Assignment #3: due May 22

1. **C^+ cooling**

One of the most important atomic coolants in the interstellar medium is singly-ionized carbon, C^+. This ion has fine structure in its ground-state, via the transition ${}^2P_{3/2} \rightarrow {}^2P_{1/2}$, which produces an emission line at a wavelength of 158 μm. The spontaneous radiative decay rate for this transition is given by $A_{21} = 2.4 \times 10^{-6}\,\mathrm{s}^{-1}$.

(a) Write down expressions for the number density of C^+ ions in the lower state (n_1) and the upper state (n_2) in terms of the total number density of ions (n_{C^+}). Assume that the effects of absorption and stimulated emission can be ignored.

(b) We next assume that collisional excitation of the upper state is due primarily to collisions between C^+ and electrons. The collisional de-excitation rate coefficient can be written as:

$$q_{21} = \int_0^\infty \sigma_{21}(E) v f(v) 4\pi v^2 \mathrm{d}v, \tag{1}$$

where $E = \mu v^2/2$ is the energy of the collider in the rest-frame of the C^+ ion, μ is the reduced mass of the system and

$$f(v) = \left(\frac{\mu}{2\pi kT}\right)^{3/2} \exp\left(\frac{-\mu v^2}{2kT}\right) \tag{2}$$

is the velocity distribution function.

Show that if the collisional de-excitation cross-section scales with energy as $\sigma_{21} \propto E^{-1}$, then q_{21} scales with the temperature as $q_{21} \propto T^{-1/2}$.

(c) Assume now that $q_{21} = 3.9 \times 10^{-7} \left(\frac{T}{100}\right)^{-1/2}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$. Use this to compute the critical density of electrons for which collisional de-excitation becomes as important as radiative de-excitation.

(d) Assume that $n_{\mathrm{e}} \ll n_{\mathrm{e,crit}}$. Use this fact plus the information given above to construct the cooling rate due to C^+ fine-structure emission, in terms of the number densities of C^+ ions and electrons.

(e) Suppose that the gas is being heated by photoelectric emission from dust grains, at a rate

$$\Gamma_{\mathrm{pe}} = 5 \times 10^{-26} n\,\mathrm{erg\,s^{-1}\,cm^{-3}}, \tag{3}$$

where n is the number density of H nuclei. If $n = 100\,\mathrm{cm}^{-3}$, $n_{C^+} = 2 \times 10^{-2}\,\mathrm{cm}^{-3}$ and $n_{\mathrm{e}} = 0.1\,\mathrm{cm}^{-3}$, what is the equilibrium temperature at which fine structure cooling balances photoelectric heating?

(f) If the gas temperature is initially 6000 K, how long does it take to cool to the equilibrium temperature you derived in part (e)?

2. **Excitation temperature**

Consider a two-level atom with lower level l and upper level u. Show that the excitation temperature ($T_{\rm ex}$) of this atom is given by

$$\frac{T}{T_{\rm ex}} - 1 = \frac{kT}{E_{ul}} \ln\left(1 + \frac{n_{\rm cr}}{n}\right), \tag{4}$$

where E_{ul} is the energy separation of the two levels, $n_{\rm cr}$ is the critical density of the transition $u \to l$, T is the kinetic temperature of the gas and n is the gas number density.