Assignment #1: due April 30

1. **Two-level atom**

   Consider a two-level model atom with lower level 0 and upper level 1, differing in energy by an amount E_{10} and with statistical weights g_0 and g_1. Suppose that we have a diffuse gas that contains many such model atoms and that this gas is in local thermodynamic equilibrium (LTE).

   (a) Give an expression for the number density of atoms in the upper level in terms of the total number density of atoms, n, and the kinetic temperature of the gas, T.

   (b) Calculate the rate per unit volume at which energy is radiated by the gas in terms of A_{10}, the spontaneous radiative transition rate between level 1 and level 0. Assume that the external radiation field is negligible (i.e. $I_{10} \simeq 0$).

   (c) Repeat part (b), but for the case where the external radiation field is given by the Planck function, with temperature $T_{\rm r}$. Show that when $T_r > T$, more energy is absorbed by the gas than is radiated away.

2. **Cooling via the 21 cm line of hydrogen**

   Suppose we have a gas consisting of pure atomic hydrogen, with temperature $T = 100$ K, number density $n = 1\ {\rm cm}^{-3}$, and an internal energy density $\epsilon = \frac{3}{2}nkT$, where k is the Boltzmann constant. Estimate how long it would take for this gas to cool to 50 K via the emission of radiation in the 21 cm hyperfine line. Is this process ever important?

   Note: $A_{10} = 2.869 \times 10^{-15}\ {\rm s}^{-1}$ for the 21 cm line.