Astron. Astrophys. 69, 407—410 (1978)

ASTRONOMY AND ASTROPHYSICS

Acoustic Dissipation and H⁻ Radiation in the Solar Chromosphere I

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Received December 17, 1977

Summary. The radiative energy loss of the solar chromosphere due to H^- transitions in statistical equilibrium is estimated from the empirical model of Vernazza et al. (1976), and the energy input due to dissipation of acoustic sawtooth type shock waves is computed for various energy fluxes and wave periods. The two energy rates show similar dependence on height for waves with periods near 20 s only. This suggests that the chromosphere is heated mainly by short period acoustic waves.

Key words: chromosphere — acoustic heating — H^- radiation loss

1. Introduction

Recently Praderie and Thomas (1976) criticized the computation by Ulmschneider (1970, 1974) of the H^- radiation loss in the solar chromosphere as well as the conclusion regarding the short period nature of the acoustic heating mechanism, which was based on this loss. Apart from a dispute about the difference by a factor of 2 in the equation for the radiation loss from a gas that may be either in LTE or in statistical equilibrium (which will be discussed by Kalkofen and Ulmschneider, 1978), the main criticism of Praderie and Thomas was the neglect of departures from LTE in H^- by Ulmschneider (1970, 1974). The model of Vernazza et al. (1976) allows now a detailed computation of the H^- losses including non-LTE effects. It is found that the calculation of the H^- losses in statistical equilibrium for the Vernazza et al. model does not significantly change the earlier results, which were based on the HSRA (Gingerich et al., 1971) and assumed LTE, contrary to the claims of Praderie and Thomas (1972, 1976). Our present calculations confirm the previous result of Ulmschneider (1970, 1974) that the assumption of heating by acoustic waves necessarily demands that the waves have short periods.

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2. Chromospheric Radiation by H⁻ in Statistical Equilibrium

Departures from LTE in H^- affect the chromospheric radiation loss in two ways. They change the H^- opacity, since the population of the bound level of H^- is different from that in LTE, and they change the source function.

Under the assumption that the low chromosphere loses energy mainly by means of H^- transitions the net rate of radiative cooling may be written as (after Kalkofen and Ulmschneider, 1978)

$$\frac{dF_R}{dh} = 4\pi N_{\mathrm{H}^-}\left\{\int_0^\infty \alpha_{\mathrm{bf}}\left(\frac{W_\nu}{b} - \frac{W_\nu^\circ}{b^\circ}\right)\left(1 + \frac{J_\nu}{2h\nu^3/c^2}\right)d\nu \right.$$
$$+ \int_0^\infty \left(\alpha_{\mathrm{ff}}\frac{W_\nu}{b} - \alpha_{\mathrm{ff}}^\circ\frac{W_\nu^\circ}{b^\circ}\right)\left(1 + \frac{J_\nu}{2h\nu^3/c^2}\right)d\nu$$
$$\left. + \int_0^\infty \left(\frac{\alpha_{\mathrm{ff}}^\circ}{b^\circ} - \frac{\alpha_{\mathrm{ff}}}{b}\right)J_\nu d\nu\right\}, \quad (1)$$

where α_{bf} and α_{ff} are the cross-sections for bound-free and free-free transitions of H^-, W_ν is the Wien function,

$$W_\nu = \frac{2h\nu^3}{c^2}\exp(-h\nu/kT) \quad (2)$$

and b is the H^- departure coefficient, i.e., the ratio of the actual H^- density N_{H^-} and the LTE population density $N_{\mathrm{H}^-}^*$, which is defined by the Saha equation in terms of the hydrogen and electron densities. (Note that in order to avoid awkward symbols for the mechanical flux our radiative flux is described by F_R. The same quantity in previous work is designated by πF or $4\pi H$).

The flux Equation (1) is obtained from the transfer equation by assuming that in the layers of interest the opacity is due to bound-free and free-free transitions of H^- and by subtracting from the flux equation the corresponding conservation equation that would be valid if the atmosphere were in radiative equilibrium. This subtraction cancels the bound-free heating term containing the mean intensity. The mean intensity remains in the equation only from the induced emissions and the free-free heating terms. The superscript ° of the Wien function, the departure coefficient and α_{ff} indicates that these quantities are taken at the position of the temperature minimum and refer to the hypothetical radiative equilibrium atmosphere. Equation (1) thus describes the excess emission due to mechanical wave dissipation. Since the energy equation depends only weakly on the intensity of the photospheric radiation field it may be represented by a dilute Planck function.

The sources of uncertainty in Equation (1) are the assumptions that only H^- contributes to the opacity in the low chromosphere and that the conditions at the temperature minimum in the empirical model of Vernazza et al. (1976) describe the radiative equilibrium atmosphere, in which wave heating would be absent. The latter assumption is also discussed by Kalkofen (1977).

Following Gingerich (1964) we combine the Saha equation with the cross-sections and write the bound-free and free-free H^- opacities as

$$K_\nu^{bf} = N_{H^-}\alpha_{bf} = a_{\lambda bf}(T) N_H P_e \quad (3)$$

$$K_\nu^{ff} = N_{H^-}^{*}\alpha_{ff} = a_{\lambda ff}(T) N_H P_e, \quad (4)$$

where the opacities per unit hydrogen density and electron pressure are modified versions of expressions given by Gingerich (with λ in Å):

$$a_{\lambda ff}(T)\cdot 10^{26} = (0.0053666 - 0.011493\theta + 0.027039\theta^2) + (-3.2062 + 11.924\theta - 5.939\theta^2)(\lambda/10^6) + (-0.40192 + 7.0355\theta - 0.34592\theta^2)(\lambda^2/10^9), \quad (5)$$

$$a_{\lambda bf}(T)\cdot 10^{26} = 0.4158 K \theta^{5/2} e^{1.736\theta} b, \quad (6)$$

where for $14200 < \lambda < 16419$ Å

$$K = 0.269818L + 0.220190L^2 - 0.0411288L^3 + 0.00273236L^4, \quad (7)$$

with $L = (16419 - \lambda)/1000$, and where for $\lambda < 14200$ Å

$$K = 0.00680133 + 0.178708L + 0.164790L^2 - 0.0204842L^3 + 5.95244\cdot 10^{-4}L^4, \quad (8)$$

with $L = \lambda/1000$. Here $\theta = 5040/T$, N_H is the neutral hydrogen density and P_e the electron pressure. (Note that our Equations (6) and (7) correct misprints in the work of Gingerich (1964).)

We have computed the H^- radiation losses based on the Vernazza et al. (1976) model for which values (see Table 1) of b have kindly been provided to us by the authors. The frequency integration of Equation (1) was performed using 64 frequency points. The parameters of the atmosphere at the temperature minimum are $T = 4150$ K for the kinetic temperature and $b = 0.93$ for the H^- departure coefficient (cf. Table 1). We represented

Table 1. Height (h in km), temperature (T in K), gas pressure (p in dyn/cm²) and H^- departure coefficient b

h	T	p	b
525	4150	1.241E3	0.930
550	4170	9.688E2	0.915
600	4350	5.932E2	0.900
650	4600	3.730E2	0.960
700	4890	2.414E2	1.11
750	5150	1.596E2	1.31
800	5360	1.076E2	1.53
1000	6100	2.580E1	2.55

the mean monochromatic intensity J_ν as a dilute Planck radiation field: $J_\nu = \frac{1}{2}B_\nu(T_{eff})$ with $T_{eff} = 5800$ K. Since J_ν appears only as a correction for stimulated emission and in an integral for free-free transitions the flux gradient is insensitive to the choice of J_ν. The results are shown in Figure 1. We find for the total radiative loss of the chromosphere in H^-

$$F_R = 2.6\text{E}6 \text{ erg/cm}^2\text{ s} \quad (9)$$

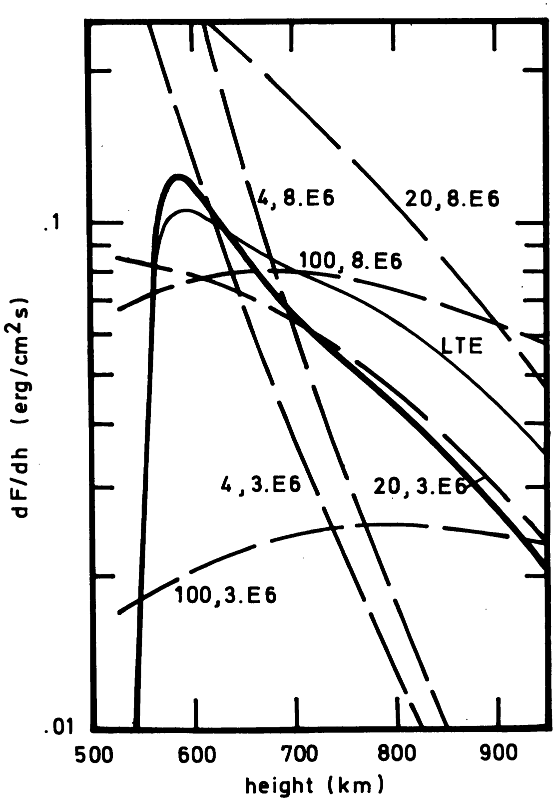

Fig. 1. H^- radiation loss rates (erg/cm³ s) with departures from LTE (drawn heavy) and in LTE (drawn thin) as functions of geom. height in the Vernazza et al. (1976) model. Dissipation rates of shock waves of indicated period (in s) and energy (in erg/cm² s) are shown dashed. Note the good agreement of the radiation and dissipation curves for the period 20 s where the radiation and the mechanical fluxes are both 3E6 erg/cm² s

For $b = 1$ we find $F_R = 3.2E6$ erg/cm^2 s. The discrepancy between these values is nowhere near the factor of 4 to 8 estimated by Praderie and Thomas (1972, 1976). These new values of chromospheric energy loss due to H^- based on the Vernazza et al. (1976) model are only slightly different from the values for previous solar models (compare Ulmschneider, 1974 Table 1). Computing F_R assuming LTE and using Equation (13) of Ulmschneider (1974) we found only a one percent increase compared to the LTE value of the present work.

3. Dissipation of Acoustic Shock Waves

Using Equations (14) and (18) of Ulmschneider (1974), fully developed weak acoustic shock waves may be computed on the basis of the Vernazza et al. (1976) model. For energies of the order of 10^6 to 10^7 erg/cm^2 s, however, the weak shock assumption cannot always be made for the solar chromosphere. We therefore derive more appropriate formulae. Let u_1, u_2 be the gas velocities in the laboratory frame, c_1, c_2 the sound velocities, p_1, p_2 the gas pressures and ρ_1, ρ_2 the densities in front of and behind the shock, respectively. Fully time dependent numerical studies of acoustic shock waves in the solar atmosphere (Ulmschneider et al., 1978) show that

$$u_1, u_2 < c_1, c_2; \qquad u_2 - u_1 < c_1, c_2 \tag{10}$$

but not

$$u_1, u_2 \ll c_1, c_2 \tag{11}$$

as is the case for weak shocks.

Defining the shock Mach number

$$M_S \equiv \frac{U - u_1}{c_1} \tag{12}$$

where U is the shock velocity in the laboratory frame we find (Landau, Lifshitz 1959, p. 331)

$$\phi \equiv \frac{p_2}{p_1} = \frac{2\gamma M_S^2 - (\gamma - 1)}{\gamma + 1}, \tag{13}$$

$$\theta \equiv \frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_S^2}{(\gamma - 1)M_S^2 + 2}. \tag{14}$$

Assuming linear sawtooth type shock waves the acoustic flux is given by (Ulmschneider, 1970)

$$F_M = \frac{1}{P}\int_0^P (p - p_0)u\,dt = \frac{p'_m u'_m}{12}, \tag{15}$$

where P is the wave period and

$$p_0 = (p_2 + p_1)/2, \tag{16}$$

$$p'_m = p_2 - p_1, \tag{17}$$

$$u_m = u_2 - u_1. \tag{18}$$

With Equations (12) to (14) we find

$$F_M = \frac{1}{6}\frac{\phi - 1}{\phi + 1}\frac{\theta - 1}{\theta} M_S \left(\frac{2}{\phi/\theta + 1}\right)^{1/2} p_0 c_0. \tag{19}$$

Here

$$c_0^2 = (c_2^2 + c_1^2)/2. \tag{20}$$

For waves of large period ($P \geqq 100$ s) the linear sawtooth assumption is probably not entirely justified. A slightly different shape however will give only a constant correction factor of the order of unity for Equation (19). That this correction factor is constant is due to the principle of shape similarity invariance found in experimental studies (Cole, 1948, p. 7; Brinkley and Kirkwood, 1947) and apparent in our time-dependent numerical work (Ulmschneider et al., 1978).

The dissipation of a shock wave occurs at the shock front where the entropy is raised by an amount ΔS. The dissipation rate in erg/cm^3 s is thus given by

$$\frac{dF_M}{dh} = -\frac{1}{P}\rho_1 T_1 \Delta S \tag{21}$$

where T_1 is the temperature in front of the shock. We thus find

$$\frac{dF_M}{dh} = -\frac{2\ln(\phi\theta^{-\gamma})}{(\phi + 1)(\gamma - 1)}\frac{p_0}{P}. \tag{22}$$

Note that the ratio of specific heats γ is assumed to be constant. This is valid at the temperature minimum and in the low chromosphere.

Equations (19) and (22) together with (13) and (14) now allow a computation of fully developed sawtooth shock waves. It is easily demonstrated that these equations for small amplitudes go over into the corresponding weak shock formulae used by Ulmschneider (1970, 1974). We have neglected reflection (cf. Ulmschneider, 1970, 1974) in our present work as we did not find any reflection in our time-dependent (Ulmschneider et al., 1978) studies.

We have computed waves with periods of 4, 20 and 100 s and acoustic fluxes F_M of 3.0E6 and 8.0E6 erg/cm^2 s. The dissipation rate dF_M/dh in erg/cm^3 s is shown in Figure 1. It is seen that because the shock waves tend towards a limiting shock strength (Ulmschneider, 1970), waves with the same period but different initial energies have similar dissipation rates.

Waves with very short periods (4 s) dissipate too rapidly. They are therefore not able to balance the empirical radiation losses. Waves with long period (100 s) dissipate only slowly; energy conservation in the wave and the decreasing density of the atmosphere lead to a growth of the wave amplitude and thus to the growing strength of the shock. The result is an *increasing* rate of dissipation of the wave with height. These waves therefore cannot balance the empirical radiation loss, which is a *decreasing* function of height. Short period waves with periods near 20 s however have the required dissipation rate as a function of height and thus are able to balance the H^- radiation loss rate. Figure 1 shows also that, because of the tendency of the shockwaves to approach a limiting strength, the conclusion regarding the short period nature of the heating mechanism is not dependent on the initial energy of the wave. The discrepancy between acoustic dissipation and radiation losses at low heights in Figure 1 may be explained by the fact that shortly after formation the shock waves are not yet fully developed. Our method of computation of shock waves will therefore overestimate the acoustic dissipation.

4. Conclusions

We have found that departures from LTE in H^- in the computation of the H^- radiation loss for the model of Vernazza et al. (1976) only minutely change the total radiation loss compared with an LTE calculation. This contradicts the estimate of a factor of 4 to 8 difference by Praderie and Thomas (1972, 1976). Moreover, and most significantly for the identification of the main chromospheric heating mechanism, the decreasing behaviour of the H^- radiation loss rate as a function of height is not affected by departures from LTE. A comparison with acoustic dissipation rates shows that under the assumption of an acoustic heating mechanism only short period waves with periods near 20 s are able to balance the empirical radiation loss. This confirms the work of Ulmschneider (1970, 1974) and together with independent evidence (Ulmschneider and Kalkofen, 1977) argues convincingly for short period acoustic waves as the main chromospheric heating mechanism.

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