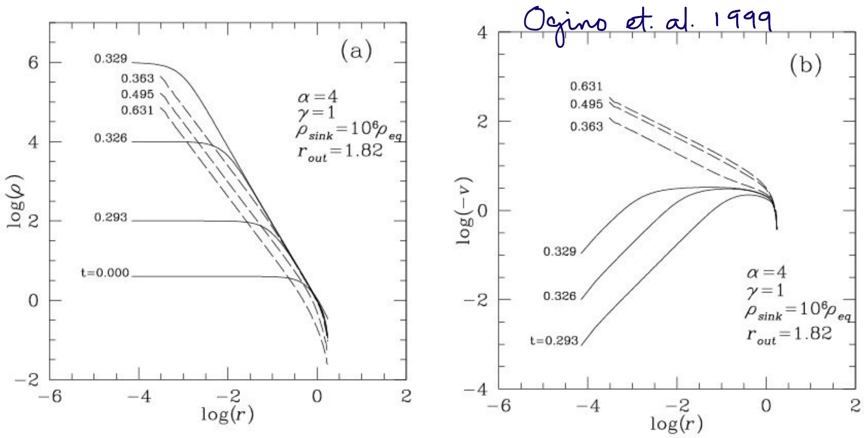
Lecture 13: Star Formation

13.1 The Larson-Penston solution

Consider an initially static uniform isothermal sphere of gas which undergoes gravitational collapse. The density evolution can be followed numerically as follows.

Ogino et. al. 1999



The distribution becomes strongly centrally condensed and approaches a density profile $\rho \propto r^{-2}$.

The evolution can be described by a self similarity solution before a protostar is formed $(t<0)$

$$\rho \propto (r^2 + r_0^2)^{-1} \quad \text{where } r_0 \to 0 \text{ as } t \to 0 \qquad v_r \propto r/t \text{ as } t \to 0, \quad v \sim -3.3 c_s \text{ as } r \to \infty$$

This solution has a singularity at its centre implying that collapse will runaway at the centre of the core. The central density peak becomes opaque to thermal radiation from dust at densities of $2 \times 10^{10}\ cm^{-3}$ and the temperature rises above its initial value of ~ 10 K.

$P \propto \rho^\gamma$ ratio of specific heats $\gamma \to 7/3$ for diatomic gas
pressure ↑ faster than gravity

At $\rho_c = 2 \times 10^{-10}\ g\ cm^{-3}$ the collapse stops.

This first "hydrostatic core" has a mass of only $\sim 0.01\ M_\odot$

The first core is a transient feature, when temperatures of $T > 2000$ K are reached H_2 molecules dissociate collisionally

$$\begin{aligned} H_2 + H &\rightarrow H + H + H \\ H_2 + H_2 &\rightarrow H + H + H_2 \qquad \text{endothermic} \\ H_2 + He &\rightarrow H + H + He \end{aligned}$$

The gas is now monatomic rather than diatomic and $\gamma < 4/3$ at which point the pressure increases more slowly than gravity so collapse resumes.

Collapse continues until the gas is fully ionised and $\gamma \rightarrow 5/3$ typical of the interior of a star. At this point collapse is permanently halted with the end result being a hydrostatic core bounded by an accretion shock. The remainder of the gas will be accreted on to this core

At $t > 0$ the remainder of the gas follows the profile

$$\rho \propto r^{-3/2} \text{ as } r \rightarrow 0 \qquad v \propto r^{-1/2} \text{ as } r \rightarrow 0$$
$$\rho \propto r^{-2} \text{ as } r \rightarrow \infty \qquad v \approx -3.3\, c_s \text{ as } r \rightarrow \infty$$

yielding an accretion rate of $\dot{M} = 47\, c_s^3 / G$

1.3.2 The Shu model

Motivated by the inadequate treatment of angular momentum in the above scenario, and the presence of magnetic fields ($\sim 3\,\mu G$ though at the time thought to be up to an order of magnitude higher.)

When we add magnetic fields into the equations of virial balance we find a critical mass-to-flux ratio for collapse

$$\left(\frac{M}{\phi}\right)_{cr} = \frac{\xi}{3\pi}\left(\frac{5}{G}\right)^{1/2} = 490\ g\ G^{-1}\,cm^{-2} \qquad \xi = 0.53 \text{ for uniform sphere}$$

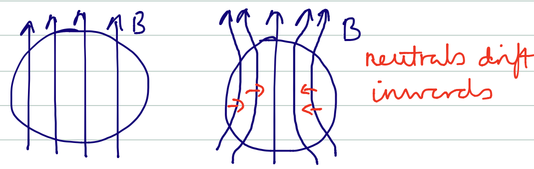
if $\dfrac{M}{\phi} < \left(\dfrac{M}{\phi}\right)_{cr}$ subcritical - stable

if $\dfrac{M}{\phi} > \left(\dfrac{M}{\phi}\right)_{cr}$ supercritical - unstable

If cores are mostly subcritical how do stars form at all.

- ambipolar diffusion
  ↳ ion-neutral drift due to differential forces felt by the two species

$$\tau_{AD} = 25\,\mathrm{Myr} \left(\frac{B}{3\mu G}\right)^{-2} \left(\frac{n_n}{10^2\,\mathrm{cm}^{-3}}\right)^2 \left(\frac{R}{1\mathrm{pc}}\right)^2 \left(\frac{x}{10^{-6}}\right)$$



Slow starformation.

Shu proposed that as the field diffused a singular isothermal sphere ($\rho \propto r^{-2}$) would develop quasistatically. Once the central regions became supercritical the core would undergo an inside out collapse.

at $t<0$ before hydrostatic core

$$\rho \propto r^{-2} \qquad v \equiv 0$$

at $t>0$

$$\rho \propto r^{-3/2} \quad r \le c_s t \qquad v \propto r^{-1/2} \quad r \le c_s t$$
$$\rho \propto r^{-2} \quad r > c_s t \qquad v \equiv 0 \quad r > c_s t$$

accretion rate $\dot{M} = 0.975\, c_s^3 / G$
much lower than the above

However there are problems with this model
- observed cores have flat inner profiles
- no strongly subcritical cores (Nakano 1998)
- field strongest on the inside (Crutcher 2009)
- infall motions observed on scales than predicted rarefaction wave.

The original dynamical collapse model seems the better description.
- this implies that SF must be very fast, but the observed galactic SF rate is only a few $M_\odot\,\mathrm{yr}^{-1}$

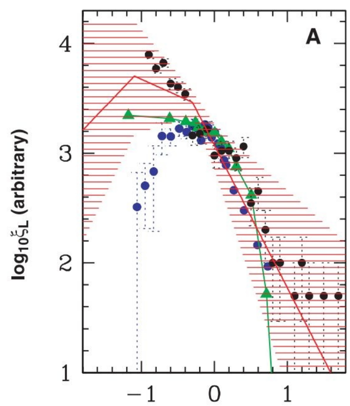
13.3 The IMF and CMF

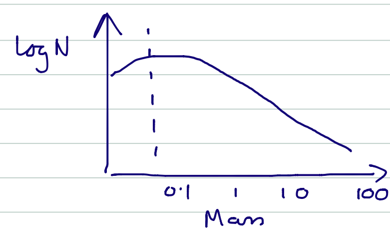
The stellar initial was first measured by Salpeter in 1955 who showed that the number of stars $\xi(m)\,dm$ which have masses between m & $m+dm$ can be described as

$$\xi(m)\, dm \approx m^{-\alpha}\, dm$$

where $\alpha = 2.35$ for $0.4 < m < 10$

With more modern observations we see the IMF more closely resembles a lognormal (Chabrier 2002) or 3 component power law (Kroupa 2002).



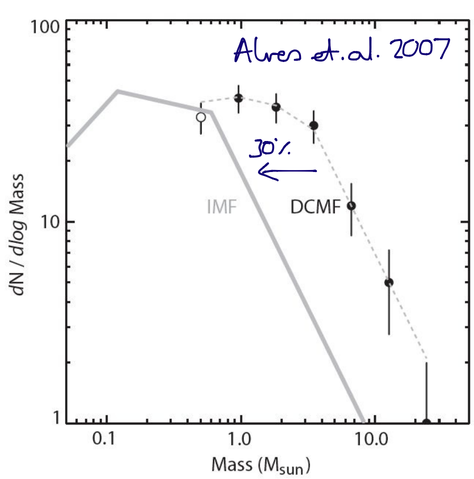


$$\xi(m) = \begin{cases} 0.26\, m^{-0.3} & 0.01 \leq m < 0.08 \\ 0.035\, m^{-1.3} & 0.08 \leq m < 0.5 \\ 0.019\, m^{-2.3} & 0.5 \leq m < \infty \end{cases}$$

For log scales subtract 1 from powers.

The IMF is common to all regions of star formation and so must be explained by any theory of SF.

In a similar way to this the mass function of the "clumps" or "cores" in molecular clouds can be plotted. This bears a strong resemblance to the IMF.



This has lead many authors to propose that the physics of SF is the physics of core formation.

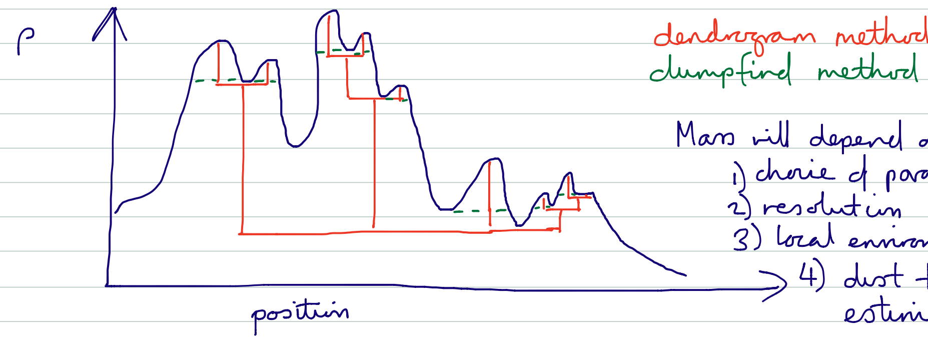
Constant efficiency

Alves 2007 eff = 30% ± 10%.

Supporting this theory
- → stars form in dense cores
- → similarity of shapes.

Problems
- → lognormal is a very general profile is insensitive to the transformation applied
- → multiplicity
- → how do you define a core mass.

Algorithms which 'identify structure have to draw abitrary boundaries.

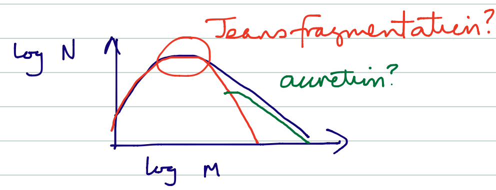


Mass will depend on
1) choice of parameters
2) resolution
3) local environment
4) dust temperature estimate

13.4 Accretion and Massive Stars

At densities of 10^5 Mo and temperatures of ~ 10K as is typical in dense molecular cores the Jeans mass is ~ 0.6 Mo.

This corresponds well to the peak of the IMF. At $n \sim 10^5$ the gas temperature no longer cools with increasing density. It has been suggested that this "kink" in the equation of state determines the peak stellar mass.

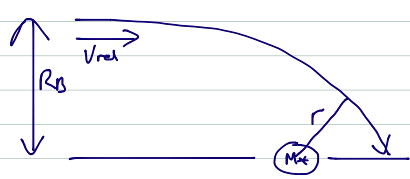


If there is a 1-1 correlation between cores and stars how do you support a massive core?

Turbulence : replace c_s with $\sigma(v_{turb})$
⇒ to support enough mass to form massive star turbulence must be supersonic from Larsons laws.
⇒ supersonic turbulence encourages fragmentation.

Another way of forming a more massive star is to start with a smaller object and grow it through accretion.

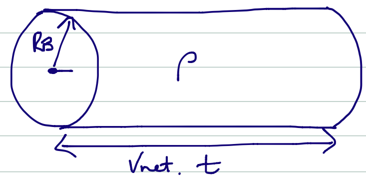
Bondi-Hoyle-Lyttleton accretion



star moving at constant v through a uniform gas cloud

$$R_B = \frac{2GM_*}{v_{rel}^2 + c_s^2}$$ v_{rel} relative velocity of the gas and star
c_s sound speed of the gas

From this we can derive an accretion rate for protostars in molecular clouds.

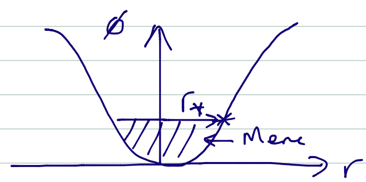


M accreted per unit time

$$M = Vol \times \rho$$
$$M = \pi R_B^2 v_{rel} t \rho$$
$$\Rightarrow \quad \dot{M} = \pi \rho \sqrt{v_{rel}^2 + c_s^2}\ R_B^2$$

Once again when applied to real molecular clouds there are problems with this estimate
1) the medium is highly inhomogeneous
2) the relative velocity between star & gas is small
3) in a cluster there is a more complicated tidal field

For a more accurate estimate use the tidal radius (equivalent to Roche lobe) set by the cluster potential. Describes how strong stars gravity is compared to the local potential.

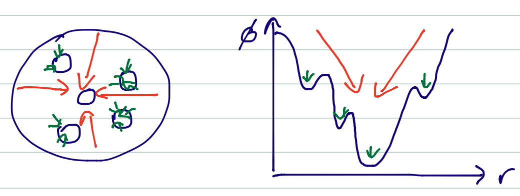


$$R_t = C_{tidal}\left(\frac{M_*}{M_{enc}}\right)^{1/3} r_*$$

$$\dot{M}_t = \pi \rho v_{rel} R_t^2 \qquad C_{tidal} \sim 0.5$$

In typical simulated young clusters $\dot{M} \sim 10^{-4}\ M_\odot yr$ as the protostellar mass increases.
$\rightarrow$ If $t \sim 10^5 yr$ $\quad M_* > 10\ M_\odot$

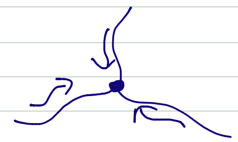
Moreover the very shape of the molecular cloud will aid the accretion of objects at the centre of the potential of a collapsing cloud.



Molecular clouds with net collapse gradients of a few $km\,s^{-1}$ are observed

$$M_{flux} = 10^5\ cm\ s^{-1} \cdot 10^3\ cm^{-3} \qquad n = 1000$$
$$= 10^8\ \text{particles}\ cm^{-2} s^{-1}$$

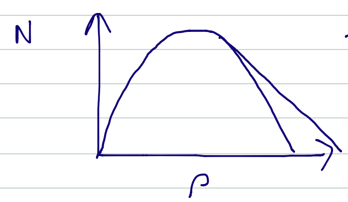
The filamentary geometry of molecular clouds may be another way of acheiving continuous high accretion rates



High accretion rates where the filaments converge.

13.5 Turbulence controlled star formation

The density distribution in molecular clouds is lognormal with a power law tail that is usually attributed to gravity.



The properties of this pdf can be related to that of the turbulence in the medium using numerical simulations

$$s = \log(\rho/\bar{\rho}) \quad \text{logarithm of the density}$$

$$p_s\, ds = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left[\frac{-(s-s_0)^2}{2\sigma_s}\right] ds$$

where $s_0 = -\sigma_s^2/2$

The density variance for a lognormal distribution is equivalent to

$$\sigma_0^2 = \ln(1 + b^2 M^2)$$

where $b = 1$ for compressive forcing $\nabla \times \underline{f} = 0$
$b = 1/3$ for solenoidal forcing $\nabla \cdot \underline{f} = 0$
$b = 0.4$ for a natural mix of modes

When magnetic fields are present the density variance is lower than the unmagnetised case.

Given this density distribution you can use the Press Schechter formalism to predict the mass distribution of structures defined by a given density threshold.

NB ASIDE

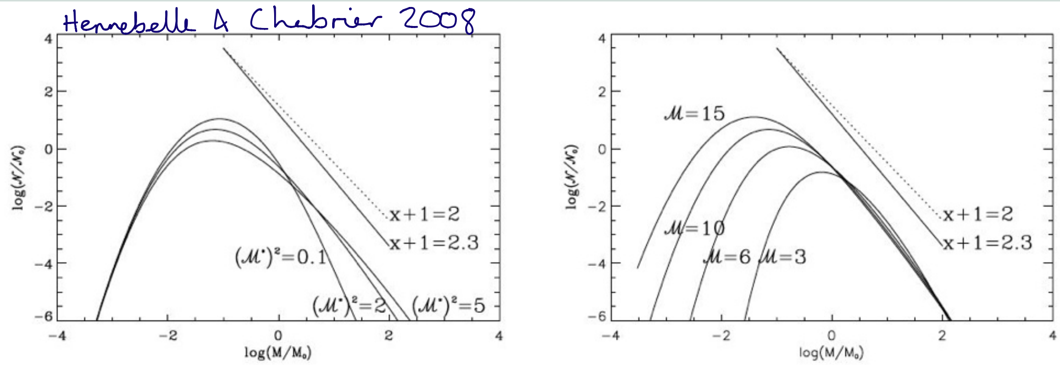
Press-Schechter formalism predicts that the number of objects with a mass between M and $M + dM$ is

$$N(M)\, dM = \frac{1}{\sqrt{\pi}} \left(1 + \frac{n}{3}\right) \frac{\bar{\rho}}{M^2} \left(\frac{M}{M_*}\right)^{3+n/6} \exp\left(-\left(\frac{M}{M_*}\right)^{\frac{3+n}{3}}\right) dM$$

where n is the index of the power spectrum of fluctuations $P(k) \propto k^n$
and M_* is a critical mass.

Hennebelle and Chabrier 2008 find that the resulting CMF depends on
→ $\mathcal{M}$ the mach number at the cloud scale
→ $\mathcal{M}^*$ the mach number at the Jeans scale

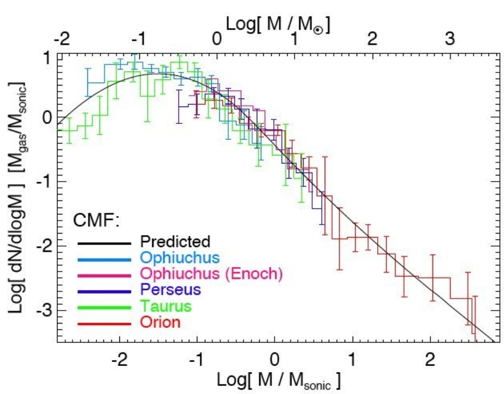
Hennebelle & Chabrier 2008



This bears a strong resemblence to the CMF.

However there still remains the "cloud in cloud" problem. Essentially this arises because structures can be bound on a variety of scales.

This analysis has been further improved by Hopkins 2012 who used a probabilistic method to derive a function of the "last crossing" mass, which is the smallest self-gravitating mass.



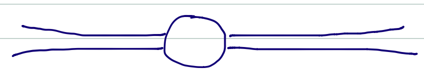
Hopkins 2012

However in this formalism there can still be accretion from the larger bound scales onto the smaller ones. So the transformation between massive cores and massive stars is still unclear.

Hower in smaller cores without accretion we can make an estimate.

13.6 Efficiency of low mass star formation

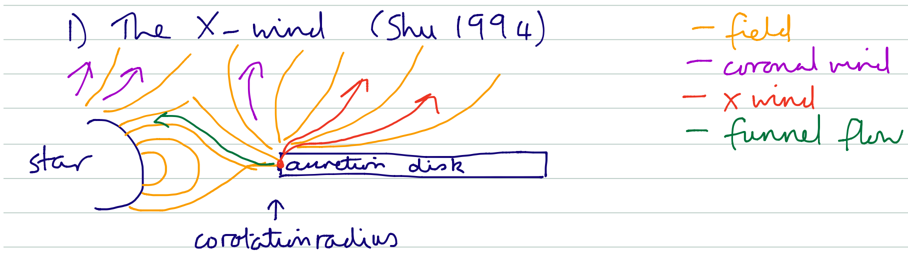
Cores in molecular clouds will have some initial rotation due to the random turbulence in the cloud. Conservation of angular momentum will increase this as the core collapses.



$\Rightarrow$ Most of the core will be accreted onto the star via a disk.

This disk will be threaded by magnetic fields that can generate outflows. There are two major models

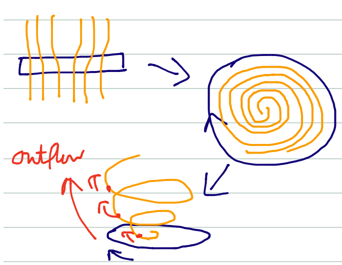
1) The X-wind (Shu 1994)



The corotation radius is regulated by the magnetic field
- if star rotates faster than the disk the field would wind up pulling on the disk and making it rotate faster

In a true steady state the excess angular momentum brought towards the star will be transferred outwards by magnetic torques to the footpoints of the magnetic field in the disk.

2) The magnetic tower model



As the disk rotates it winds up the field.

This creates a region dominated by magnetic pressure above the disk

The twisting field of the magnetic tower lifts material off the surface of the disk if it is trapped in the field as it rotates

Using such models one can estimate how much mass is lost to the jet.

Method 1: 30-70% off Matzner & McKee 2000
Method 2; $\frac{M_{jet}}{M_{acc}} \sim \frac{1}{3}$ Banerjee & Pudritz 2006

An efficiency of ~30% best matches the observations.