Assignment #10:

1. **Magnetic Support and Star Formation**

(a) The equation of virial equilibrium for a uniform sphere threaded by a constant magnetic field B can be written as follows if all surface terms apart from a constant pressure term P_0 are neglected,

$$4\pi R^3 P_0 = 3\frac{Mk_BT}{\mu} - \frac{1}{R}\left(\frac{3}{5}GM^2 - \frac{1}{3}R^4B^2\right) \tag{1}$$

where $M = (4/3)\pi R^3\rho$ is the mass of the region, k_B is the Boltzmann constant, T is the temperature and μ is the molecular weight. The terms in parenthesis represent gravity and magnetic forces. Show that the critical mass above which gravity can overwhelm magnetic repulsion is

$$M_{cr} = \frac{5^{3/2}}{48\pi^2}\frac{B^3}{G^{3/2}\rho^2} = 4\times10^6\mathrm{M}_\odot\left(\mathrm{n}/1\mathrm{cm}^{-3}\right)^{-2}\left(\frac{\mathrm{B}}{3\mu\mathrm{G}}\right)^3 \tag{2}$$

(b) The critical mass can also be expressed as a typical mass to flux ratio

$$\left(\frac{M}{\Phi}\right)_{cr} = \frac{\zeta}{3\pi}\left(\frac{5}{G}\right)^{1/2} = 490\mathrm{gG}^{-1}\mathrm{cm}^{-2} \tag{3}$$

For a typical interstellar field of 3μG what number density does this correspond to in a sheet of thickness 1pc for $\mu = 1.3$?

(c) If the ionisation is sufficiently high that the field is frozen in the magnetic field increases linearly with density as the cloud collapses what does this imply for the critical mass? Comment on the likelihood of this scenario.

2. **Accretion**
Consider a 1 $\mathrm{M}_\odot$ protostar traveling on a 3 kms^{-1} orbit at a radius of 0.5 pc within a uniform molecular cloud. The cloud has a mean temperature of 20K and a molecular weight of 2.3.

(a) How dense would the cloud have to be for the protostar to have a Bondi-Hoyle-Littleton accretion rate above 10^{-4} $\mathrm{M}_\odot\mathrm{yr}^{-1}$?

(b) How massive a molecular cloud does this correspond to? Comment on the likelihood of this scenario for forming massive stars in 10^5 yr.

(c) What would the tidal accretion rate be for a molecular cloud density of $n = 10^3$ cm^{-3}?

(d) If the molecular cloud was centrally condensed how would this change the tidal accretion qualitatively?