Assignment #4: due June 5

1. **Photoionization equilibrium**

   Consider a small parcel of gas illuminated by ionizing radiation with a mean specific intensity

   $$J_\nu = J_0 \left(\frac{\nu_0}{\nu}\right)^2, \tag{1}$$

   where $J_0 = 10^{-17}\,\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,sr^{-1}}$ and $h\nu_0 = 13.6$ eV.

   (a) Compute the ionization rate in this region as a function of the hydrogen number density n_{H}. For simplicity, you may adopt the approximation

   $$\sigma_\nu \simeq \sigma_0 \left(\frac{\nu_0}{\nu}\right)^3 \tag{2}$$

   for the hydrogen photoionization cross-section.

   (b) Assume that the gas is composed of pure hydrogen and is initially fully atomic. Estimate the time taken for the gas to reach photoionization equilibrium.

   (c) Compute the heating rate of the gas due to photoionization.

   (d) Now assume that the number density of hydrogen nuclei is $n = 100\,\mathrm{cm^{-3}}$. Using the following simple approximation to the recombination rate coefficient

   $$\alpha_{\mathrm{rec}} = 2 \times 10^{-10} T^{-0.64}\,\mathrm{cm^3\,s^{-1}}, \tag{3}$$

   estimate the equilibrium fractional ionization of the gas as a function of temperature.

   (e) With the help of your results to parts (c) and (d), calculate the equilibrium temperature of the ionized gas to the nearest 100 K. You may assume that the dominant heating and cooling processes are photoionization heating and Lyman-α cooling, respectively.

2. **A recombination paradox**

   When an ionized gas first begins to recombine, its temperature *increases*, despite the fact that it is emitting energy in the form of recombination radiation. Explain why this is the case.