Assignment #2: due May 15

1. **Optical depth of Lyman-α line**

   (a) Consider a gas composed of pure atomic hydrogen, with a temperature $T = 10^4\,\mathrm{K}$ and a number density $n_{\mathrm{H}} = 1\ \mathrm{cm}^{-3}$. How large must the column density of this gas be for it to produce an optical depth $\tau = 1$ at the centre of the Lyman-α line?

   Note: you may assume that all of the hydrogen atoms are in their ground state, and that the line profile is given by pure Doppler broadening. The Einstein A coefficient for the Lyman-α transition is: $A_{21} = 6.3 \times 10^8\ \mathrm{s}^{-1}$.

   (b) Suppose that this gas has an optical depth in the Lyman-α line that is much greater than one, i.e. $\tau \gg 1$. What does this imply for the ability of the gas to cool via Lyman-α radiation?

2. **21 cm emission**

   (a) Consider the same cloud of hydrogen as in question 1. Suppose that the cloud is illuminated from behind by a radiation field with a Planck spectrum with temperature $T_{\mathrm{r}} = 6000$ K. Sketch the shape of the spectrum that we would observe from the cloud in the vicinity of the 21 cm line.

   (b) Repeat part (a), but this time assume that the temperature of the hydrogen in the cloud is only 100 K.

   (c) For the cloud in part (b), compute the optical depth at line centre of the 21 cm line as a function of the column density of atomic hydrogen. Assume that the hyperfine levels of hydrogen have a thermal (i.e. Boltzmann) distribution.

   Note: $A_{10} = 2.869 \times 10^{-15}\ \mathrm{s}^{-1}$ for the 21 cm line.

3. **Source function**

   The source function for a two-level atom at a frequency ν can be written as

$$S_\nu = \frac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}}, \tag{1}$$

   where n_u and n_l are the number densities of the upper and lower levels, respectively. Use this equation to show that when the energy levels of the atom are in local thermodynamic equilibrium with a temperature T, the source function becomes $S_\nu = B_\nu(T)$, where B_ν is the Planck function:

$$B_\nu(T) = \frac{2h\nu_{ul}^3/c^2}{\exp\left(\frac{E_{ul}}{kT}\right) - 1}. \tag{2}$$