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Theoretical Temperature Minima for Arcturus (K 2 IIIp), a Possible Explanation of the Wilson-Bappu Effect

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Summary. Theoretical temperature minima for Arcturus are determined on basis of the acoustic heating theory using $T_{\mathrm{eff}} = 4250$ K, $\log g = 1.7$ and $\alpha = 1.0, 1.5$. With a stellar envelope code and the Lighthill theory, acoustic fluxes are determined and periods estimated. Acoustic waves are piston initiated and followed with a non linear radiation hydrodynamic code. The minimum of the mean temperature was determined at $m = 2.4 \pm 1.2$ g/cm^2 in good agreement with the empirical value of $m = 1.8$ g/cm^2 found by Ayres and Linsky (1975a). Likewise the acoustic flux at the base of the chromosphere of Arcturus, $F_{MT} = 2.4\,\mathrm{E}\,6 \pm 1.2\,\mathrm{E}\,6$ erg/cm^2 s, was found in good agreement with empirical values of the chromospheric radiation loss given by Ayres (1975). Using a similar agreement for the Sun we propose an explanation for the Wilson-Bappu effect.

Key words: Arcturus – stellar chromospheres – Wilson-Bappu effect – radiative damping

Almost exactly 20 yr ago Wilson and Bappu (1957) discovered the linear correlation between the logarithm of the width W_0 of the emission core of the Ca II K line and the absolute visual magnitude M_v for late type stars. This so called Wilson-Bappu effect was found to hold over the fascinating range of 15 mag from $M_v = -6$ to $M_v = +9$ and over one order of magnitude in the width W_0. Together with the Sun most stars are found to satisfy the Wilson-Bappu relation independent of their spectral type, luminosity class or strength of Ca II core emission

In this paper we propose an explanation of the Wilson-Bappu effect using our results for the Sun and for Arcturus. On the basis of the mixing length theory of convection, the Lighthill theory of acoustic sound generation and computations of acoustic wave propagation we determine the location of the temperature minimum in the stellar photosphere. We assume that the Ca II K line is formed as a result of "abundance broadening" (Wilson and Bappu, 1957, see also Cram, 1979 and Ayres, 1979) in an atmosphere of large line optical depth where the monochromatic flux in the line simply reflects the temperature distribution in the outer stellar layers. From the location of the stellar temperature minimum we predict the width W_1 of the flux minima in the K line and because of the strong correlation between W_1 and W_0 the width W_0 of the emission core of this line. The well known determination of M_v from the stellar parameters completes our argument.

We thus propose that the Wilson-Bappu effect is governed by the mass location of the photospheric temperature reversal which is caused by shock dissipation of acoustic waves that are generated in the convection zone.

There are many uncertainties associated with this mechanism. Although universally used in stellar structure calculations the mixing length theory of convection, after more than twenty years, is still considered to be rather inaccurate. In addition the Lighthill theory of sound generation developed for the computation of noise in jet engines, depends on the eighth power of the mean turbulent velocity $\bar{v}$ and is thought to considerably amplify the uncertainty of the acoustic energy generation predicted by the combination of both theories. For a collection of somewhat pessimistic arguments concerning the uncertainty of the acoustic sound generation in stars see Cram (1977). In our work we take the mixing length and Lighthill theories as they stand and consider only the uncertainty of the parameter α, the ratio of the mixing length to the pressure scale height.

Further uncertainties arise from the fact that the theory of formation of the K line adopted by us is still not universally accepted as can be seen in recent discussions of K line theories by Cram (1979) and Ayres (1979). Considerable uncertainties finally arise from various simplifications employed in the computation of the acoustic wave propagation. These uncertainties are discussed in Sect. 4.

Our discussion in Sects. 2 and 3 starts with a theoretical determination of the temperature minimum and of the acoustic flux at the base of the chromosphere for Arcturus. Together with the values for the Sun derived elsewhere we use these results in Sect. 4 for a tentative explanation of the Wilson-Bappu effect and for a discussion of the question of additional chromospheric heating mechanisms which are suggested by the secular variability of the Ca II K_2 emission.

2. Method of Computation

a) Physical Parameters of Arcturus

For the computation of the theoretical position of the temperature minimum certain physical parameters, particularly the effective temperature T_{eff} and the surface gravity g, have to be known. For Arcturus (α Boo, K2III p) these parameters have recently been the center of a controversy especially after Mäckle et al. (1975a, b) on basis of a complete spectrum analysis relative to the Sun found a surprisingly low value of the gravity. This implied however a very low mass of the star that would be difficult to explain by stellar evolution theory. In the last few years as shown in Table 1 there seems however a growing consensus for the larger value of the gravity. In view of this and because we want to compare our

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Table 1. Recent determinations of effective temperature $T_{\rm eff}$ (K) and surface gravity g (cm/s^2) for Arcturus. For variable error limits the larger of the errors is given

Authors	$T_{\rm eff}$ (K)	$\log g$ (cm/s^2)
Martin (1977)	4300±90	1.74±0.2
Ayres and Johnson (1977)	—	1.6 ±0.2
Johnson et al. (1977)	4250±100	—
Blackwell and Willis (1977)	4400±60	1.48±0.15
Blackwell et al. (1975)	4500±120	—
Mäckle et al. (1975a, b)	4260±50	0.90±0.35
Van Paradijs and Meurs (1974)	4350±50	1.95±0.25
Gustafsson et al. (1974)	4030	1.9

theoretical results with the empirical chromosphere models of Ayres and Linsky (1975a, b) we use the recent determinations of Martin (1977), Johnson et al. (1977) and Ayres and Johnson (1977). Thus we adopt the values $T_{\rm eff}=4250$ K and $\log g=1.7$. As shown in Sect. 4 we feel that our theoretical approach independently indicates a higher value of the gravity.

b) Acoustic Flux and Period

The methods to compute the acoustic flux and the acoustic period of a star have been discussed by Renzini et al. (1977). The total acoustic flux is calculated following Lighthill (1952, 1954) and Proudman (1952) who find

$$F_{M_0}=\frac{1}{2}\int 38\,\rho\,\frac{\bar{v}^8}{\alpha H v_s^5}\,dx. \tag{1}$$

Here ρ is the density, $\bar{v}$ the mean velocity of rising convection elements, v_s the sound velocity, x the geometrical height and $\alpha=l/H$ the ratio of mixing length to pressure scale height. The integration is carried over the small height interval at the top of the convection zone where the contribution to $\bar{v}^8$ is significant. Note that in order to avoid the awkward notation πF_M, the mechanical flux, contrary to earlier work, is now denoted by F_M.

The acoustic frequency spectrum for the Sun under various assumptions about the turbulence spectrum has been calculated by Stein (1968). Calculations for a series of other stars indicate (Stein, 1970) that the acoustic spectra have a maximum near a period

$$P_{\rm Max}\simeq\frac{1}{10}P_A=\frac{1}{10}\,\frac{4\pi v_s}{\gamma g} \tag{2}$$

where P_A is the acoustic cut off period at the height of maximum sound generation, γ the ratio of specific heats and g the surface gravity. For our calculations periods are chosen which correspond to the maximum of the spectrum, $P=P_{\rm Max}$.

For given $T_{\rm eff}$ and $\log g$ the atmospheric parameters entering Eqs. (1) and (2) are computed with a stellar envelope code described by Renzini et al. (1977). The flux results and periods for Arcturus are given in Table 3 and are compared with values for the Sun taken from Ulmschneider et al. (1977b).

c) Atmosphere Model and Acoustic Wave Propagation

Given a suitable atmospheric model Ulmschneider et al. (1977a, b), Kalkofen and Ulmschneider (1977) as well as Ulmschneider and Kalkofen (1977) have shown how heights of shock formation in a periodically pulsed atmosphere may be computed for a given star. In this work the non linear hydrodynamic equations and the two stream approximation of the radiative transfer equation were solved using the grey approximation.

These calculations showed (cf. Table 3) that the shock formation heights (in Langrangian mass) in the Sun coincided well with empirical heights of the temperature minimum. A similar comparison for Arcturus however (Ulmschneider et al., 1977b, Cram and Ulmschneider, 1978) showed large discrepancies. These discrepancies, as the present work shows, were primarily due to the misconception carried over from solar work that the shock formation height should coincide with the temperature minimum height. Unlike the situation for the Sun, shock formation for Arcturus occurs in the radiative damping zone (cf. Table 3 and Figs. 1, 2) where the rapid growth of the shock wave and a consequently large shock dissipation is prevented by radiation damping. Thus as shown by Table 3 and Figs. 1, 2 the temperature

Table 2. Grey two stream radiative equilibrium model used as initial atmosphere for Arcturus. a is the geometric height in the Lagrange frame, m mass column density, T temperature, S_0 entropy, τ optical depth and t_R radiative relaxation time after Eq. (6) based on the Kurucz (1978) opacities. Note that the gas pressure can be obtained multiplying m with gravity $g=50.12$ cm/s^2

a(km)	m(g/cm^2)	T(K)	S_0(erg/gK)	τ	t_R(s)
−2.26 E 4	2.02 E 2	5439	−6.43 E 8	3.00 E 0	3.18 E 1
1.56 E 4	1.05 E 2	3942	−6.52 E 8	4.10 E-1	3.98 E 2
5.80 E 4	4.24 E 1	3532	−6.12 E 8	5.89 E-2	2.40 E 3
9.80 E 4	1.63 E 1	3465	−5.54 E 8	1.18 E-2	5.45 E 3
1.43 E 5	6.24 E 0	3451	−4.93 E 8	2.57 E-3	1.01 E 4
1.85 E 5	2.38 E 0	3449	−4.32 E 8	5.93 E-4	1.74 E 4
2.28 E 5	9.07 E-1	3448	−3.70 E 8	1.47 E-4	2.85 E 4
2.70 E 5	3.46 E-1	3448	−3.08 E 8	3.95 E-5	4.30 E 4
3.13 E 5	1.32 E-1	3448	−2.47 E 8	1.16 E-5	5.93 E 4
3.55 E 5	5.02 E-2	3448	−1.85 E 8	3.59 E-6	7.60 E 4
3.98 E 5	1.92 E-2	3448	−1.23 E 8	1.12 E-6	8.93 E 4

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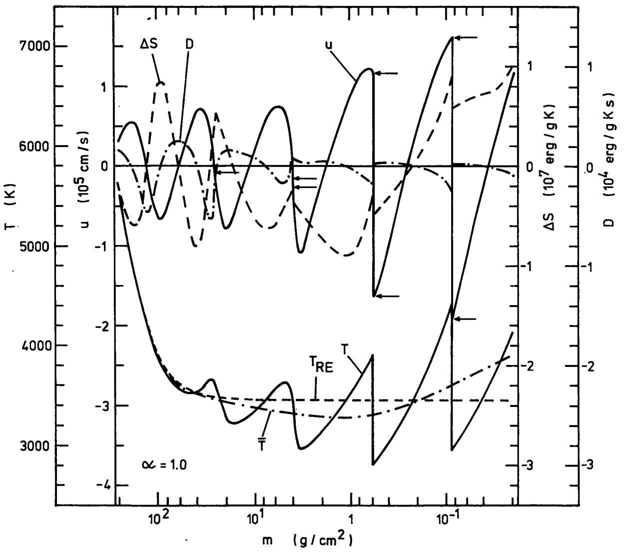

Fig. 1. Acoustic wave in the atmosphere of Arcturus shown on a mass scale m at time $t=1.94$ E 5 s. The ratio of mixinglength to pressure scale height is $\alpha=1.0$. T indicates the temperature, $\bar{T}$ the mean temperature, T_{RE} the radiative equilibrium temperature. u is the gas velocity, $\Delta S=S-S_0$ is the entropy difference to the initial atmosphere of Table 2, $D=dS/dt$ is the radiative damping function. Arrows indicate the shock discontinuities

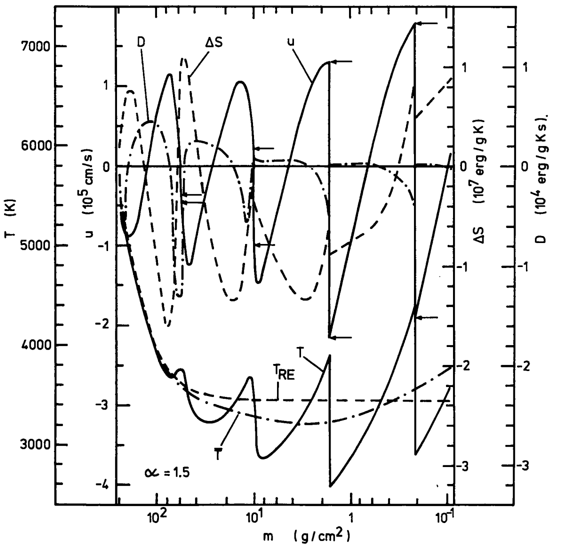

Fig. 2. Same as Fig. 1, however at $t=2.01$ E 5 s, with $\alpha=1.5$

minimum in this star is moved to much greater height (lower m) compared to the point of shock formation.

Before we describe our results we have to mention one difficulty. In the above cited work the perturbation method (Ulmschneider and Kalkofen, 1977) was used to compute acoustic wave propagation. This method tried to prevent the serious limitation imposed by the restriction to a grey calculation. A grey radiation hydrodynamic code with given parameters of T_{eff} and $\log g$ in absence of wave motion will always produce a grey radiative equilibrium atmosphere model which is of poor quality compared with recent non grey models e.g. for Arcturus (Johnson et al., 1977) or for the Sun (Kurucz, 1974). If non grey models are used as ambient atmosphere through which acoustic waves travel, zeroth order radiation terms must be removed from the energy equation in order to ensure the time independence of this atmosphere. This perturbation method (Ulmschneider and Kalkofen, 1977) is valid in cases where the wave amplitudes are small. However, in our present case where relatively large amplitude shock waves are followed into the chromosphere the perturbation method cannot be used. Here we are forced to use the full radiation hydrodynamic method, that is, we retain terms of zeroth order in the energy equation. A discussion of similar calculations for the Sun is given by Ulmschneider et al. (1978). The initial (grey radiative equilibrium) atmosphere used for Arcturus is exhibited in Table 2. As described by Ulmschneider et al. (1977a) an acoustic wave train is introduced in the atmosphere by specifying at time t a piston velocity at the bottom of the atmosphere such that

$$u_1=-u_0 \sin\left(\frac{2\pi}{P}t\right), \tag{3}$$

with

$$u_0=\left(\frac{2F_{N_0}}{\rho_1 c_1}\right)^{1/2}, \tag{4}$$

where F_{M_0} and P are given by Table 3 and ρ_1 and c_1 are density and sound velocity at the piston boundary. In addition the outgoing specific intensity of the two stream approximation

$$I_1^+=\frac{\sigma}{\pi}T_1^4+\frac{3\sigma T_{\text{eff}}}{4\pi}\mu \tag{5}$$

must be specified for $\mu=+1/\sqrt{3}$ at this boundary where T_1 is the

Table 3. Initial acoustic flux F_{Mo}, acoustic period P, shock formation height on a mass scale m_S, height of the temperature minimum m_T, empirical height of the temperature minimum m_E, acoustic flux at shock formation F_{MS}, and at the temperature minimum F_{MT} as well as the empirical chromospheric flux F_C for Arcturus and the Sun. Values in brackets are extrapolated on basis of Ulmschneider et al. (1978). (1) indicates taken from Ulmschneider et al. (1977b). For m_E, HSRA and VAL respectively indicate the empirical heights of the temperature minima on the mass scale taken from Gingerich et al. (1971) and from Vernazza et al. (1976). T_T is the time averaged temperature at the temperature minimum

	Arcturus		Sun	
	$\alpha=1.0$	$\alpha=1.5$	$\alpha=1.0$	$\alpha=1.5$
F_{Mo} (erg/cm^{2}s)	2.5 E 7	7.5 E 7	1.6 E 7	4.9 E 7
P(s)	1.4 E 4	1.4 E 4	27	26
$m_S^{(1)}$(g/cm^2)	21	56	1.8 E-2	6.2 E-2
m_S(g/cm^2)	25	59	—	—
m_T(g/cm^2)	1.4	3.5	(1.4 E-2)	(5.8 E-2)
m_E (g/cm^2)	1.8		HSRA: 3.2 E-2 VAL: 5.2 E-2	
$F_{MS}^{(1)}$ (erg/cm^{2}s)	5.0 E 6	2.2 E 7	2.7 E 6	7.8 E 6
F_{MS} (erg/cm^{2}s)	8.7 E 6	4.0 E 7		
F_{MT} (erg/cm^{2}s)	1.2 E 6	3.6 E 6	(2.2 E 6)	(7.3 E 6)
F_C (erg/cm^{2}s)	1.8 E 6		6.0 E 6	
T_T (K)	3240	3170	4030	4230

temperature and σ the Stefan Boltzmann constant. At the top of the atmosphere we took a transmitting boundary condition as discussed by Ulmschneider et al. (1977a).

3. Results

a) *Resulting Acoustic Waves*

The result of computations of acoustic waves for Arcturus with F_{M_0} and P given by Table 3 using the Kurucz (1978) opacity table and taking the grey radiative equilibrium model of Table 2 as initial state is shown in Figs. 1 and 2 as well as in Tables 3 and 4. With about 10 shocks transmitted through the upper boundary both calculations have approximately reached a steady state. The heights of the temperature minima determined as the position of the minimum of the time averaged temperature $\bar{T}$ are given in Table 3 together with shock formation heights and similar values for the Sun taken from Ulmschneider et al. (1977b). The position of the minimum of $\overline{T^4}$ was found to coincide with that of the $\bar{T}$ minimum.

A number of similarities with the case of waves in the solar atmosphere (cf. Ulmschneider et al., 1978) is seen. Due to the nonlinearity of the Planck function there is a depression in the photosphere of the time averaged temperature $\bar{T}$ below the radiative equilibrium temperature. The shock Mach numbers of Fig. 1 from left to right are $M_S = 1.00056$, 1.0113, 1.39, 1.51 while the Mach numbers of Fig. 2 are $M_S = 1.0031$, 1.094, 1.53, 1.53. In the chromosphere these Mach numbers are similar to the solar values. This is understood from the limiting strength behavior, e.g. Eq. (16) of Ulmschneider et al. (1978), and our present Eq. (2). Note that in Figs. 1, 2 the positions of the shock discontinuities are marked by arrows.

b) *Influence of Radiative Damping*

The influence of radiative damping is apparent from the different shape of the waves compared with the Sun. Because radiative damping prevents the rapid growth of the shock, the discontinuity remains small up to the temperature minimum region, resulting as seen by Figs. 1, 2 and Table 3 in a great height range between shock formation and full development. Even at the temperature minimum region the shock has not yet reached a full sawtooth shape. Note e.g. in Fig. 1 that at $m = 0.6$ g/cm^2 the temperature behind the shock decays rapidly because of radiation and does not follow the velocity profile which indicates compression. This behaviour in Arcturus is quite different from the Sun where radiation damping at the region of shock formation is much less important and where the transition from shock formation to the full sawtooth profile of the wave is quite rapid (Ulmschneider et al., 1978).

The influence of radiative damping on an acoustic wave can be elucidated by considering the radiative relaxation time for which Schmitz (1979) gives the expression

$$t_R = \frac{2.5\, c_v}{16\, \bar{\varkappa} \sigma T_0^3} \tag{6}$$

where T_0 is the mean temperature in the atmosphere.

Note that apart from a constant factor this expression is similar to those given by Spiegel (1957) or Oster (1957) for the case of a homogeneous medium where the energy terms are of first order. As shown however by Ulmschneider et al. (1978), first order radiation terms in acoustic waves describe only the reversible exchange of photons between high and low temperature areas while radiative damping of the acoustic energy is connected with second order terms.

In cases where t_R is small compared to the wave period P we expect strong influence of radiation damping on the wave. Table 2 gives values of t_R for the initial radiative equilibrium model. It is seen that the optical depth range in Arcturus where $t_R < P$ is much extended to $\tau_L = 1.1$ E-3 compared with the Sun where this zone reaches only to $\tau_L = 5.4$ E-2. The reason for this is the different dependence of t_R and P on gravity and T_{eff}. Lower opacity and lower temperature in Arcturus lead to an increase of t_{Rad} by a factor of about 80 relative to the Sun while the acoustic period after Table 3 increases by a factor of 470.

The dependence of t_R on the stellar parameters was recently discussed by Schmitz (1979). In Eq. (6) he has replaced the opacity $\bar{\varkappa}$ by an H$^-$ approximation formula after Ulmschneider et al. [1978, Eq. (26)] or Stein (1966, p. 20) and has expressed t_R following Stein (1966, p. 18) as a function of T_{eff} and gravity g. Together with the wave period P after Eq. (2) he finds

$$\frac{t_R(\tau)}{P} = \frac{t_{R\odot}(\tau_\odot)}{P_\odot} \left(\frac{g}{g_\odot}\right)^{0.57} \left(\frac{\tau}{\tau_\odot}\right)^{-0.43} \left(\frac{T_{\mathrm{eff}\odot}}{T_{\mathrm{eff}}}\right)^{6.4}. \tag{7}$$

Here the ratio t_R/P of Arcturus is seen to be roughly by a factor of 5 smaller than the solar ratio showing significantly increased radiative damping in Arcturus. Quite generally Eq. (7) shows that for hot and low gravity stars increasingly efficient radiative damping is expected for short period acoustic waves. The optical depth range of significant radiation damping for acoustic waves can be derived from Eq. (7) assuming that the left hand side is unity and solving for the limiting optical depth τ_L using solar values $t_{R\odot} = 30$ s at $\tau_\odot = 5.4$ E-2 and $P_\odot = 30$ s

$$\tau_L = 5.4\ \text{E-2} \left(\frac{g}{g_\odot}\right)^{1.33} \left(\frac{T_{\mathrm{eff}\odot}}{T_{\mathrm{eff}}}\right)^{14.9} \tag{8}$$

$\tau_L = 1.3$ E-3 from this equation is in reasonable agreement with the value 1.1 E-3 found from Table 2. Radiation damping of acoustic waves in hot, low gravity stars is not only more severe but occurs also in a much extended range of optical depth.

The radiative damping function $D = dS/dt$ (Ulmschneider et al., 1978) in Arcturus is considerably (factor 20) smaller than for the Sun but much less concentrated towards the stellar surface. The cycle averaged damping function $\bar{D}$ shown in Table 4 is essentially zero below the height of shock formation x_S as $\bar{S}$ is constant in steady state. At heights greater x_S we have $\bar{D} = \Delta S/P$ where $\Delta S \simeq 5.1$ erg/g K is the entropy jump at the shock in the low chromosphere.

4. Discussion

a) *The Height of the Temperature Minima*

The theoretical heights of the temperature minimum (T.M.) of Arcturus given in Table 3 may be compared with heights determined empirically from the K_1 minima of the Ca II K line by Ayres and Linsky (1975a, b). It is seen that for both Arcturus and the Sun the empirical heights fall within the uncertainty expressed by the choice of α. Higher values of α, because of larger mixing length lead to larger convection velocities, larger acoustic flux and consequently low shock formation heights (high values of m). The sensitivity to T_{eff} and $\log g$ of the height of the T.M. is reduced considerably compared to that for the shock formation heights (Ulmschneider et al., 1977b). We estimate that an uncertainty of ± 100 K in T_{eff} probably leads to ± 0.05 in $\log m$ while an error of ± 0.2 in $\log g$ to ± 0.1 in $\log m$. This is considerably smaller

Table 4. Temperature $\bar{T}$, pressure $\bar{p}$, entropy change relative to the initial atmosphere $\overline{\Delta S}$, damping function $\bar{D}$, height shift $\bar{x}-a$ and acoustic flux $\overline{F_M}$ as function of Lagrange height a averaged over 2 wave periods at time 2.7 E 5 s for the wave of initial acoustic flux $F_{M_0}=7.5$ E 7 erg/cm^{2}s and period $P=1.4$ E 4 s

a(km)	$\bar{T}$(K)	$\bar{p}$(dyn/cm^2)	$\overline{\Delta S}$(erg/gK)	$\bar{D}$(erg/gKs)	$\bar{x}-a$(km)	F_M(erg/cm^{2}s)
−2.26 E 4	5392	1.01 E 4	−9.777 E 5	−7.89 E 0	−1.84 E 3	6.86 E 7
1.56 E 4	3901	5.25 E 3	−9.834 E 5	−5.40 E 0	−1.59 E 3	5.81 E 7
5.80 E 4	3441	2.13 E 3	−3.481 E 6	2.06 E 1	−1.05 E 3	3.06 E 7
9.80 E 4	3279	8.21 E 3	−7.932 E 6	8.23 E 0	−1.86 E 3	1.24 E 7
1.43 E 5	3209	3.13 E 3	−1.045 E 7	−9.87 E 1	−3.02 E 3	6.20 E 6
1.85 E 5	3189	1.20 E 2	−1.140 E 7	−2.98 E 2	−4.60 E 3	2.96 E 6
2.28 E 5	3289	4.60 E 1	−6.642 E 6	−3.79 E 2	−5.43 E 3	1.19 E 6
2.70 E 5	3459	1.78 E 1	5.845 E 5	−4.81 E 2	−5.06 E 3	4.72 E 5
3.13 E 5	3554	6.39 E 0	8.577 E 6	−4.49 E 2	−1.65 E 3	1.76 E 5
3.55 E 5	3625	2.16 E 0	1.952 E 7	−5.26 E 2	6.46 E 3	6.74 E 4

than the uncertainty in α. A very tentative estimate of the T.M. of Arcturus for $\log g=0.9$ is $m=4\pm 2$ g/cm^2 which is only barely in agreement with Ayres and Linsky (1975a). If $\alpha=1.3$ is chosen however our theoretical results indicate that the value $\log g=0.9$ is too low. The mean temperature at the T.M. for Arcturus as seen in Table 3 is lower for the more energetic wave ($\alpha=1.5$). This is the influence of the nonlinearity due to a larger amplitude. Note that the perturbation method used for the sun does not show this effect. There the higher T.M. temperature of the $\alpha=1.5$ wave is due to an earlier shock formation in the ambient atmosphere.

b) Uncertainty of the Initial Acoustic Flux

The uncertainty of the acoustic flux because of the low accuracy of the mixing length theory and because of the $\bar{v}^8$ dependence in Eq. (1) is usually assumed to be very large in the order of a factor of 1000 (Cram, 1977). We restrict our discussion to the uncertainty of the parameter α. Recent attempts for accurate solar interior models by Christensen-Dalsgaard and Gough (1976) as well as Gough and Weiss (1976) narrow the range of α to between 1.1 and 1.3. A similar narrow range of between 1.0 and 1.5 for the Sun is found by Michaud (1977, p. 184) in order to prevent underabundances of helium due to diffusion. In Table 4 we show the acoustic flux F_M as function of height. Due to the large effect of radiative damping, phase shifts of the pressure oscillation relative to the gas velocity are introduced at the lower boundary such that the atmosphere accepts only 90% of the flux that is generated in the convection zone.

c) Uncertainties in the Computation of Acoustic Wave Propagation

Aside of the fact that we cannot account for the spatial distribution of acoustic energy on the stellar surface and its three dimensional transmission there are numerous uncertainties associated with purely one dimensional acoustic wave propagation. The present computation is monochromatic while in reality one has an acoustic flux spectrum. Our period chosen is that of the maximum of the spectrum. For the Sun Ulmschneider and Kalkofen (1977) have shown that these waves form shocks first. Thus we expect the height of the temperature minimum to be little affected by other components of the acoustic flux. However the chromospheric temperature rise is certainly (Ulmschneider et al., 1978) influenced especially by long period components of the acoustic spectrum. The factor $\frac{1}{10}$ in Eq. (2) together with Stein's (1968) assumptions on the turbulence spectrum may be in error. Solar observations by Deubner (1976) show however significant power near Stein's SE or EE flux maxima i. e. at periods between 20 and 40 s. Other uncertainties arise from our grey calculation which approximates only crudely the actual radiation field. These uncertainties can however only be assessed when non grey calculations are made. Uncertainties due to non-LTE effects (Cayrel effect) are most likely as small for Arcturus as for the Sun because of similar number density at the T.M. (Ulmschneider and Kalkofen, 1978, Kalkofen and Ulmschneider, 1979).

d) The Acoustic Flux at the Temperature Minimum, Chromospheric Heating

In Table 3 we give the acoustic flux at the T.M. for Arcturus together with values for the Sun taken from Ulmschneider et al. (1977b). These values may be compared with the empirically determined radiation loss from the chromosphere given by Ayres (1975) or Linsky and Ayres (1978). It is seen that the theoretical fluxes in both stars agree with the empirical values in the same sense as found for the heights of the T.M. Waves with large flux (large α) produce shocks earlier and have more energy at the T.M. The sensitivity to T_{eff} and $\log g$ is difficult to assess. Here we must await future work. Crudely however we estimate for $\Delta T_{\mathrm{eff}}=\pm 100$ K an error $\Delta \log F_{MT}=\pm 0.1$ and for $\Delta \log g=\pm 0.2$ we find $\Delta \log F_{MT}=\pm 0.05$. Knowing well that these above results need to be supported by investigations for a much larger number of stars it is nevertheless tempting to discuss consequences for the heating mechanism. If we assume that the empirical chromospheric fluxes are realistic, the good agreement between theoretical and empirical fluxes indicates that the acoustic heating theory is not only correct but that it is the main heating mechanism. This is quite unexpected as already Wilson and Bappu (1957) found that the intensity of the K_2 emission is not correlated with the Wilson-Bappu effect. There are stars which for the same T_{eff} and $\log g$ show different K_2 intensities which presumably indicates different total chromospheric emission. A large part of the scatter found for the Mg II $h+k$ line emission of a number of stars by Linsky and Ayres (1978) is probably due to this excess chromospheric emission. Our theory for given T_{eff}, $\log g$ and α finds only one value for the acoustic flux F_{MT} at the T.M. Thus very likely an additional, possibly magnetic (Alfvén wave) heating mechanism is at work which may also work in the solar corona. This mechanism is also suggested by the correlation between age, magnetic field, rotation and Ca II K emission found by Skumanich (1972). Such a mechanism can only be tied down observationally if the effect of the acoustic heating mechanism is subtracted. We thus propose that the systematic dependence on T_{eff} and log g of the Mg II $h+k$ emission found by Linsky and Ayres (1978, Fig. 3) is due to the acoustic while the scatter to a magnetic heating mechanism. These conclusions are however tentative as in addition we presently have still to suppose large errors in the determination of the total chromospheric radiation loss.

e) The Wilson-Bappu Effect

With two stars a linear relation between M_v and log W_0 can be established. We have seen that the acoustic heating theory for the Sun and Arcturus can account for the empirical height of the T. M. Thus as the empirical heights by Ayres and Linsky (1975a, b) were derived from the width W_1 of the Ca II K_1 minima we are able to account for the width W_1. With the strong correlation between W_0 and W_1 found empirically by Cram et al. (1979) which undoubtedly can also be supported theoretically by the methods of Ayres and Linsky (1975a) we are thus able to explain the width W_0. With the known values of M_v of the Sun and Arcturus and the known fact (Wilson, Bappu, 1957; Wilson, 1970) that both stars satisfy the Wilson Bappu relation we are thus able to tentatively explain the Wilson-Bappu effect.

5. Conclusions

We have seen that the acoustic heating theory not only is able to explain the height of the temperature minimum of the Sun (G2V) but also that of Arcturus (K2IIIp). This theory further fairly accurately predicts the empirical chromospheric radiation loss which shows that the acoustic heating mechanism is very likely the main energy supplier of the temperature minimum region and the low chromosphere. Because Arcturus and the Sun lie on the Wilson-Bappu relation and because the height of the temperature minimum can be translated into the width W_0 of the emission core of the Ca II K line, the acoustic heating mechanism is able to tentatively explain the Wilson-Bappu effect.

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