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THE EFFECT OF MECHANICAL WAVES ON EMPIRICAL SOLAR MODELS

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Abstract. Empirical solar models contain the effect of heating due to radiative energy loss from acoustic waves. We estimate here the temperature difference between the radiative equilibrium model and the empirical model. At optical depth $\tau_{5000} = 0.1$ this difference is small, but near the temperature minimum ($\tau_{5000} = 10^{-4}$) it reaches between 53 and 83 K. The temperature difference between the equator and the poles caused by a hypothetical difference in the heating is estimated.

1. Introduction

Empirical models of the solar atmosphere giving temperature and pressure versus height (e.g., Gingerich and de Jager, 1968; Gingerich *et al.*, 1971) are constructed from the observed frequency dependence of the intensity at the center of the solar disk and from the center-to-limb variation at various frequencies: We have tried to find the relation between temperature and pressure versus height that reproduces the observations best. In view of the multitude of wave phenomena that occur at every height, the resulting models give the time average of temperature and pressure fluctuations at every height.

One might now ask how these empirical models are affected by the presence of wave phenomena. This question was recently raised, for instance, by Ingersoll and Spiegel (1971) (see also Durney and Werner, 1973), who tried to explain the observed solar oblateness by differences in the amount of wave energy dumped in the directions toward the poles and the equator. On the other hand, a theoretical model atmosphere that excludes the effect of mechanical waves must ask for a comparison with a temperature-vs-height relation that is not affected by wave phenomena. Thus, again, we ask how the empirical solar models change if wave phenomena are 'switched off'.

Finally, in the field of solar wave propagation and chromospheric heating, we need an unperturbed atmosphere model on which the waves propagate, not a model that already includes the effect of wave dissipation.

At this point we want to distinguish two important effects. If we take the term 'heating of a mechanical wave' to mean the derivative of the mechanical flux of the wave with respect to geometrical height, then there are two ways of heating. The first, through the formation of shock waves, occurs at and beyond the temperature mini-

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mum. The second, through radiative losses suffered by the wave, occurs in much deeper layers than does shock heating. This can be seen from the following argument: It is estimated that the chromosphere above the temperature minimum emits a flux of about 2×10^6 erg cm^{-2} s^{-1} (Athay, 1966; Ulmschneider, 1970). On the other hand, an acoustic flux of about 7×10^7 erg cm^{-2} s^{-1} is produced in the convection zone (Stein, 1968) and propagates without appreciable reflection toward the temperature minimum. As there is no sink of acoustic energy beyond the temperature minimum other than the observed chromospheric and coronal emission, nearly all the mechanical flux of 7×10^7 erg cm^{-2} s^{-1} has to be converted into radiation in the photospheric layers below the temperature minimum.

Although this flux is small compared to the total solar flux of 6.4×10^{10} erg cm^{-2} s^{-1}, it is very important where it is emitted. If, for example, we assume that the radiative loss rate from the wave is depth independent, then we would obtain for a relevant height distance of 500 km an emission rate of 1.5 erg cm^{-3} s^{-1}. This is small compared to that of about 600 erg cm^{-3} s^{-1} at 100 km, but large compared to 0.1 erg cm^{-3} s^{-1} at 500 km.

If waves are present, the increased emission that is seen in the upper photosphere can be attributed to higher temperatures. This overestimate of the temperature in empirical solar models can be seen physically as follows: A wave enters a volume element with a positive temperature perturbation and leaves it, because of radiation losses, with a smaller negative temperature perturbation. Thus, the time-averaged temperature, T_{emp}, of the volume element is slightly higher than the temperature T_0 of the unperturbed atmosphere.

2. Computation

The difference

$$\Delta T = T_{\mathrm{emp}} - T_0 \tag{1}$$

between the temperatures of the empirical and the unperturbed atmospheres can be computed on the basis of a given relation between the mechanical flux πF_{mech} and the height. Assuming that the radiation appears through the dominant H^- radiation loss mechanism, we have the energy equation

$$\frac{\mathrm{d}\pi F_{\mathrm{mech}}}{\mathrm{d}h} = -4\pi \int_0^\infty K^{\mathrm{H}^-} \left[B_\nu(T_{\mathrm{emp}}) - B_\nu(T_0)\right] \mathrm{d}\nu \approx -16\sigma \bar{K} T_{\mathrm{emp}}^3 \Delta T\,, \tag{2}$$

where $B_\nu(T)$ is the Planck function, and σ, the Stefan-Boltzmann constant. For the mechanical flux πF_{mech} in Equation (2) we took values from Ulmschneider (1971) based on two acoustic noise spectra of Stein (1968).

The Harvard-Smithsonian Reference Atmosphere (HSRA) (Gingerich *et al.*, 1971) was taken as the empirical solar model. Because of the uncertainties associated with the mechanical fluxes, we used the published opacity at 5000 Å instead of Rosseland's

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TABLE I

Mechanical flux πF_{mech}, the empirical temperature T_{emp}, and the temperature difference ΔT between the empirical and an unperturbed solar atmosphere model as a function of optical depth, for different initial acoustic flux spectra. The last two columns show values for one-fifth Stein's SE flux

		EE spectrum		SE spectrum		One-fifth SE spectrum	
τ_{5000}	T_{emp} (K)	πF_{mech} (erg cm^{-2} s^{-1})	ΔT (K)	πF_{mech} (erg cm^{-2} s^{-1})	ΔT (K)	πF_{mech} (erg cm^{-2} s^{-1})	ΔT (K)
1.00E-4	4170	2.19E6	52.9	2.56E6	83.3	4.95E5	9.97
1.26E-4	4175	2.26E6	37.1	2.65E6	58.3	5.10E5	9.28
1.59E-4	4190	2.33E6	28.7	2.78E6	51.2	5.30E5	8.20
2.00E-4	4205	2.40E6	27.0	2.90E6	47.6	5.50E5	7.94
2.51E-4	4225	2.50E6	24.6	3.08E6	36.9	5.80E5	7.39
3.16E-4	4250	2.60E6	19.8	3.20E6	32.0	6.10E5	6.98
3.98E-4	4280	2.70E6	16.9	3.40E6	41.8	6.50E5	6.30
5.01E-4	4305	2.82E6	15.0	3.75E6	38.8	6.92E5	5.40
6.31E-4	4330	2.95E6	12.1	4.05E6	34.8	7.40E5	4.58
7.94E-4	4355	3.08E6	9.13	4.50E6	27.2	7.90E5	3.85
1.00E-3	4380	3.20E6	7.97	4.80E6	23.6	8.45E5	3.54
1.26E-3	4405	3.35E6	11.0	5.30E6	18.0	9.10E5	3.22
1.59E-3	4430	3.70E6	10.9	5.60E6	11.7	9.90E5	2.51
2.00E-3	4460	4.00E6	11.0	6.00E6	12.3	1.06E6	2.05
2.51E-3	4490	4.50E6	9.59	6.50E6	7.55	1.14E6	2.01
3.16E-3	4525	4.90E6	7.51	6.70E6	4.18	1.25E6	1.83
3.98E-3	4550	5.40E6	12.8	7.00E6	5.79	1.36E6	1.56
5.01E-3	4575	6.90E6	13.3	7.60E6	7.87	1.49E6	1.52
2.51E-3	4490	4.50E6	9.59	6.50E6	7.55	1.14E6	2.01
3.16E-3	4525	4.90E6	7.51	6.70E6	4.18	1.25E6	1.83
3.98E-3	4550	5.40E6	12.8	7.00E6	5.79	1.36E6	1.56
5.01E-3	4575	6.90E6	13.3	7.60E6	7.87	1.49E6	1.52
6.31E-3	4600	8.10E6	15.8	8.60E6	7.32	1.67E6	1.39
7.94E-3	4630	1.10E7	13.3	9.50E6	7.24	1.85E6	1.54
1.00E-2	4660	1.25E7	10.6	1.10E7	8.27	2.18E6	1.77
1.26E-2	4690	1.55E7	16.7	1.30E7	9.25	2.60E6	1.89
1.59E-2	4720	2.15E7	16.0	1.60E7	8.71	3.20E6	2.10
2.00E-2	4750	2.65E7	14.2	1.90E7	9.66	4.05E6	2.39
2.51E-2	4790	3.40E7	11.6	2.45E7	9.44	5.30E6	2.19
3.16E-2	4840	4.00E7	7.99	3.00E7	6.98	6.60E6	1.94
3.98E-2	4895	4.60E7	6.70	3.50E7	5.16	8.20E6	1.40
5.01E-2	4950	5.30E7	5.41	4.00E7	4.64	9.30E6	0.70
6.31E-2	5010	6.00E7	3.59	4.70E7	3.89	1.00E7	0.30
7.94E-2	5080	6.50E7	2.30	5.30E7	1.84	1.03E7	0.12
1.00E-1	5160	7.00E7	1.77	5.50E7	0.35	1.05E7	0.04
1.26E-1	5240	7.50E7	0.94	5.50E7	0	1.05E7	0
1.59E-1	5330	7.70E7	0.20	5.50E7	0	1.05E7	0
2.00E-1	5430	7.70E7	0	5.50E7	0	1.05E7	0

opacity $\bar{K}$. The resulting temperature differences are shown in Table I for Stein's EE and SE spectra.

The pressures and geometrical heights were computed using T_0 vs τ_{5000} and

$$\mathrm{d}p = \frac{g}{K_{5000}} \mathrm{d}\tau_{5000}, \tag{3}$$

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$$\mathrm{d}h = -\frac{1}{K_{5000}}\,\mathrm{d}\tau_{5000}\,. \tag{4}$$

This integration was performed for us by Duane Carbon. We found that because the opacity changes very little, pressures and geometrical heights change very little, occasionally increasing the last digit by one. As the values of pressure and height are practically identical with those of the HSRA, they are omitted from the table.

3. Conclusion

It can be seen from Table I that the temperature difference between the empirical and an inferred unperturbed solar atmosphere is small, increasing in magnitude toward smaller optical depth. Because the radiative heating effect is excluded, the unperturbed atmosphere has the lower temperature. Near the temperature minimum, a maximum temperature difference of between 53 and 83K is computed. The factor-of-two difference between the results for the EE and the SE spectra represents the uncertain state of knowledge of the wave generation. A wave spectrum of smaller initial flux, because it has to penetrate the same radiative loss region, will have smaller radiativt heating and consequently smaller temperature differences. This can be seen in the las two columns of Table I. A hypothetical wave flux in the extreme case of no flux toward the pole and a full flux toward the equator, can produce at most the temperature difference indicated in the fourth or sixth column of Table I. In an intermediate case, where one-fifth of the full equatorial flux goes toward the pole, the temperature difference is reduced to the difference between columns 6 and 8. Other flux distributions can be interpolated from the table. Because of increasing uncertainties due to shock dissipation, our computation was not carried beyond the temperature minimum. The oscillations in ΔT of Table I are due to the arbitrary splitting of the flux spectra into monochromatic waves.

After this work was completed, the work of Praderie and Thomas (1972) became known to us, indicating that in Equation (2) the numerical factor should be 8 instead of 16. This decrease of a factor of two would correspondingly increase ΔT by a factor of two if $\mathrm{d}\pi F_{\mathrm{mech}}/\mathrm{d}h$ were constant. However, in the computation of πF_{mech} vs height, the flux is fitted such that at 800 km, $\pi F_{\mathrm{mech}} = 1.0 \times 10^6$ erg cm^{-2} s^{-1}. This fit was necessary in order to take into account the effect of optical depth on radiative relaxation suffered by the acoustic waves. It specified at which height the boundary, separating regions of complete optical thinness and thickness needed to be set. If radiative losses due to H^- in the atmosphere were computed using Praderie and Thomas' numerical factor, then the fit at 800 km needs to be only $\pi F_{\mathrm{mech}} = 5 \times 10^5$ erg cm^{-2} s^{-1}. Comparing the ΔT's of the two SE spectrum computations in columns 6 and 8 of Table I, we find that a decrease of the flux by a factor of 2 would decrease the ΔT's roughly by a factor of 2. Thus, summing up, we find that the ΔT's shown in Table I are roughly unchanged by the reduction of the numerical factor in Equation (2).

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