ON FREQUENCY AND STRENGTH OF SHOCK WAVES IN THE SOLAR ATMOSPHERE

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Abstract. Comparison of computed radiative energy losses of several new empirical chromospheric models with heating by shock wave dissipation gives information on the frequency and strength of shock waves in the solar chromosphere. A mechanical flux of around 2.5×10^6 erg/cm^2 sec is found for the base of the chromosphere. The shocks are weak and the wave period is around 10 sec.

1. Introduction

In recent attempts (Kuperus, 1965; Ulmschneider, 1967; Kopp, 1968) to compute theoretical models of the chromosphere and the chromosphere-corona transition regions a high degree of uncertainty exists about the frequency ν and strength $\bar{\eta}$ of shock waves. Because in the chromosphere the radiative losses are balanced by the dissipation of shock waves and because shock heating in turn is directly proportional to the frequency of the shock wave, an uncertainty in the frequency will immediately lead to an uncertainty of the theoretical model. It is moreover puzzling that although it is currently fairly certain (e.g. Kuperus, 1969) that shock dissipation provides the ultimate energy input for chromosphere and corona, no travelling shocks have been observed. Note that although in the chromosphere-corona transition layer thermal conduction is the main heating mechanism, conduction only transports energy from the shock heated upper transition layer and low corona down into the transition zone.

For the period $P = 1/\nu$ of the shock waves Kuperus (1965, p. 39) took 300 sec, because oscillations of the 300 sec type are observed. Ulmschneider (1967) took $P =$ 110 sec, based on the theoretical reasoning of Osterbrock (1961) that the sound frequency spectrum produced in the convection zone should have a maximum at the frequency $\nu = \bar{v}/H$ where H is the scale height at the top of the convection zone and $\bar{v}$ the mean velocity of rising turbulence elements. Kopp (1968, p. 228) computed cases with $P = 100$, 300 and 600 sec. These values with a sound velocity of 7 km/sec in the upper photosphere give wavelengths between 700 and 4200 km. Waves of this wavelength should be observed however as periodic frequency shifts of spectral lines.

A possibility to account for the failure of observation would be that these waves have much smaller periods of around 10 sec. The wavelength of 70 km would then fall well within the region of formation of almost any spectral line contributing to line broadening but not to any frequency shift. Because shock waves represent the main energy input into the chromosphere this broadening would seem to be the main contribution to the observed microturbulence.

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To test this short period suggestion we may use recent empirical models of the chromosphere, the Bilderberg model (BCA) (Gingerich and De Jager, 1968), the new Harvard reference atmosphere (HRA) (Noyes *et al.*, 1969) and the two models of Athay (1969a), which are based on new UV and submillimeter as well as eclipse observations. These models also give information on the total radiative flux in the chromosphere to be balanced by mechanical heating. Thus the mechanical flux and the strength of the shockwave can be inferred. This settles the question whether equations for strong or for weak shock waves may be used.

In Section 2 we compute the radiative losses as function of height in the empirical models. Integration of these loss curves gives the total radiative flux and therefore mechanical flux necessary to balance the atmosphere. The shock strength and relevant shock equation, the development and rate of dissipation of shock waves superposed over the empirical models is shown in Section 3. We start with various initial mechanical fluxes and treat the frequency as a free parameter. Because the dissipation of shocks changes drastically with frequency and not nearly as much with initial flux, a comparison with the radiative loss curves proves to be a sensitive method to determine the frequency.

2. Computation of the Radiative Energy Losses

The radiative energy losses in the lower chromosphere are shown by Athay (1966) to be mainly due to the H^- ion and the Lyman and Balmer series of Hydrogen.

A. THE H^- LOSSES

The total energy flux πF in erg/cm^2 sec due to H^- has been computed by Athay (1966) using

$$\Delta\pi F = 16\,\sigma T_0^3 (T - T_0)\,\Delta\bar{\tau} \tag{1}$$

given also by Osterbrock (1961). Here T_0 is the boundary temperature of the sun, T the local kinetic temperature and $\Delta\bar{\tau}$ the total optical thickness of the chromosphere from the temperature minimum outward using Rosseland's mean opacity. $\sigma = 5.669 \times 10^{-5}$ erg/cm^2 sec deg^4 is the Stefan-Boltzmann constant. Note that both Athay (1966) and Osterbrock (1961) used a different value of σ (Athay, 1969b). Because the H^- losses are most important we use a different way to compute the H^- losses. Assuming LTE and considering only the H^- contribution we may write for the transfer equation in a plane parallel atmosphere

$$\mathrm{d}I_\nu = (B_\nu - I_\nu)\,\kappa_\nu^{H^-}\,\frac{\mathrm{d}h}{\mu} \tag{2}$$

where $\kappa_\nu^{H^-}$ is the H^- opacity and B_ν the Planck function. Assuming that the atmosphere is optically thin we may neglect the term $\sim I_\nu$ in Equation (2).

Note now that dissipation by mechanical waves balances only those radiative losses which occur because of the rise of the kinetic temperature above the boundary temperature T_0. Integrating Equation (2) over solid angle and over frequency we obtain

$$\left.\frac{\mathrm{d}\pi F}{\mathrm{d}h}\right|_{\mathrm{H}^-} = 4\pi \int_0^\infty \left(B_\nu(T) - B_\nu(T_0)\right) \kappa_\nu^{\mathrm{H}^-}\, \mathrm{d}\nu \qquad \mathrm{erg/cm^2\,sec.} \tag{3}$$

Setting $\kappa_\nu^{\mathrm{H}^-} = \bar{\kappa}$ we regain Equation (1) after doing the frequency integration and assuming $(T - T_0) \ll T$. Using the H^- opacity given by Gingerich (1964) we may integrate Equation (3) for any given solar model.

B. THE H LOSSES

The losses due to the Lyman and Balmer series of hydrogen may be computed following Athay (1966).

$$\left.\frac{\mathrm{d}\pi F}{\mathrm{d}h}\right|_{\mathrm{Ly}} = \langle h\nu_{u1}\rangle\, N_1 C_{12} \qquad \mathrm{erg/cm^3\,sec}, \tag{4}$$

$$\left.\frac{\mathrm{d}\pi F}{\mathrm{d}h}\right|_{\mathrm{Ba}} = \langle h\nu_{u2}\rangle\, N_2 C_{23} \qquad \mathrm{erg/cm^3\,sec}, \tag{5}$$

where N_1 and N_2 are the number densities of hydrogen atoms in the first and second levels, C_{12} and C_{23} the collisional excitation rates of the Lyα and Hα transitions which may be computed with help of Equation (6.24) of Jefferies (1968), $\langle h\nu_{u1}\rangle = h\nu_{12}$ and $\langle h\nu_{u2}\rangle = 4.10^{-12}$ are mean energies of the Lyman and Balmer series respectively as given by Athay. As in the low chromosphere in Athay's (1969a) models $b_1 = b_2$, we use C_{13} instead of C_{12} in Equation (4) and subtract emission due to the boundary temperature for his models.

C. RADIATION LOSSES IN THE SOLAR MODELS

Figure 1 shows the total radiative energy loss assuming different boundary temperature T_0 for the Bilderberg model (BCA), the Harvard reference atmosphere (HRA) and for both of Athay's models (Athay I, II). Because for the HRA model only the $T(\tau)$ relation is given, a pressure and height integration was done using H^-, H, Rayleigh and Thompson scattering as opacity sources (HRA1). Same as in the BCA model LTE was assumed and the initial gas pressure was zero. Because the initial pressure should at least be the coronal pressure and departures from LTE affect the opacity a second pressure and height integration was done using $b_1 = 10$ and starting with $p = 10^{-1}$ dyn/cm^2 (HRA2). Values obtained this way are given in Table I.

Integrating the total radiative energy loss over height one obtains the total flux (see Table II) which has to be balanced by a mechanical flux.

Note that the values given by Osterbrock (1961) and Ulmschneider (1967) for the total mechanical flux of acoustic waves produced by the convection zone indicate either the severe damping of acoustic waves before the temperature minimum or the inaccuracy of the Lighthill (1952) method used to compute the total flux or both.

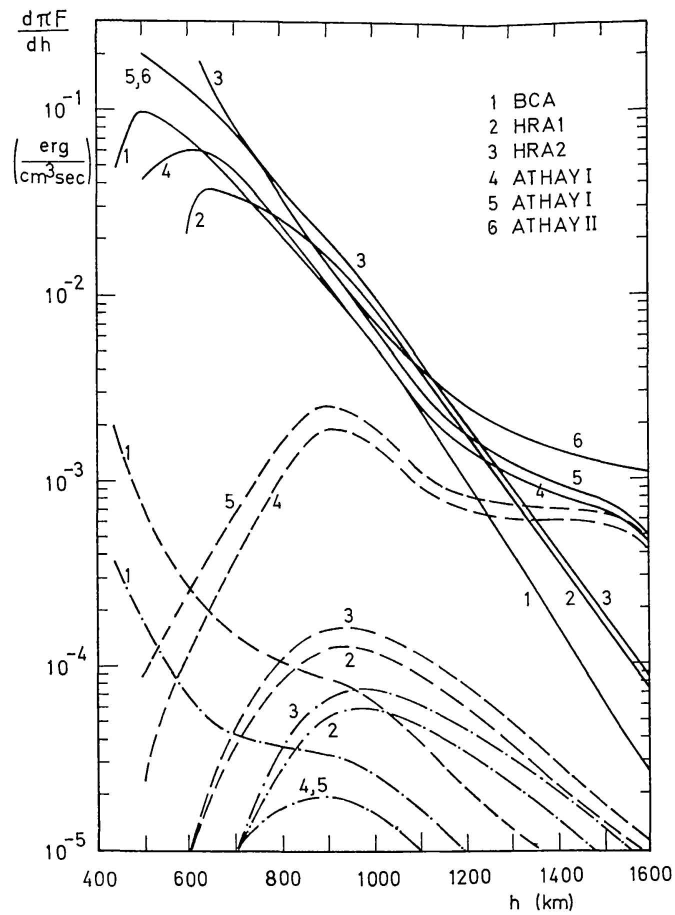

Fig. 1. Radiative energy losses as function of geometrical height (zero at $\tau_{5000} = 1$) are shown computed for different empirical models. These models are described in the text. The boundary temperatures are given in Table II (Curve 4 has $T_0 = 4600$ K). To the height scale of Athay's models 300 km have been added arbitrarily. The total losses are shown (drawn), the Ba series losses (dashed) and the Ly series contribution (dash-point-dash).

TABLE I

Chromospheric models HRA1 and HRA2 computed from the $T(\tau)$ relation of Noyes *et al.* (1969). Zero height level is at $\tau_{5000} = 1$

HRA1:

H (km)	T (K)	N_H (cm^{-3})	N_e (cm^{-3})
591	4255	1.04×10^{15}	6.09×10^{10}
820	5070	1.07×10^{14}	6.08×10^{10}
1109	5580	9.41×10^{12}	7.23×10^{10}
1474	6000	5.42×10^{11}	4.82×10^{10}

HRA2:

H (km)	T (K)	N_H (cm^{-3})	N_e (cm^{-3})
617	4255	1.31×10^{15}	4.05×10^{10}
800	5070	1.81×10^{14}	4.04×10^{10}
1027	5580	2.64×10^{13}	4.37×10^{10}
1345	6000	2.28×10^{12}	3.20×10^{10}

TABLE II

Total radiative flux πF_{rad} of the solar chromosphere computed from different models with different boundary temperatures T_0 taken. The last two entries show computed noise fluxes in the convection zone

F_{rad} (erg/cm^2 sec)	T_0 (K)	Model
2.2×10^6	4600	BCA (1968)
1.1×10^6	4255	HRA1 (1969)
2.1×10^6	3755	HRA2 (1969)
2.0×10^6	4600	Athay I (1969a)
3.5×10^6	4000	Athay I (1969a)
3.6×10^6	4000	Athay II (1969a)
5.6×10^6	4300	Athay (1966)
3.3×10^7	–	Osterbrock (1961)
1.6×10^7	–	Ulmschneider (1967)

3. The Energy Dissipation of Shock Waves

A. THE TYPE OF SHOCK WAVE

Conductive heating is unimportant in the lower chromosphere (see Table III).

There might however be an additional source of energy in the dissipation of the 300 sec type resonant oscillations discovered by Leighton (1960). However as the oscillations possibly derive their energy again from shock waves (Schmidt, 1969) it seems that their inclusion acts in such a way as to increase slightly the dissipation rate of shock waves at each height level. Because the dissipation rate of shocks is proportional to the frequency ν this yields an effective frequency $\nu' > \nu$ in the dissipation term.

TABLE III

Conductive flux πF_{cond}, coefficient of thermal conductivity K, temperature gradient $\mathrm{d}T/\mathrm{d}h$ vs. height in the BCA (1968) model

πF_{cond} (erg/cm^2 sec)	K (erg/cm sec K)	$\mathrm{d}T/\mathrm{d}h$ (K/km)	height (km)
0.4	1.3×10^5	0.3	451
1.4	1.5×10^5	0.93	1090
5.76	4×10^4	14	2210

To get an idea what type of shock waves we may expect on basis of the computations of the radiative flux of Section 2 we construct a wave which carries just an equal amount of mechanical flux πF_{mech}.

The shock strength $\bar{\eta}$ is defined (Osterbrock, 1961) as

$$\bar{\eta} = (\varrho_2 - \varrho_1)/\varrho_1\,, \tag{6}$$

where ϱ_2, ϱ_1, are the densities behind and in front of the shock respectively.

Ulmschneider (1967) has given πF_{mech} for a shock wave of arbitrary strength. Setting $\pi F_{\text{mech}} = \pi F_{\text{rad}}$ at arbitrary altitudes in the empirical models we find that always $\bar{\eta} \ll 1$ that is, weak shocks (see Table IV).

For weak shocks developing out of sound waves we expect a sawtooth shape (Osterbrock, 1961; Kuperus, 1969). For these the mechanical flux may be written

$$\pi F_{\text{mech}} = \nu \int_0^P (p - p_0)\, u \, \mathrm{d}t = \tfrac{1}{3}\, p_{00} u_{00} = \tfrac{1}{12}\, \gamma p_0 c \bar{\eta}^2 \,, \tag{7}$$

where $P = 1/\nu$ is the period of the wave, p the gas pressure, $p_0 \gg p$ the equilibrium pressure of the atmosphere, $2p_{00} = p_2 - p_1$ the pressure difference and $2u_{00}$ the velocity difference at the shock front.

Note that based on various reasonings the numerical factor in front of Equation (7)

TABLE IV

Shock strengths η obtained by setting the mechanical flux at various heights equal to the radiative flux

$\bar{\eta}$	F_{rad} (erg/cm^2 sec)	T_0 (K)	height (km)	Model
0.11	1.58×10^6	4600	529	BCA (1968)
0.174	1.18×10^5	4600	924	BCA (1968)
0.159	9.46×10^5	4255	647	HRA1 (1969)
0.111	2.00×10^6	4000	200	Athay I, II (1969)

is given to be $\frac{1}{8}$ by Schatzman (1949, p. 210), $\frac{1}{12}$ by Weymann (1960, p. 454), $1/8\sqrt{3}$ by Osterbrock (1961, p. 373) and $\frac{1}{3}$ by Kuperus (1965, p. 30, 31).

To derive Equation (7) we have used the connecting formulas (Landau and Lifshitz, 1959, p. 331) across the shock front and the time behavior of velocity and pressure,

$$p = p_0 + p_{00} - \frac{2p_{00}}{P}\, t, \quad u = u_{00} - \frac{2u_{00}}{P}\, t \,. \tag{8}$$

The connecting formulas yield for weak shocks

$$2p_{00} = \gamma p_0 \bar{\eta}, \quad 2u_{00} = c\bar{\eta} \,. \tag{9}$$

B. THE SHOCK EQUATION

The shock equation governs the growth and decay of $\bar{\eta}$ due to dissipation and changes in the atmosphere. It was first derived from the semi-empirical approach of Brinkley and Kirkwood (1947) who made use of the 'principle of shape similarity invariance' based on experiments of underwater and atomic explosions. A completely different approach treating shocks as the result of the development of large amplitude sound waves has been given by Landau and Lifshitz (1957, p. 372). They find shape similarity rigorously valid for sawtooth and triangle pulses. This approach has been shown

(Ulmschneider, 1966) to yield an identical shock equation as the Brinkley-Kirkwood theory thus increasing the confidence in their approach.

This has been adopted by Schatzman (1949), Weymann (1960), Osterbrock (1961), Uchida (1963) and Kuperus (1965) to the solar atmosphere. Ulmschneider (1966, 1967) has modified a third approach to a shock equation given by Bird (1964) to incorporate dissipation, to include the variability of γ and to increase the range of validity to arbitrarily strong shocks, because at that time it was thought that the shocks might be rather strong. The derivation of Bird and subsequently Ulmschneider neglected irreversible processes in front of and behind the shock relative to those in the shock front itself. These have been included by Kopp (1968). Kopp has however not included refraction, and the variability of γ which becomes important close to ionization of hydrogen. These shock equations may be simplified very much if we restrict ourselves to the chromosphere.

First we may neglect all effects due to a spherical geometry as we are only interested in the height region of 400–4000 km above the solar reference level at $\tau_{5000\,\text{Å}}=1$. We have an essentially plane atmosphere.

Second and most significantly we may neglect all influence of the solar wind flow. If we assume an ion density of 2/cm^2 and a wind velocity of 500 km/sec as measured by Mariner II (Neugebauer and Snyder, 1962) we get a mass flux at the solar surface of $\varrho_0 u_0 = 8 \times 10^{-12}$ g/cm^2 sec. Applying the steady state continuity equation in plane geometry,

$$\varrho u = \varrho_0 u_0 , \tag{10}$$

e.g. to the HRA1 (1969) model we obtain the solar wind velocity as function of height (see Table V).

Euler's equation reads in steady state

$$u \frac{\mathrm{d}u}{\mathrm{d}h} = - \frac{1}{\varrho} \frac{\mathrm{d}p}{\mathrm{d}h} - g . \tag{11}$$

TABLE V

Influence of the solar wind flow in the Euler equation on basis of the HRA1 (1969) model and with an empirical mass flux of 8×10^{-12} g/cm^2 sec at the solar surface

u (cm/sec)	u (du/dh) (1/cm)	ϱ (g/cm^3)	T (K)	height (km)
3.21×10^{-3}	1.05×10^{-12}	2.49×10^{-9}	4225	591
1.64×10^{-1}	1.51×10^{-9}	4.87×10^{-11}	5450	1015
8.97	4.70×10^{-6}	8.92×10^{-13}	6090	1537
1.51×10^{2}	1.02×10^{-3}	5.30×10^{-14}	8165	1993

The left hand term in this equation may now simply be computed by differentiating Equation (10). The term $u(\mathrm{d}u/\mathrm{d}h)$ should be compared with $g = 2.736 \times 10^4$ in Table V. It is easily seen that we have hydrostatic equilibrium throughout. The thermal energy of the solar wind may also be neglected in the energy balance. The energy flux due to the thermal energy of the solar wind is

$$\varrho u H \approx \varrho u k T / m_H \mu = 2.75 \times 10^2 \quad \mathrm{erg/cm^2\,sec}, \tag{12}$$

where H is the enthalpy per gram, k the Boltzmann constant, m_{H} the mass of the H atom, $\mu=0.6$ the mean molecular weight of fully ionized solar gas and $T=10^5$ a temperature in the transition region. With these approximations the shock equation reduces to a very simple form.

The dissipation of a shock wave may be computed following Brinkley and Kirkwood (1947), going over to weak shocks

$$\frac{\mathrm{d}\pi F_{\mathrm{mech}}}{\mathrm{d}h} = -\, v\varrho \Delta H = -\, v\varrho \frac{c^2}{\gamma(\gamma-1)} \ln\left(\frac{p_2}{p_1}\left(\frac{\varrho_2}{\varrho_1}\right)^{-\gamma}\right) \approx -\tfrac{1}{12}\,\gamma(\gamma+1)\, p\bar{\eta}^3 v,$$

where ΔH is the enthalpy difference per gram across the shock front. We may use the independent result of Landau and Lifshitz (1959, p. 377) for sawtooth waves,

$$\frac{1}{\pi F_{\mathrm{mech}}} \frac{\mathrm{d}\pi F_{\mathrm{mech}}}{\mathrm{d}h} = -\,(\gamma+1)\frac{\bar{\eta}v}{c} \tag{14}$$

which by using Equation (7) becomes identical with Equation (13). Setting now the derivative of Equation (7) with respect to height equal to this dissipation we obtain the shock equation

$$\frac{\mathrm{d}\bar{\eta}}{\mathrm{d}h} = \frac{\bar{\eta}}{2}\left(-\frac{1}{\gamma}\frac{\mathrm{d}\gamma}{\mathrm{d}h} + \frac{\gamma g}{c^2} - \frac{1}{2c^2}\frac{\mathrm{d}c^2}{\mathrm{d}h} - \frac{(\gamma+1)\,\bar{\eta}v}{c}\right). \tag{15}$$

In assuming that the numerical factor in front of Equation (13) is independent of height we made explicit use of shape similarity invariance. Equation (15) is in agreement with more general formulas of Ulmschneider (1967) and except the $(1/\gamma)(\mathrm{d}\gamma/\mathrm{d}h)$ term with Kopp (1968) in the limit of vanishing solar wind flow.

C. INCLUSION OF REFRACTION

Osterbrock (1961) has shown that in analogy to optics, where in a medium with the index of refraction n the quantity I/n^2 is conserved, I being the intensity, the quantity $\pi F_{\mathrm{mech}} c^2$ should be conserved for weak shocks. Considering a contact discontinuity and an incident shock, Kuperus (1965) found the same result. This gives an additional term yielding

$$\frac{\mathrm{d}\bar{\eta}}{\mathrm{d}h} = \frac{\bar{\eta}}{2}\left(-\frac{1}{\gamma}\frac{\mathrm{d}\gamma}{\mathrm{d}h} + \frac{\gamma g}{c^2} - \frac{3}{2c^2}\frac{\mathrm{d}c^2}{\mathrm{d}h} - \frac{(\gamma+1)\,\bar{\eta}v}{c}\right). \tag{16}$$

D. INTEGRATION OF THE SHOCK EQUATION

With a starting value of $\bar{\eta}$ the shock equation may be integrated easily on basis of an empirical solar model. γ was computed in LTE similarly to Ulmschneider (1967) where $c_v=(\partial E/\partial T)_\varrho$ and $c_p=(\partial H/\partial T)_p$ have now to be computed separately. Here E is the internal energy per gram and c_p, c_v the specific heats. Results are shown in Figures 2 to 5.

In Figures 2 to 5 the shock dissipation $d\pi F/dh$ in erg/cm^3 sec is shown obtained by integrating Equation (16) on basis of the empirical models and using Equation (13). The curves are shown with the wave period $P = 1/\nu$ and the initial mechanical flux as free parameter. $\bar{\eta}_{\text{initial}}$ is then computed via Equation (7). The radiative losses are

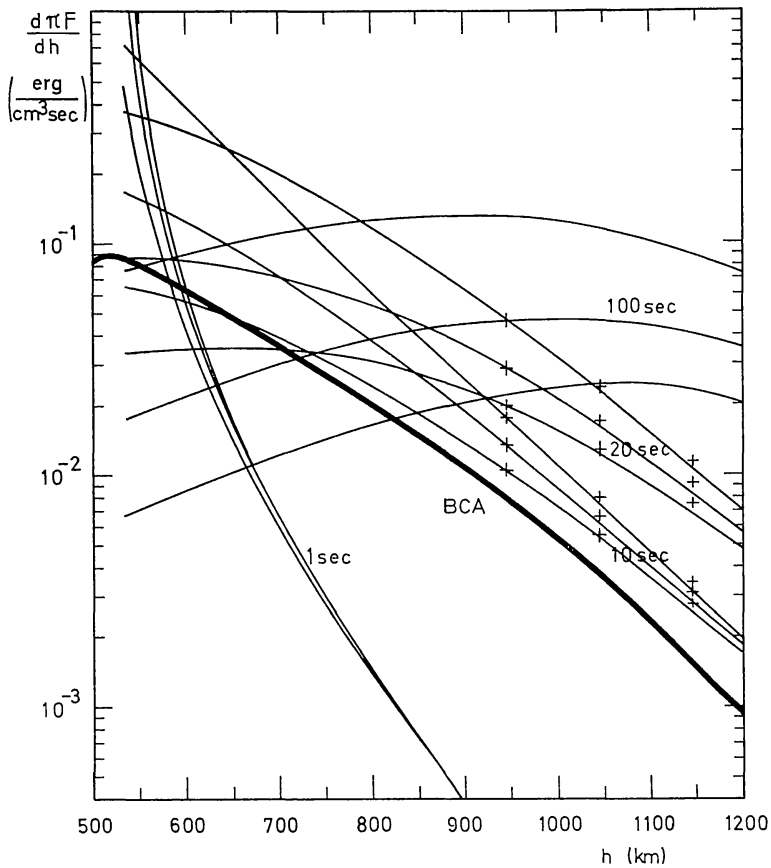

Fig. 2. Radiative energy loss (heavily drawn) compared with shock heating in the BCA model. There are 4 triplets of curves differing by the indicated wave periods P. The initial flux at $h = 544$ km in each triplet is from bottom to top: 1.6×10^6, 3.0×10^6, 8.0×10^6 erg/cm^2 sec. Influence of neglect of wave refraction is shown as crosses.

indicated in all figures. In Figure 2 we have in addition used Equation (15) instead of (16) to show the neglect of refraction. Because the temperature does not change very rapidly in the chromosphere this influence is small.

4. Conclusions

Although the difference in the empirical models are quite large in this analysis we find that the total mechanical flux input needed to balance the radiative losses in the chromosphere is about 2.5×10^6 ergs/cm^2 sec with an uncertainty of about a factor of 2. With fluxes of this type it is consistent in all models to have sawtooth type weak shock waves. Strong shocks are excluded. Most significantly we find that regardless of how much initial mechanical flux we have and how much the empirical models differ, a comparison of shock dissipation with radiative losses shows that the period of the shock waves should be around 10 sec with an uncertainty of about a factor of two or better, because shock heating has to balance radiative losses at every height lower than the transition layer.

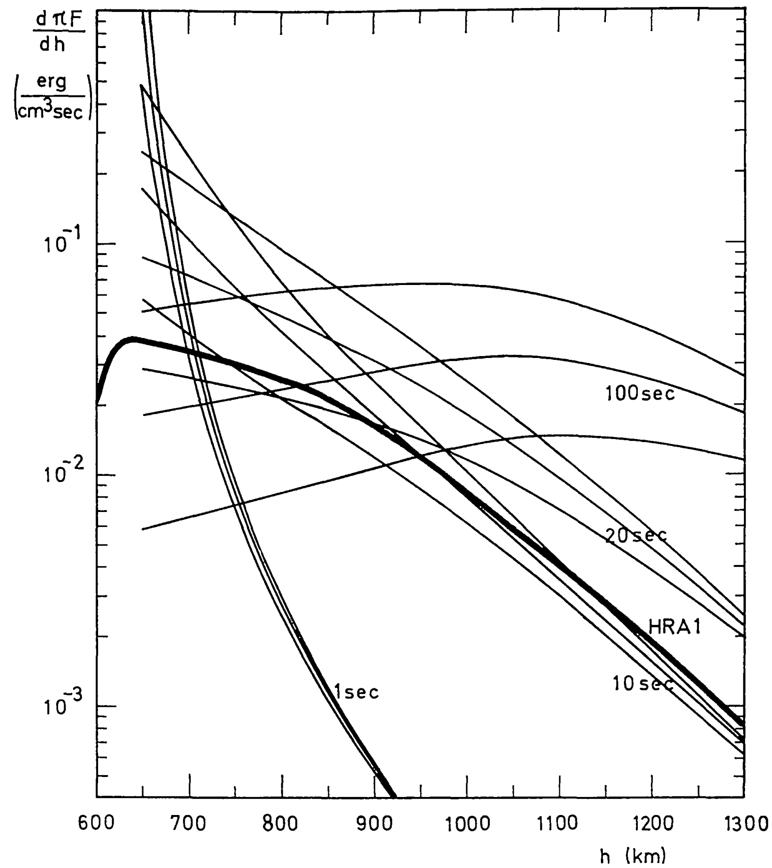

Fig. 3. Radiative energy loss (heavily drawn) compared with shock heating in the HRA1 model. There are 4 triplets of curves differing by the indicated wave periods P. The initial flux at $h = 640$ km in each triplet is from bottom to top: 9.5×10^5, 2.0×10^6, 4.0×10^6 erg/cm^2 sec.

The convergence toward greater height of curves of the same shock frequency but different initial fluxes exhibited in Figures 2–5 is due to the fact that asymptotically an equilibrium is reached between the steepening influence of the atmosphere and the influence of dissipation. For a not too big temperature gradient one sees from Equation (16) that $\bar{\eta}$ reaches at great heights a constant value

$$\bar{\eta} = \gamma g/(\gamma + 1)\, cv\,. \tag{17}$$

This is a well known property of shock waves in an isothermal gravitational atmosphere. With Equation (13) one gets asymptotically

$$\frac{\mathrm{d}\pi F_{\text{mech}}}{\mathrm{d}h} = \frac{1}{12} \frac{\gamma^4}{(\gamma + 1)^3} \frac{g^3 p}{c^3 v^2}\,, \tag{18}$$

independent of the initial flux, which may be used directly to get the frequency from radiative losses.

The uncertainty about how much energy feeds the 300 sec type oscillations is probably included in the uncertainty of the shock frequency. Because the radiative

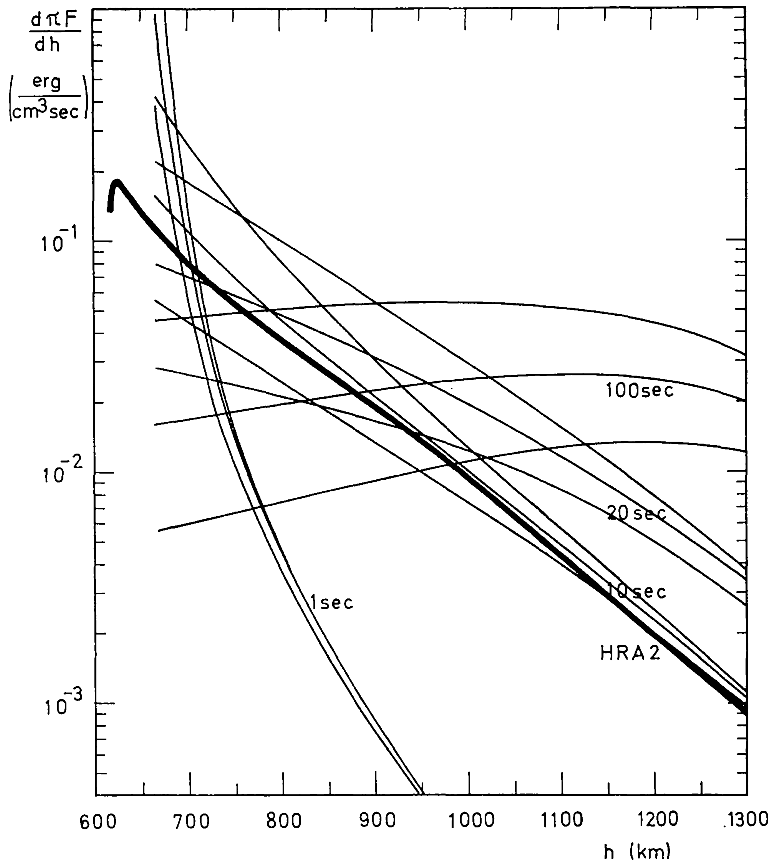

Fig. 4. Radiative energy loss (heavily drawn) compared with shock heating in the HRA2 model. There are 4 triplets of curves differing by the indicated wave periods P. The initial flux at $h = 660$ km in each triplet is from bottom to top: 1.0×10^6, 2.0×10^6, 4.0×10^6 erg/cm^2 sec.

losses of these oscillations are included in the empirical models, this just changes the above determined frequency into an effective frequency.

The question of a different heating mechanism may also be answered along these lines. A more efficient heating mechanism seems equivalent to using the Brinkley-Kirkwood mechanism with a shorter frequency.

There remains the question of a mechanism capable of producing sound waves with periods P of around 10 sec. Looking for the maximum of the spectrum of acoustic noise generation in the convection zone (Stein, 1968, Figure 4) one finds for Spiegel's turbulence spectrum $P = 29$ sec. and for the exponential spectrum $P = 39$ sec. Even granting a large uncertainty in this theoretical result it nevertheless points towards the sharp reduction of the currently adopted value of $P = 300$ sec.

For observational evidence on high frequency waves see Howard (1967).

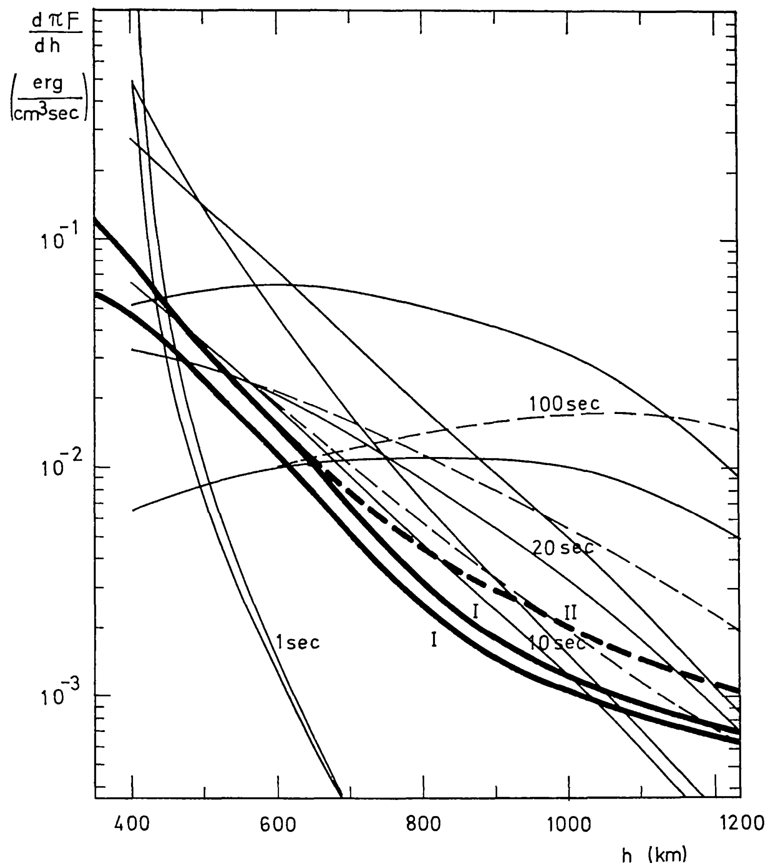

Fig. 5. Radiative energy loss (heavily drawn) compared with shock heating in Athay's models. I labels Athay I with $T_0 = 4600$ K top) and $T_0 = 4000$ K (bottom). Athay II curves are shown dashed. At $h = 400$ km two initial fluxes, 1.0×10^6 and 4.0×10^6 erg/cm^2 sec were used. Wave periods are indicated.

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