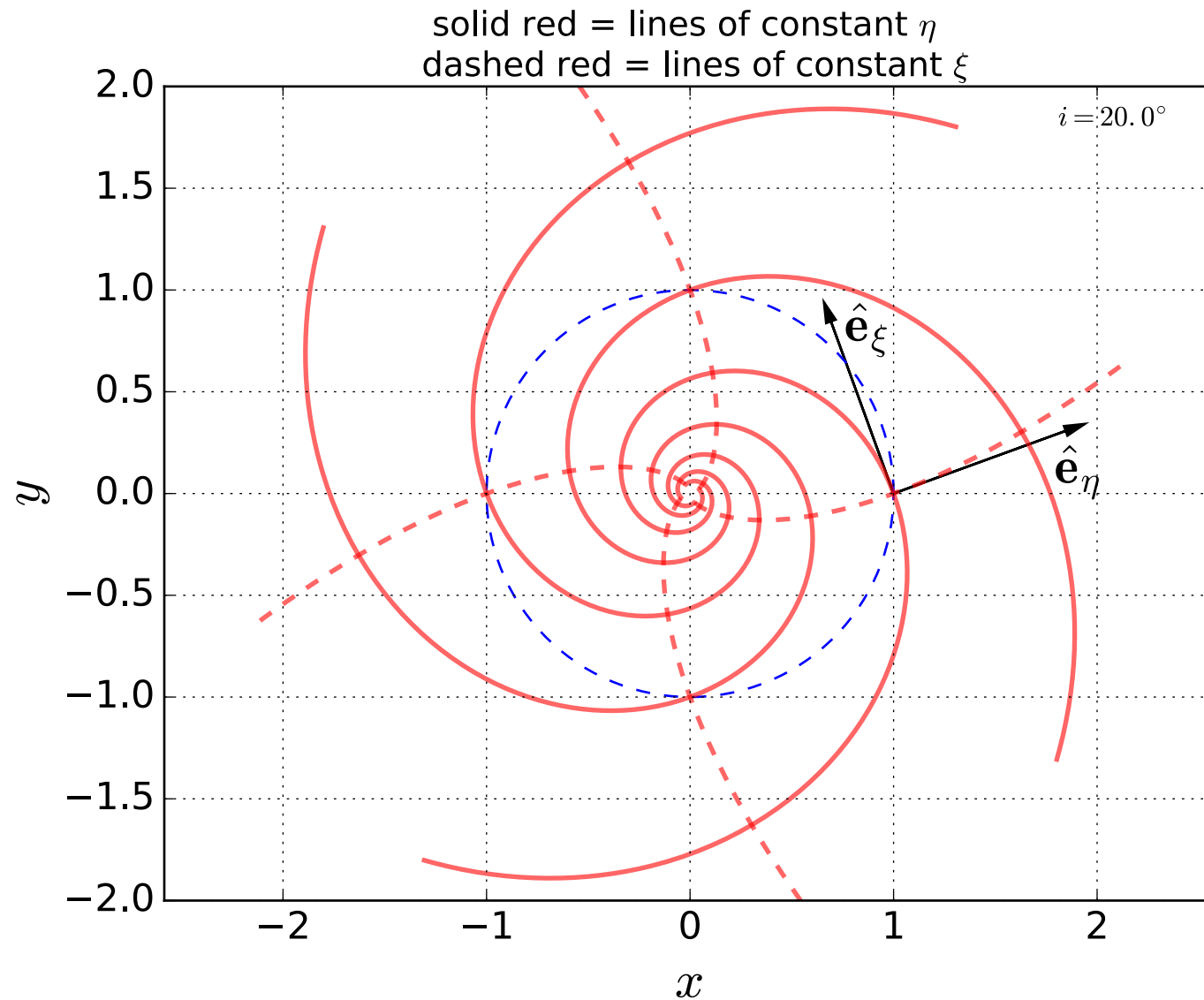
# How to derive equations

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla P}{\rho} - \nabla (\Phi_0 + \Phi_s) - 2\Omega_p \times \mathbf{v} - \Omega_p \times (\Omega_p \times \mathbf{r})$$



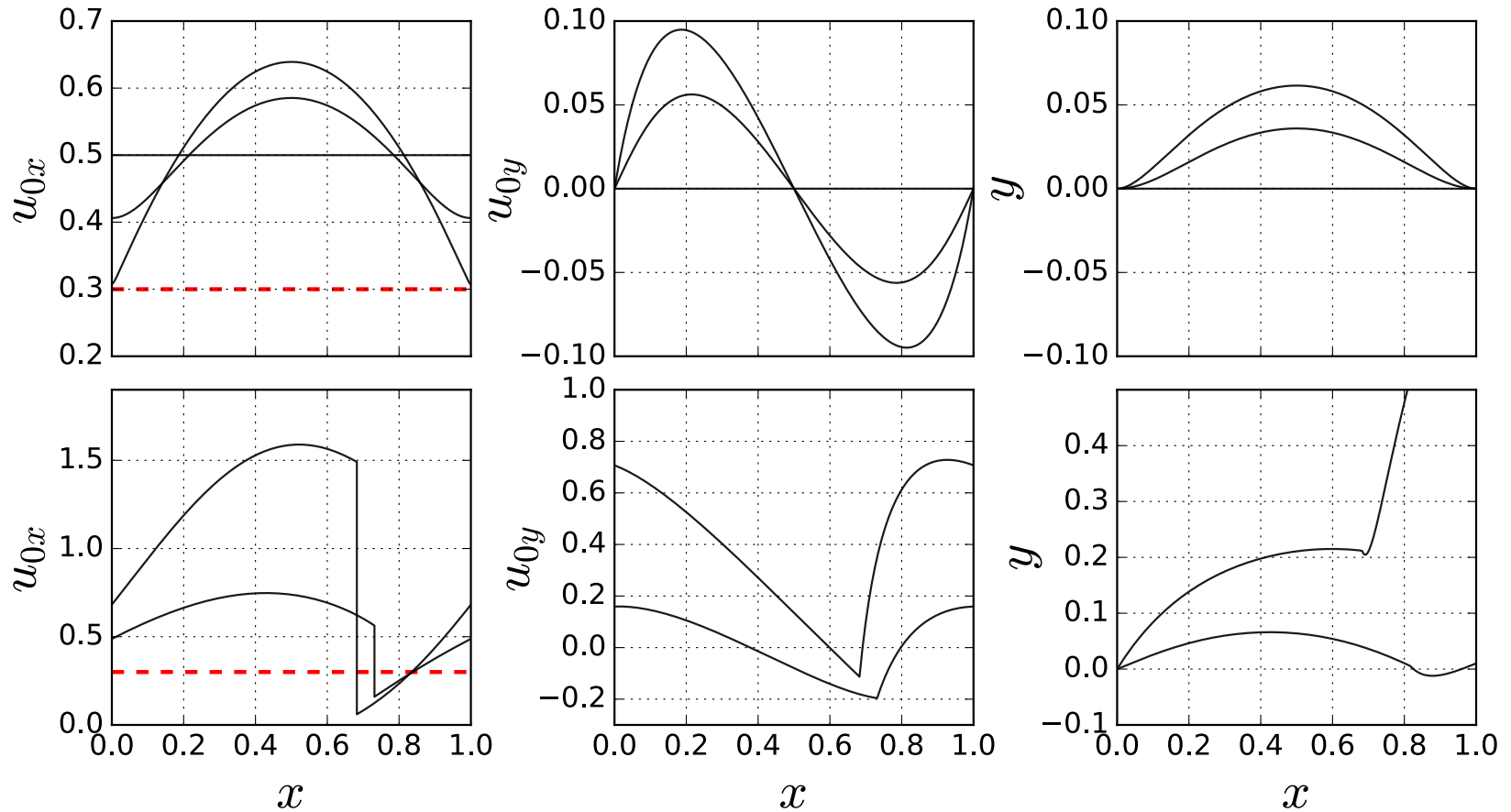
$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla P}{\rho} - \nabla \Phi - 2\Omega \times \mathbf{v} + \mathbf{F}$$

# Spiral coordinate system



# Steady state - cs03

$c_s = 0.3$

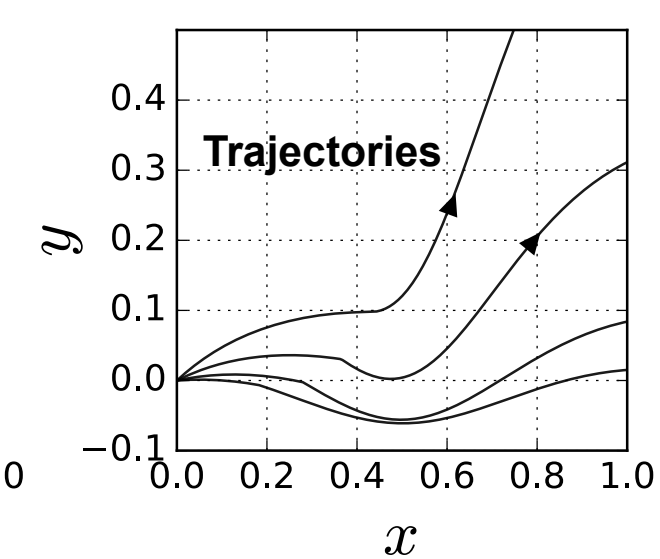
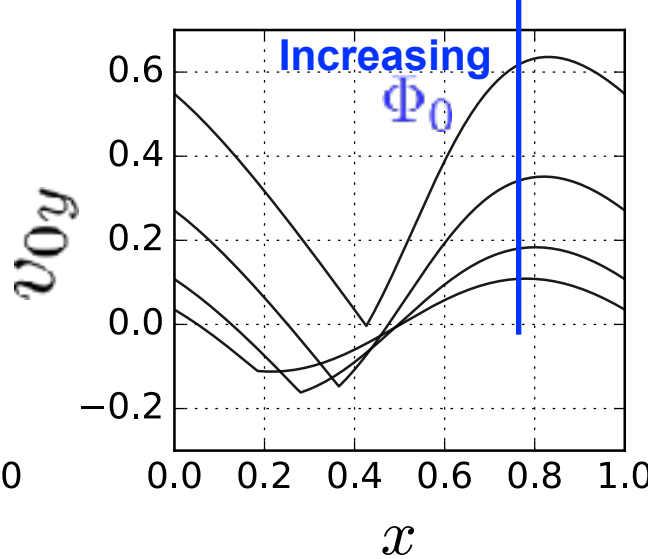
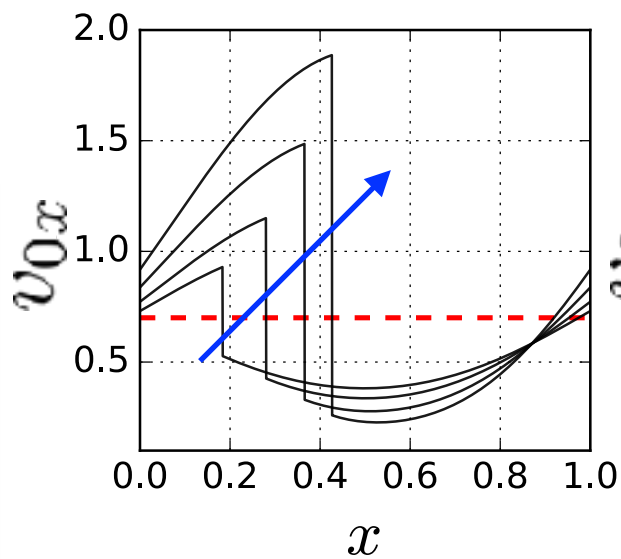
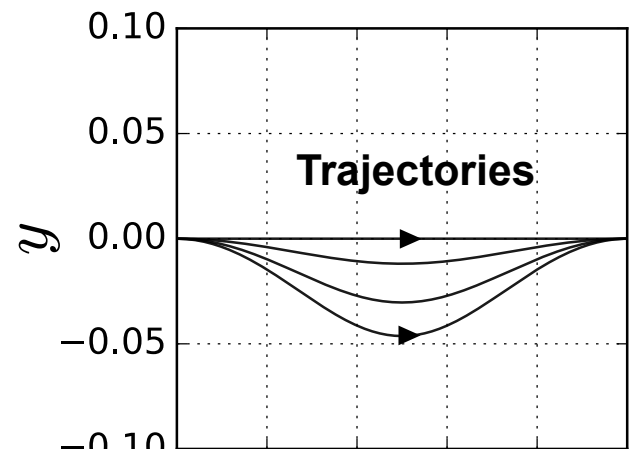
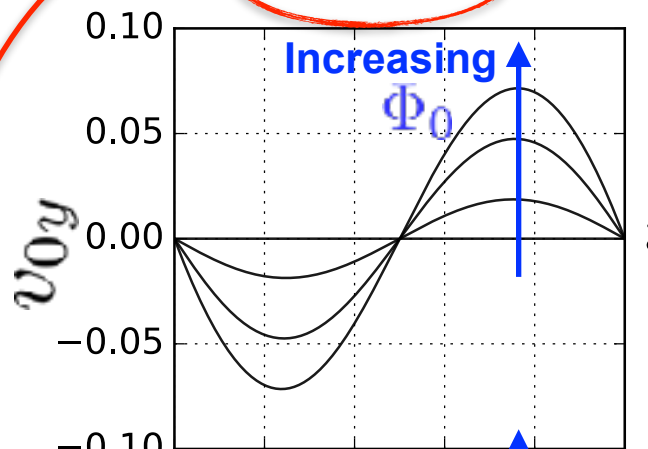
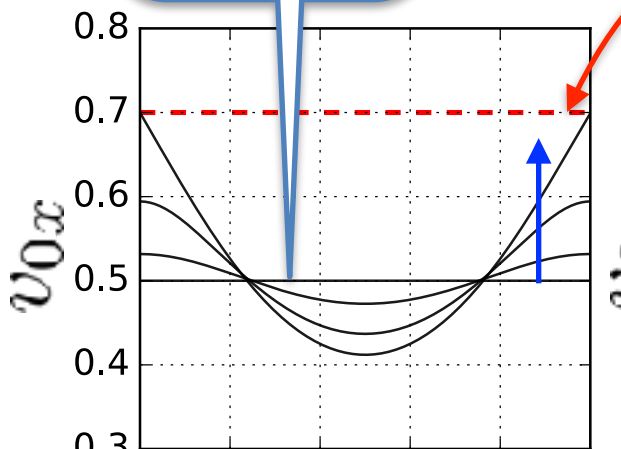


# Steady states

- Periodic in  $x$
- Do not depend on  $y$

$\Phi_0 = 0$   
↓  
 $\mathbf{F} = 2\boldsymbol{\Omega} \times \mathbf{v}$

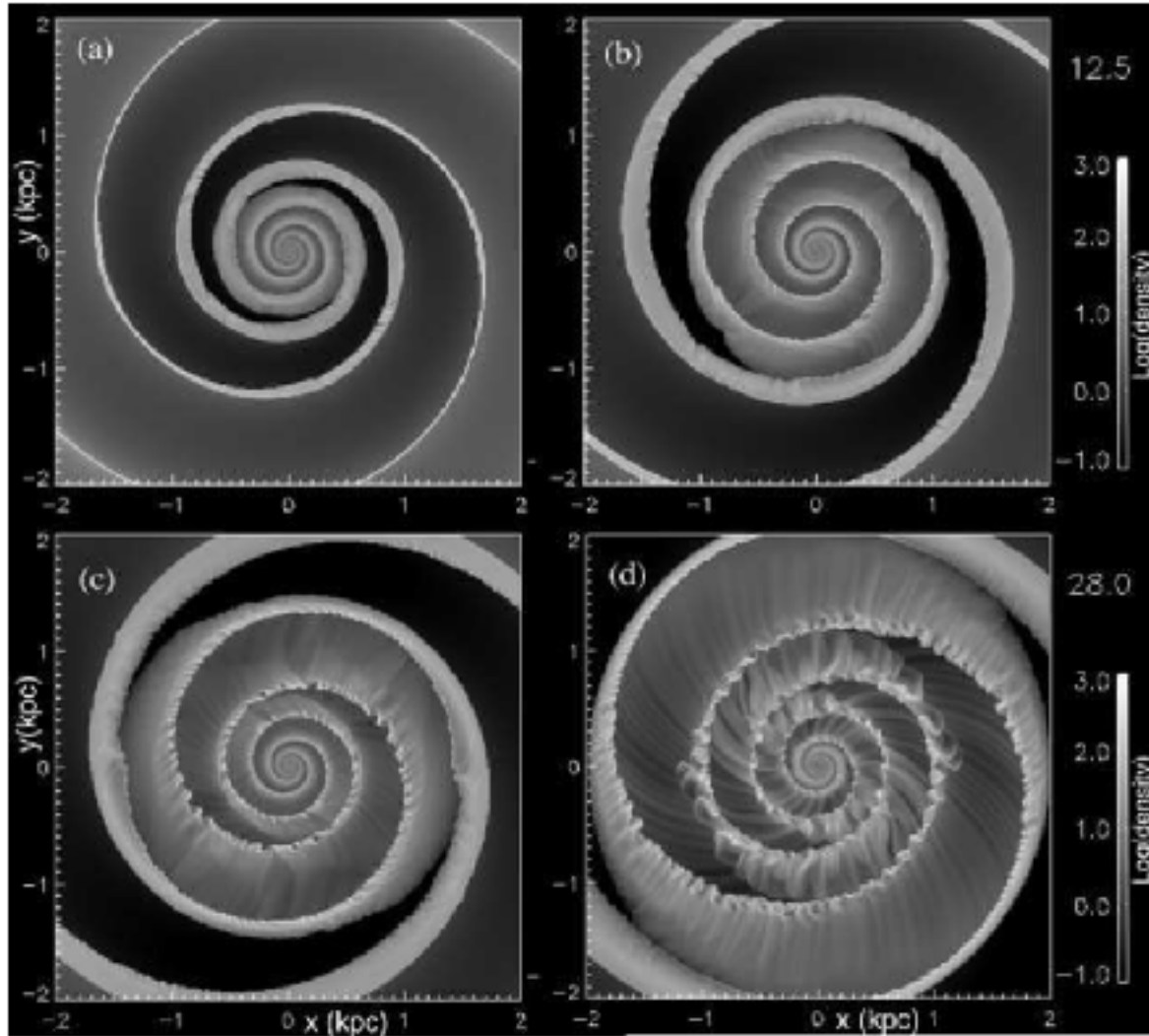
$c_s = 0.7$



# Wada & Koda 2004

- External spiral potential
- 2D
- Isothermal
- No self-gravity

**Why is unstable??**



# Consider the following problem

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla P}{\rho} - \nabla \Phi - 2\Omega \times \mathbf{v} + \mathbf{F}$$

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$P = c_s^2 \rho$$

$$\Phi(x) = \Phi_0 \cos\left(\frac{2\pi x}{L}\right)$$

Four forces act on the fluid:

- Pressure force  $-\nabla P / \rho$
- Coriolis force  $-2\Omega \times \mathbf{v}$
- Constant force  $\mathbf{F}$
- External potential  $-\nabla \Phi$

Take:  $L = 1$ ,  $\Omega = 1$ ,  $\mathbf{F} = (0, 1)$ ,  $c_s = 0.3$  or  $0.7$ ,  $\Phi_0 = 0-0.4$