Lecture 12: Molecular Clouds II
Turbulence & Dense structures

<u>Outline</u>

12·1 Turbulence

"Turbulence is an irregular condition of flow in which the various quantities show a random variation with time and space co-ordinates so that statistically distinct average values can be discerned"

Hinze 1975

The Reynolds number and Mach number characterises the nature of a flow

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{V\,L}{\nu}$$

V - flow velocity
L - typical length scale
ν - viscosity of the fluid
C_s - sound speed

$$M = \frac{\text{inertial forces}}{\text{pressure forces}} = \frac{V}{C_s}$$

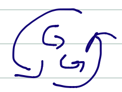
The dimensionless quantities describe the relative strength of the terms in the Navier Stokes equations

inertial term - $(\underline{v}\cdot\nabla)\underline{v}$ pressure term $\Rightarrow \nabla P/\rho$ viscous dissipation $\Rightarrow \nu\nabla^2\underline{v}$

At high Reynolds numbers there is a transition between laminar flow and turbulent flow. (In a pipe this occurs for values above ~2000.) In the cold ISM Reynolds numbers are high $Re \sim 10^5 - 10^7$ so the medium is highly turbulent.

The wavelengths at which there is turbulence range from a minimum value set by viscous dissipation up to the length scale of the turbulent driving.

Kolmogorov turbulence: typical in the subsonic incompressible case



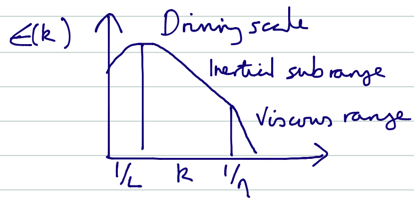
solenoidal motions

Kinetic energy cascades down the scales due to vortex stretching. Turbulence can be thought of as a series of eddies with intense vorticity. Vortexes will be stretched if alligned in such a way that mean velocity gradients will amplify it. Most of the turbulent energy is contained in large scale eddies, but most of the viscosity is on small scales.

Burgers turbulence : shock dominated. Arise from formulation of pressureless gas dynamics. Energy is dissipated in the shocks and does not necessarily pass through all the scales.

compressive motions

Turbulence has a characteristic energy spectrum



$E(k) = C_K \, \epsilon^{2/3} k^{-5/3}$ $\quad 1/L << k << 1/\eta$

for Kolmogorov turbulence where C_K is the Kolmogorov constant and ϵ is the rate at which larger eddies supply energy.

for Burgers turbulence $E(k) \propto k^{-2}$

In reality in the ISM turbulence is supersonic, highly compressible and contains a mixture of solenoidal and compressive modes.

Turbulent energy will decay in a crossing time

$E_k \propto t_c^{-\eta}$ $\quad \eta \sim 1$

meaning that turbulence must either be short lived or continually driven.

12.2 Larsons Laws

In 1981 Larson reviewed observation of substructure in molecular clouds and found the following empirical laws.

1) $\rho \propto R^{\alpha}$ $\quad \alpha = -1.15 \pm 0.15$
2) $\sigma(v) \propto R^{\beta}$ $\quad \beta = 0.4 \pm 0.1$

The first relation equates to constant column density and is probably an observational artifact. A value of $\beta = 0.5$ suggests energy equipartition

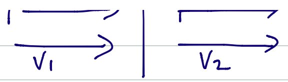
$$\frac{E_{kin}}{|E_{pot}|} = \frac{\sigma^2 R}{2GM} \sim \frac{1}{2}$$

but also arises as a natural consequence of the energy spectrum $E \propto k^{-2}$ in an ensemble of shocks.

12.3 Shocks

What is the effect of turbulent shocks on dense gas in molecular clouds?

$\rho_1 \; P_1 \rightarrow \; | \; \rho_2 \; P_2 \rightarrow$ $\qquad$ In the frame of the shock



$\frac{\partial}{\partial t} = 0$

plane parallel

Conservation Laws - no external forces

Mass: $\frac{\partial \rho}{\partial t} + \nabla . (\rho \bar{v}) = 0$

$$\frac{\partial}{\partial x}(\rho \bar{v}) = 0$$

$$\rho_1 v_1 = \rho_2 v_2$$

Momentum: $\rho \frac{\partial \bar{v}}{\partial t} + \rho(\bar{v}.\nabla)\bar{v} + \nabla P = 0$

$$\rho v \frac{\partial v}{\partial x} + \frac{\partial P}{\partial x} = 0$$

$$\rho_1 v_1^2 + P_1 = \rho_2 v_2^2 + P_2^2$$

Energy: $\frac{\partial}{\partial t}(\rho e_{tot}) + \nabla(\rho h_{tot} v) = 0$

$e_{tot} = e + \frac{1}{2}v^2 \qquad h_{tot} = h_{tot} + \frac{1}{2}v^2$

$$\Rightarrow \rho_1 h_{tot_1} v_1 = \rho_2 h_{tot_2} v_2$$

define specific volume $V_1 = 1/\rho_1 \qquad V_2 = 1/\rho_2$

mass flux $j = \rho_1 v_1 = \rho_2 v_2$

sub into momentum equation

$$j^2 = \frac{P_2 - P_1}{V_1 - V_2}$$

sub into energy equation

$$h_2 - h_1 = \frac{1}{2}(P_2 - P_1)(V_1 + V_2)$$

Rankine Hugoniot shock adiabat

Leave missing steps for a tutorial question

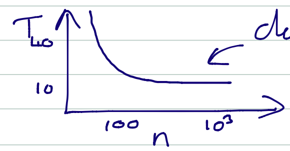
for a strong shock $P_2 >> P_1$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{\gamma+1}{\gamma-1}}$$

for $\gamma = 5/3 \quad \frac{\rho_2}{\rho_1} \Rightarrow 4 \qquad \gamma = 7/5 \quad \frac{\rho_2}{\rho_1} \Rightarrow 6$

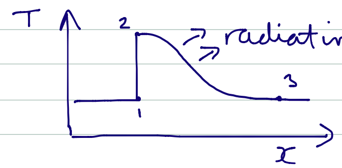
<u>However</u> this assumes energy conservation, but in molecular clouds the gas can cool.

Remember



close to isothermal

Let us look at the temperature profile across a shock wave



isothermal $T_1 = T_3$

$P \propto \rho T \qquad P_3 = \frac{\rho_3}{\rho_1} \frac{\cancel{T_3}}{\cancel{T_1}} P_1 = x_3 P_1$

$$\rho_1 v_1 = \rho_3 v_3$$

sub into momentum equation

$$\rho_1 v_1^2 + P_1 = \rho_3 v_3^2 + P_3$$
$$= \frac{\rho_1 v_1^2}{x_3} + x_3 P_1 \qquad \times\ x^3$$

$$0 = P_1 x_3^2 - (P_1 + \rho v_1^2) x_3 + \rho_1 v_1^2$$

$$0 = x_3^2 - (1 + \rho_1 v_1^2 / P) x + \rho_1 v_1^2 / P$$

isothermal $M^2 = v^2/c^2 = \rho v^2 / p$

$$0 = x_3^2 - (1 + M^2) x_3 + M^2$$

$x_3 = 1 \qquad x_3 = M^2$

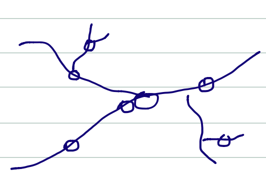
$$\Rightarrow \quad \frac{\rho_3}{\rho_1} = M^2 = \frac{P_1 v_1^2}{\rho_1}$$

Arbitrarily high compression dependent on the Mach number

$$t_{ff} \propto 1/\sqrt{\rho}$$

Shocks can trigger local instability in the gas

12·4 Filaments

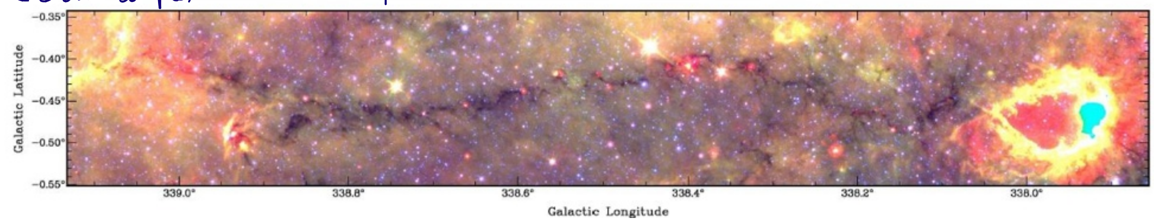


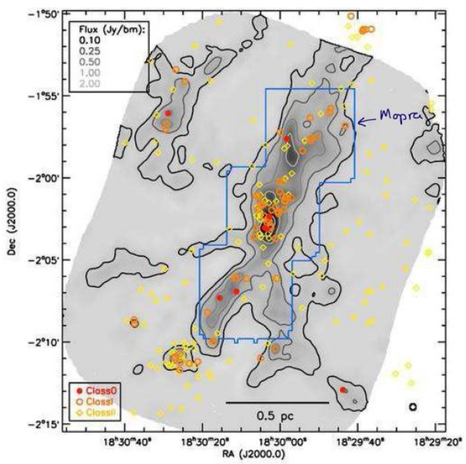
Observations of molecular clouds show that the gas has a filamentary distribution.

Filamentary structure is seen on all scales in Molecular clouds.

e.g "Nessie"

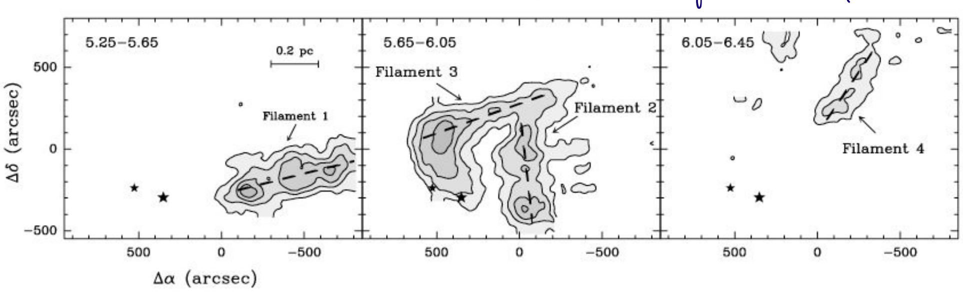
GMC cloud complex ~100 pc





GMC scale

Scale of star formation



The radial density in an isothermal filament was shown by Ostriker in 1964 to be in equilibrium

$$\rho_{eq} = \rho_c \left[1 + \left(\frac{r}{H_0} \right)^2 \right]^{-2} \qquad H_0 = \sqrt{\frac{2 c_s^2}{\pi G \rho_c}}$$

H_0 effective radius
ρ_c central density

Filamentary structures are particularly prone to collapse. Consider first the balance of pressure and gravity in a plane parallel sheet.

$$F_P = -\frac{1}{\rho}\frac{\partial P}{\partial z} \sim \frac{c_s^2}{z_h} \qquad F_G = -\frac{\partial \phi}{\partial z} = -G\Sigma$$

z_h = scale height
Σ = surface density

however in the case of a filament this becomes

$$F_P = -\frac{1}{\rho}\frac{\partial P}{\partial z} \sim \frac{c_s^2}{R} \qquad F_G = -\frac{\partial \phi}{\partial z} \sim \frac{G M_{line}}{R}$$

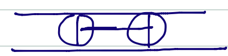
For a filament both pressure and gravity have a radial dependency

In equilibrium $2\epsilon_P = \epsilon_g$

$$\Rightarrow \frac{2c_s^2}{R} = \frac{G M_{line}}{R}$$

$$M_{crit} = \frac{2c_s^2}{G} = 16.7\left(\frac{T}{10K}\right) M_\odot\, pc^{-1}$$

Any filament above this critical value will fragment on scales of two times the width



This is a natural mechanism for producing dense cores of gas.

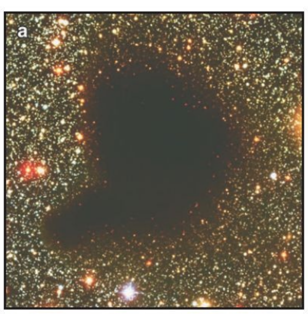
Recent observations have started characterising the width of filament using the profile

$$\Sigma(r) = A_p \frac{\rho_c R_{flat}}{[1+(r/R_{flat})^2]^{\frac{p-1}{2}}}$$

In this form the above equilibrium form is equivalent to $p = 4$

Arzoumanian et al. 2011 find $\left.\begin{array}{l} R_{flat} \sim 0.1\,pc \\ 2 < p < 1 \end{array}\right\}$ for all MC filaments

12.5 Bonnor Ebert spheres



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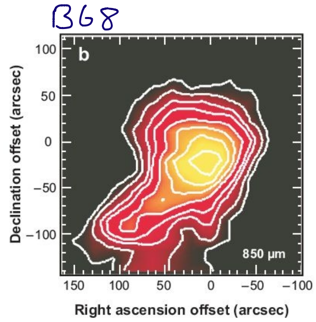
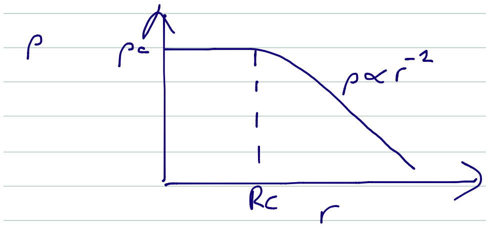


Bok globules are a special class of dense cores where a cold dense core is surrounded by a hotter more diffuse outer medium.

The temperature and density profile in such objects is described well by a class of models known as Bonnor Ebert spheres.

This is an equilibrium solution for spherical cold gas surrounded by a hot confining medium.



$$\rho(r) = \frac{\rho_c R_c}{R_c^2 + r^2}$$

$$R_c = \left(\frac{4\pi G \rho_c}{c_s^2}\right)^{1/2}$$

The core length, R_c, is used as a normalisation length for the entire core $\xi_e = r_e / R_c$ where r_e is the external radius.

r_e is determined by the balance of thermal pressure
eg. $P_e \uparrow \Rightarrow r_e \downarrow$ increases self gravity

above $\xi_e = 6.3$ there are no equilibrium models

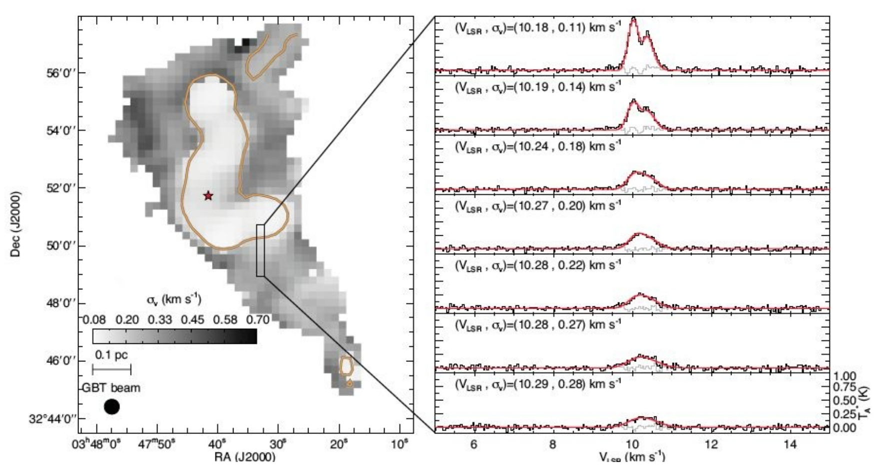
this equates to a critical mass

$$M_{BE} = 2.1 \left(\frac{T}{20K}\right)^2 \left(\frac{P/k}{10^6 \, K cm^{-3}}\right)^{-1/2} M_\odot$$

However it is not clear how well this scenario corresponds to dense cores in molecular clouds that are not surrounded by a hot medium.

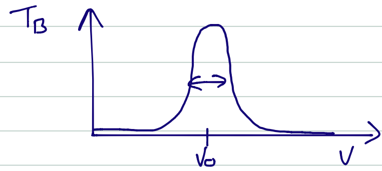
12.6 Embedded dense cores

Pineda et al 2011



Observations show a sharp transition to coherence at the boundaries of dense cores.

Information on the density field within dense cores is obtained from dense gas tracers such as N_2H^+



FWHM ~ 0.2 km s^{-1}

$\left(\equiv 2\sqrt{2\log 2}\ \sigma(v) \text{ from Gaussian fit} \right)$

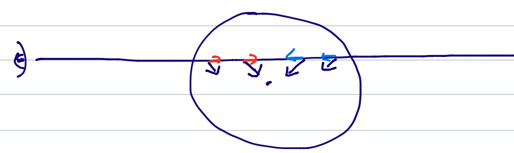
This suggests that they may equate to the stagnation points of the turbulent velocity field within a molecular cloud.

Such cores could be transient density features and may have mass flux across boundaries that are ill defined as there is no intrinsic difference between the gas inside and out.

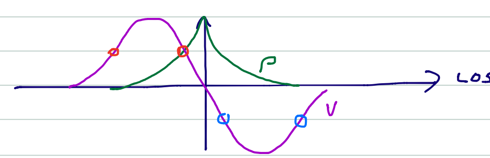
Cores like these can still be fit by Bonner-Ebert spheres however simulations have shown that even collapsing cores can be well fit by static BE spheres. Moreover observations have shown that cores are typically prolate ie elongated.

Using optically thick tracers we can determine whether a a core is expanding or contracting

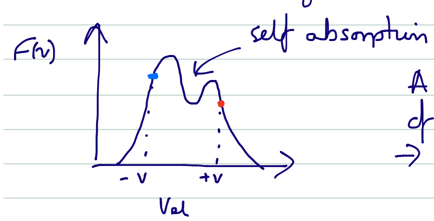
Consider a collapsing spherical core



The line of sight densities are



but in an optically thick species only the gas nearest to the observer at a given velocity is visible.



A blue skewed profile is indicative of collapse.

→ Of course there are also reasons why this would not work every time eg. complexity, geometry, line of sight contamination

The general behaviour of a sample of cores can be classified using the normalised velocity difference between the optically thick and thin components.

$$\delta V = (V_{thick} - V_{thin}) / \Delta V_{thin}$$

Generally core samples are found to have a blue excess eg. Gregersen et. al. found a blue excess of 39%. Therefore it is clear that a substantial number of cores are collapsing. These will undergo star formation as we will discuss in the next lecture.