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THE PRESENT STATE OF WAVE-HEATING THEORIES OF STELLAR CHROMOSPHERES

P. Ulmschneider

Institut für theoretische Astrophysik, Im Neuenheimer Feld 561, 6900 Heidelberg, F.R.G.

ABSTRACT

Observational constraints are summarized and the present state of the acoustic and mhd-wave heating theories for chromospheres in early and late-type stars are discussed. It is found that the slow-mode mhd-wave heating theory looks most promising but that mode-coupling from transverse or torsional Alfven waves may be significant for the upper layers.

INTRODUCTION

Ground-based and satellite observations have shown that with the possible exception of A-stars all stars possess chromospheric shells or hot areas with temperatures exceeding the effective temperature T_{eff}. Such hot regions can be explained only by mechanical heating. Yet the chromospheric heating mechanism is presently still unknown. Observationally one finds a tight correlation between chromospheric emission and magnetic fields. Any heating theory must therefore allow for the presence and the inhomogeneous distribution of magnetic fields. The question is whether the magnetic fields are involved directly in the heating of chromospheres or indirectly by modifying the generation and distribution of the mechanical energy. Concentrating here on the latter possibility, only wave heating mechanisms are discussed in this work. The present topic has recently been reviewed by Stein /31/.

OBSERVATIONAL CONSTRAINTS

Fig. 1 shows the net radiative cooling rates obtained from the solar empirical models of Avrett /1/. These models are based on Skylab observations with a 5x5 arcsec^2 spatial resolution. This resolution is not high enough to separate the magnetic flux tubes and the field free regions. Moreover these models do not show the observed cold C O radiating areas /3/ and thus give only spatially averaged quantities as function of height. Fig. 1 shows that the net radiative cooling rate becomes negative at the temperature minimum. This is very likely due to the incomplete resolution and to missing C O cooling. In a more realistic treatment, which includes the cold C O area losses, one expects that the net radiative cooling rate does not become negative at the temperature minimum region /20/. In spite of these limitations it is possible to infer stringent observational constraints on the chromospheric heating mechanism from Avrett's empirical models. The difference in the cooling rates of the models shows that the chromospheric emission varies strongly over the solar surface. In addition it is seen that in the layers above the temperature minimum the cooling rates in the (cell interior) model A' and in the (very bright network) model F' rise rapidly to values of $3\ 10^{-2}$ and $3\ 10^{-1}$ erg cm^{-3}s^{-1}, respectively. Other important constraints can be obtained from the observed stellar chromospheric line fluxes. Fig. 2 by Basri and Linsky /4/ shows the MgII k-line emission from late-type supergiant, giant, and main sequence stars observed with the IUE-satellite. It is seen that the emission flux apparently does not show any gravity dependence, while it appears to depend on the temperature as $F_{MgII\ k}/\sigma T_{eff}^4 \sim T_{eff}^{3.7}$. An additional observational constraint on the heating mechanism is the observed coronal x-ray and chromospheric emission gap at the A-star region /36/ which separates stars with a rotation - emission activity and a luminosity - emission activity correlation /22/,/33 Fig's 5 to 8/. Evidence for short-period (compared with the cut-off period of about 200 s) waves on the sun with periods as low as 40 s has been given by Deubner and Endler /11/, /12/.

ACOUSTIC HEATING THEORY

An early and most extensively developed idea for the chromospheric heating mechanism is the acoustic heating theory which goes back to Biermann /5/ and Schatzman /26/. This theory (e.g./33/) argues that the surface convection zones of late-type stars generate acoustic waves. In propagating towards the stellar surface the acoustic wave amplitudes are magnified by the rapid density decrease and steepen into shocks. The rapid onset of chromospheric

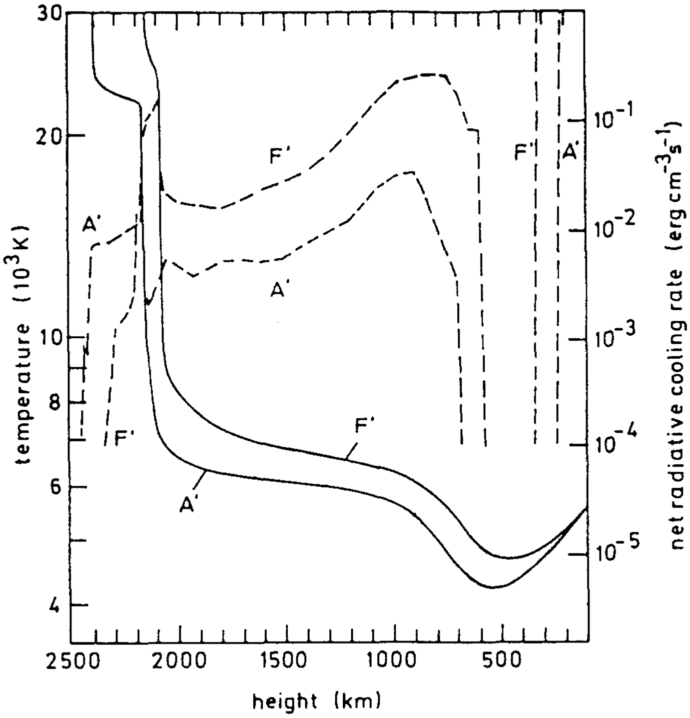

Fig. 1 Temperatures (solid) and net radiative cooling rates (dashed) after Avrett /1/

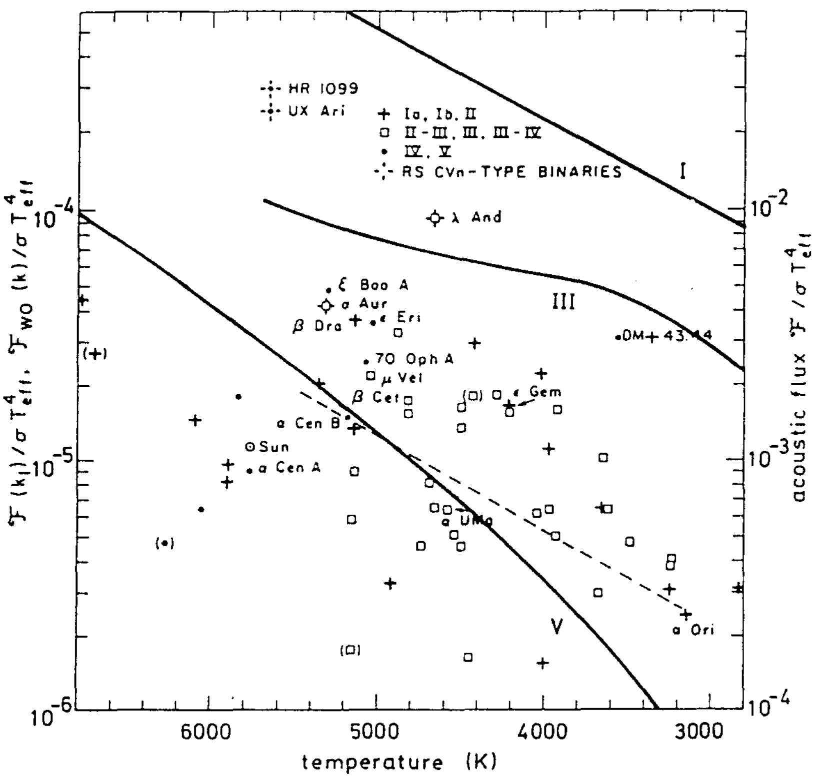

Fig. 2 MgII k-line emission fluxes after Basri et al. /4/ compared with acoustic fluxes after Bohn /6/ (α=1) for the luminosity classes I, III, V (solid).

emission is explained by the fact that shock formation and subsequent dissipation generates a sudden rise of chromospheric heating. This theory has been applied to stars later than spectral type A. However it should be realized that the acoustic heating theory is also applicable to early-type stars where the necessary acoustic energy is not produced by convection but by radiative amplification as shown by Wolf /38/. The different generation mechanisms of wave energy in early and late-type stars provides a natural explanation for the observed A-star gap: The decrease of the radiative flux when going from O-stars to later spectral type and the decreasing size of the convection zones when going from G-stars to earlier spectral type points to a deep minimum of the wave energy generation at spectral type A. That minimum however will hardly be zero.

Consideration of viscous and thermal conductive heating points to shock dissipation as the important heating mechanism for acoustic waves: Using the velocity amplitude $u=3\ 10^5$ cm sec^{-1}, temperature amplitude $\Delta T=1000$ K, characteristic length (≈scale height) $L=1.5\ 10^7$ cm, as well as coefficients of viscosity $\eta=5\ 10^{-4}$ dyn $cm^{-2}s$ and thermal conductivity $\kappa=1\ 10^5$ erg $cm^{-1}K^{-1}s^{-1}$ one finds for the viscous heating rate

$$\epsilon_V = \eta(du/dx)^2 \approx \eta u^2/L^2 = 2\ 10^{-7} erg\ cm^{-3}s^{-1} \quad . \qquad (1)$$

and for the conductive heating rate

$$\epsilon_C = d/dx(\kappa dT/dx) \approx \kappa \Delta T/L^2 = 4\ 10^{-7} erg\ cm^{-3}s^{-1} \quad . \qquad (2)$$

The values are for a wave with an acoustic flux of about $8\ 10^{-4}\sigma T_{eff}$ at the top of the convection zone (/34/, Tab. 3). Eq.'s (1) and (2) show that the rates ϵ_V and ϵ_C are six orders of magnitude smaller than the required empirical cooling rates. If one does not want to have unrealistically large wave amplitudes the only way by which the theoretical rates can be raised is by reducing the characteristic length scale L by a factor of 1000. This represents velocity and temperature variations of 6 km/s and 2000 K respectively over distances of about 150 m which essentially is a shock. We conclude that for acoustic waves only shock heating is able to balance the observed radiation losses. This implies that only acoustic waves with periods short compared to the acoustic cut-off period will be of interest for the chromospheric heating mechanism: The observed 300 s oscillations discovered by Leighton /18/ do not form shocks in the low chromosphere because of their long wavelength (>2000 km) and the observed phase shift of 90 degrees between temperature and velocity oscillations /10/.

ACOUSTIC AND MHD WAVE ENERGY GENERATION

The convection zones of stars depend only on three parameters, T_{eff}, gravity g and the ratio α of mixing length to pressure scale height. Using stellar envelope codes Renzini et al. /25/ and more recently Bohn /6/ c.f. Fig. 3 have computed acoustic wave energies for a large range of T_{eff}, g and α. These computations are based on Lighthill and Proudman's theory /19/,/24/ or Steins's theory /29/, respectively and depend on the choice of assumed turbulence spectra. Similar computations for magnetohydrodynamic waves are not yet available. However for our present discussion it is sufficient to adopt rough estimates for the different mhd wave energy fluxes. Such fluxes have been computed for homogenous magnetic fields by Stein and Ulmschneider /30/, /35/ and depend on the magnetic field strength B. Taking $B^2 \simeq 8\pi p$ valid for intense flux tubes, p being the external photospheric gas pressure, and using a simple opacity law one finds at the top of the convection zone the wave fluxes

$$F^{acoust}/\sigma T_{eff}^4 \simeq (u/c)^5 \simeq 1.9\ 10^{-38} g^{-0.959} T_{eff}^{10.6} \quad . \qquad (3)$$

$$F^{fast}/\sigma T_{eff}^4 \simeq (u/a)^5 \simeq 1.2\ 10^{-38} g^{-0.959} T_{eff}^{10.6} \quad . \qquad (4)$$

$$F^{slow}/\sigma T_{eff}^4 \simeq u/c \simeq 9.4\ 10^{-9}\ g^{-0.192} T_{eff}^{2.13} \quad . \qquad (5)$$

$$F^{Alfven}/\sigma T_{eff}^4 \simeq u/a \simeq 8.6\ 10^{-9}\ g^{-0.192} T_{eff}^{2.13} \quad . \qquad (6)$$

where c is the sound speed, a the Alfven velocity at the tube boundary (here a=1.1c) and u the mean convective velocity. The u^5-dependence of the normalized acoustic and fast wave fluxes comes from the isotropic propagation which leads to quadrupole wave generation. The slow and Alfven wave fluxes have a much smaller u-dependence which comes from the fact that these waves propagate along the magnetic field lines which leads to monopole wave generation. Fig. 3 shows that for the acoustic waves these estimates approximate the more elaborate computations of Bohn only very poorly. Bohn /6/ finds for the linear part in Fig. 3

$$F^{acoust}/\sigma T_{eff}^4 \simeq 1.2\ 10^{-22} g^{-0.5} T_{eff}^{5.75} \alpha^{2.8} \quad . \qquad (7)$$

Naturally in Eq.'s (4)-(6) the mdh-wave fluxes apply only for those areas of the star where magnetic fields occur. Note that because in the mixing-length theory of convection zones one has $\sigma T_{eff}^4 \simeq 5\rho u^3/\alpha$ if the total flux is carried entirely by convection, the total T_{eff}-dependence for acoustic and fast waves is $\sim u^8$ and for the slow and Alfven waves is $\sim u^4$.

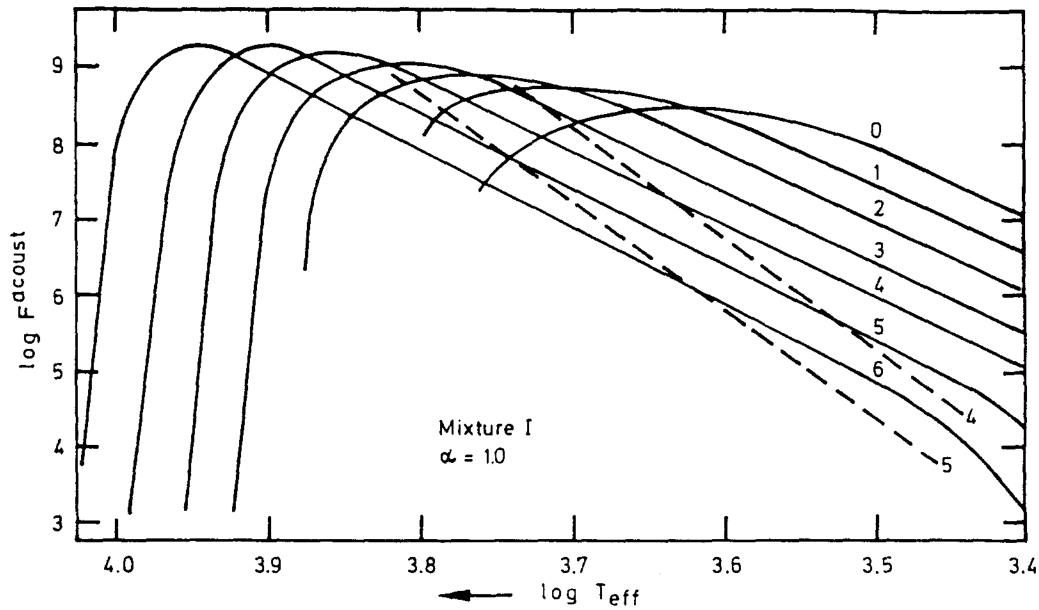

Fig. 3 Acoustic wave fluxes F^{acoust} (erg $cm^{-2}s^{-1}$) after Bohn /6/ (solid) as function of T_{eff} with log g as parameter. The rough estimate after Eq. (3) is indicated dashed.

This shows that only the surface layers where u is large are involved in the generation of wave energy. As however the mhd-waves depend on the presence of magnetic fields the depth of the convection zone indirectly enters the wave energy generation inasmuch as it affects the dynamo mechanism.

VALIDITY OF THE ACOUSTIC HEATING THEORY

For a direct comparison of the fluxes from Eq.'s (3) to (7) with Fig. 2 it should be kept in mind that the former fluxes are computed at the top of the convection zone while the MgII fluxes are observed in the chromosphere. One must take into account photospheric radiation damping of the waves and allow for the fact that the MgII k fluxes constitute only about one tenth of the total chromospheric losses. In addition for the mhd-waves one must multiply with the filling factor of the magnetic field. The acoustic wave energy calculations (Fig. 2) indicate a large g and T_{eff} dependence. This large dependence is not seen in the observations. In addition the computed acoustic wave energy depends only on the three parameters T_{eff}, g, α which are constant for a given star and thus can not explain the observed rotation-activity connection as well as the observed inhomogenous chromospheric network emission over the stellar surface. Note however that acoustic energy generation which depends strongly on the mean convective velocity is expected to show some spatial and temporal variation. Clearly the acoustic heating theory which ignores magnetic fields can only be valid for very slowly rotating stars, possibly for late-type supergiants where indeed a gravity dependence of the chromospheric emission has been found /32/ or for very late fully convective dwarf stars if these stars cannot produce or retain magnetic fields. For the bulk of the late-type stars the acoustic heating theory is not valid. However from the success of the acoustic heating theory to explain chromospheric emission by the process of shock formation and dissipation it is seen that this theory remains an important possibility for the heating of nonmagnetic areas on stars.

MHD-WAVE HEATING THEORIES

Mhd-waves are alternative mechanisms for the heating of stellar chromospheres. In homogeneous magnetic fields these are the slow-, fast- and Alfven mode waves. As the magnetic fields appear in the form of rapidly spreading flux tubes these wave modes are appropriate for regions above the middle chromosphere where the flux tubes fill out the entire available space /23/. Below this height the tube wave analogues of the homogenous field wave-forms are more appropriate. These modes are the longitudinal-, transverse- and torsional tube waves /28/. The longitudinal tube mode is a wave where cross-sectional variations occur and the gas pressure is the principal restoring force. This mode is very similar to an acoustic wave and to a slow-mode wave (only cases with a>c are considered here) if propagation only along the field lines is considered /13/. While for the slow-mode and acoustic waves the propagation speed is c, the propagation speed of the longitudinal tube wave $c_T=(c^2a^2/(c^2+a^2))^{1/2}$ is

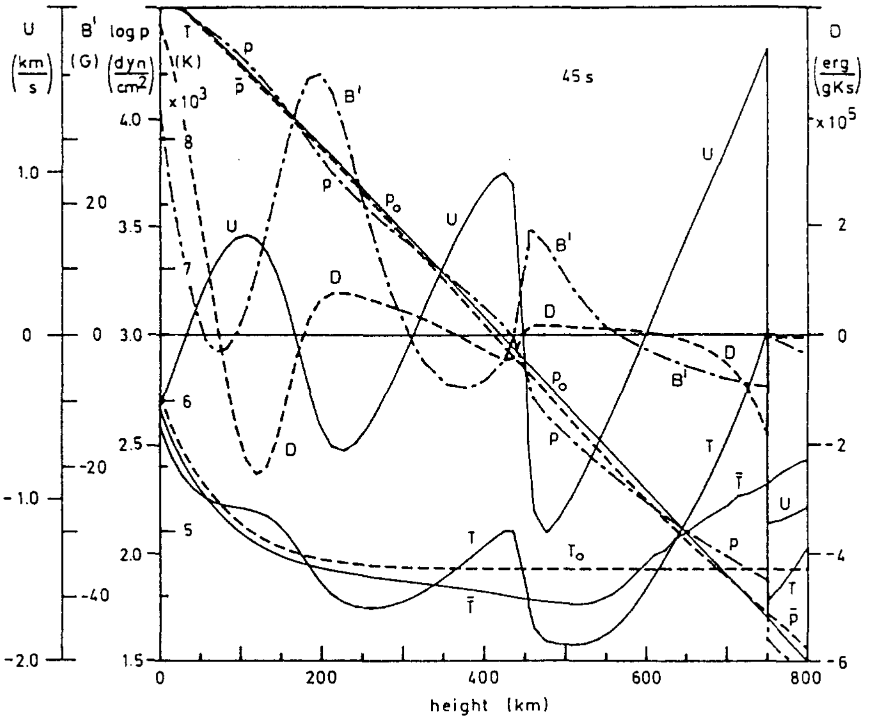

Fig. 4 Longitudinal mhd tube waves after Herbold et al. /13/.

somewhat smaller than c. The transverse tube wave or shaking mode does not show any cross-sectional variation. The same is true for the torsional tube wave. In both cases the magnetic tension is the restoring force and the propagation speed is a. The transverse wave is very similar to the Alfven wave in homogenous fields, while for the torsional wave there is no analogue in homogenous fields. Likewise if small amplitude waves in isolated thin flux tubes are considered there is no analogue of the fast-mode waves in the tube geometry.

Let me now use the term slow-mode independent of the field geometry to denote both the acoustic-like longitudinal- and the slow-mode waves with slow propagation speeds c_T or c<a, and similarly the term Alfven waves to denote both the transverse- and the Alfven modes with a fast propagation speed a. If one uses values u=2 km/s and c=10 km/s typical for the top of the solar convection zone in Eq.'s (3) to (6) it is seen that the slow-mode and Alfven mode mhd waves are produced more efficiently than acoustic or fast mode waves by a factor of about 600. This is a consequence of the much more efficient monopole sound generation. As the magnetic fields on the sun are concentrated at the boundaries of the granulation- and supergranulation cells, slow-mode and Alfven waves could readily explain the observed spatial inhomogeneity of the chromospheric emission. Moreover both waves show much smaller dependences on gravity and T_{eff} which agrees better with the behaviour of the chromospheric emission shown in Fig. 2.

The problem is whether these two mhd wave types can reproduce the rapid onset of chromospheric emission shown in Fig. 1. For Alfven waves the Joule- and viscous- as well as the ion-neutral collisional heating rates /21/ at the temperature minimum area are given by

$$\epsilon_{JV} = F^{Alfven} f 4\pi^2 (c_L^2/(4\pi\lambda)+\eta/\rho)/(P^2 a^3) = 5\ 10^{-5} \mathrm{erg\ cm^{-3} s^{-1}} \quad . \qquad (8)$$

$$\epsilon_{IN} = F^{Alfven} f 4\pi^2 \tau_n/(P^2 a) = 4\ 10^{-6} \mathrm{erg\ cm^{-3} s^{-1}} \quad . \qquad (9)$$

Here the wave period is P=40 s, the Alfven flux is $F^{Alfven} = 8\ 10^9$ erg $cm^{-2}s^{-1}$ from Eq. (6), the electrical conductivity $\lambda = 2\ 10^{12}\ s^{-1}$, the Alfven speed a=1.1 10^6 cm s^{-1}, the ion-neutral collision time $\tau_n = 2.2\ 10^{-6}$ s and the density in the flux tube $\rho = 2.3\ 10^{-9}$ g cm^{-3}. c_L is the velocity of light. For the filling factor f=1 10^{-2} we have assumed one flux tube per granulation cell and that a tube of radius 70 km spreads to fill the entire granular area of radius 700 km. In spite of a short wave period and a very large flux (the latter is necessary if one wants to heat the layers of the fully spread flux tube by injection of wave energy into the narrow area of the foot of the tube) it is seen that the heating rates are too small. In addition, as these rates vary slowly with height they do not explain the observed rapid onset of chromospheric emission. Yet heating by Alfven waves may be important if shorter periods are taken or if mode-coupling with slow-mode and fast-mode waves in higher

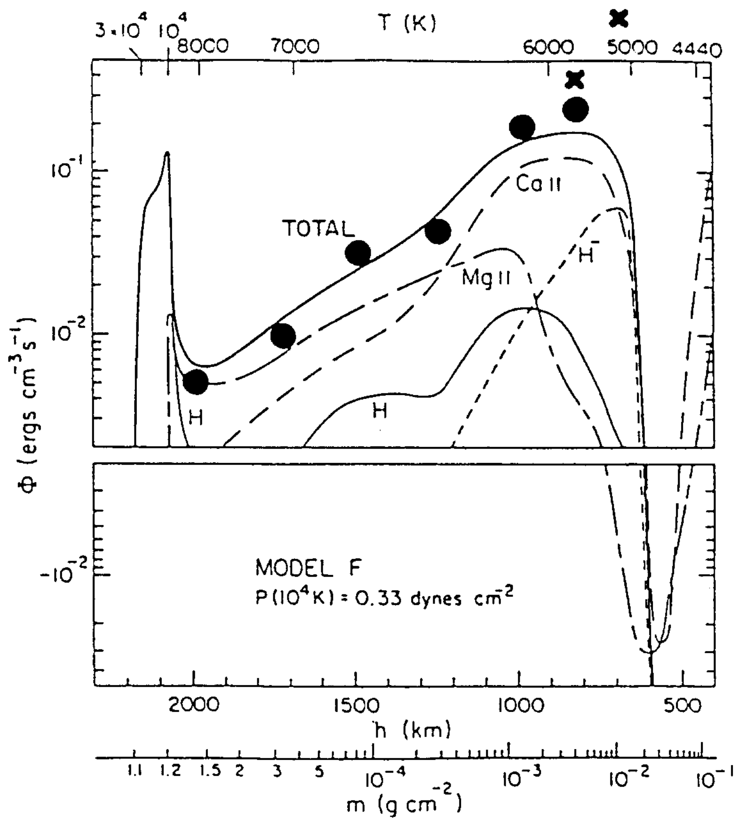

Fig. 5 Dissipation by torsional Alfven waves using a Kolmogorov-type heating law after Hollweg /16/ (dots) compared with radiative cooling rates of Vernazza et al. /37/. An extension to lower heights by the author is indicated by crosses.

chromospheric and coronal layers are considered. It should also be pointed out that it is difficult to explain the observed activity-luminosity correlation with Alfven waves. As the radiative momentum in early-type stars is in the radial direction one would need magnetic field lines inclined to the vertical if Alfven waves were generated by radiation pressure. The strong winds of these stars however act to minimize the deviations from the radial fields.

Slow-mode waves are very similar to acoustic waves propagating along the magnetic field. Herbold et al. /13/ have shown that in intense magnetic flux tubes longitudinal waves behave essentially like acoustic tube waves. The main difference compared to plane-parallel acoustic waves is the spreading geometry of the magnetic flux tube. The somewhat smaller propagation speed in the longitudinal wave is of minor importance. From this close similarity of slow-mode waves and acoustic waves it is obvious that slow-mode waves can easily explain the magnitude and sudden onset of chromosperic emission. With a slow-mode wave flux of $F^{slow} = 9\ 10^9$ erg cm^{-3} s^{-1} from Eq. (5) we have with the same filling factor f as above an average initial acoustic flux of $1\ 10^{-3} \sigma T_{eff}$ which is comparable to the value used above for acoustic waves. Fig. 4 taken from Herbold et al. /13/ illustrates how the slow-mode heating mechanism operates in principle. These calculations however need a much better treatment of the H^-, MgII, CaII and Lyman emission before a detailed comparison with observations can be made. As discussed above the efficient energy generation of slow-mode waves in the presence of magnetic fields is able to explain the inhomogeneity of the chromospheric emission over the solar surface and the strong correlation between emission and magnetic fields. In addition with presumably radial magnetic fields in early-type stars it is easy to picture this wave mode to be amplified by radiation pressure similar to acoustic waves. As rotation in late-type stars leads to greater magnetic field filling factors the rotation-activity correlation can be explained by this wave mode. For late-type stars the surface convection zones are essential for the generation of slow-mode wave energy. Finally the two generation mechanisms radiative amplification and convection give a natural explanation for the chromospheric and coronal emission gap at the A-stars.

Torsional Alfven waves as opposed to the transverse Alfven waves discussed above provide an additional possibility for heating in magnetic flux tubes. These waves are probably excited by cyclonic downflows of the non-magnetic gas just outside the flux tubes /27/. Heyvaerts and Priest /14/ as well as Hollweg /15/, /16/ have discussed the dissipation of these waves in the context of the generation of Helmholtz-Kelvin or tearing-mode instabilities for coronal active region loops. These instabilities are assumed to generate a Kolmogorov-type turbulent cascade where after reaching small enough wave numbers the energy is dissipated by viscosity or electrical resistivity. Based on such a picture a heating rate

$$\epsilon_K = \rho u_t^3/r \quad , \tag{10}$$

is given /16/, where u_t is the torsional velocity, and r the radius of the flux tube. If one identifies the observed total horizontal nonthermal velocities u_H (e.g. Canfield and Beckers /9/) with u_t, Hollweg /16/ has shown that the magnitude as well as the height distribution of the solar chromospheric losses can be balanced as shown in Fig. 5. Extending the computations of Hollweg to lower height using flux tube radii as given by Pneuman et al. /23, Fig. 4, α=.025/ it is seen that ϵ_K shows a rapidly decreasing behaviour with height which does not reproduce the sudden onset of chromospheric emission behind the temperature minimum. The efficiency of this Kolmogorov-type heating law has not yet been sufficiently confirmed by detailed computations or experiments. Moreover u_H can not be entirely torsional. From the picture presented above it must be expected that a large fraction of the horizontal nonthermal velocities is due to transverse Alfven waves. Thus the energy in the torsional waves is presently poorly known. It should be noted however that independent of the effectiveness of the Kolmogorov-type heating law there is the possibility that torsional waves heat the upper chromosphere and corona by mode-coupling to slow and fast mode waves as shown by Hollweg et al. /17/. For early-type stars it appears difficult to generate the torsional wave mode by radiative amplification and to explain the A-star emission gap.

DISCUSSION AND CONCLUSIONS

In the present discussion of chromospheric heating mechanisms based on acoustic and mhd-waves a picture emerges which seems to fit the observational constrains quite naturally. This picture explains at the one hand observations like the magnitude and the sudden onset of the radiative emission by the formation and dissipation of acoustic-wave-like shocks. The inhomogeneities of the chromospheric emission over the solar surface, the correlation of the emission with luminosity in early-type stars and the emission-rotation correlation in late-type stars on the other hand are explained by more efficient slow-mode mdh wave generation in the presence of flux-tube-like magnetic fields which have some analogy to horn-like loudspeakers. Obviously this picture, summarized in Tab. 1 for the different wave types, is

acoust waves	Alfven mhd waves	slow mhd waves	torsional mhd waves	observational constraints
no	yes	yes	yes	large surface-inhomogeneities
yes	no	yes	yes?	net cooling rate of 0.3 erg $cm^{-3}s^{-1}$
yes	no	yes	no	rapid onset of emission above temp. min.
no	yes	yes	?	small gravity-dependence of wave flux
no	yes	yes	?	moderate T_{eff}-dependence of wave flux
yes	no?	yes	no	emission gap near the A-stars
yes	no?	yes	no	activity-luminosity correl. for early-type stars
no	yes	yes	yes	activity-rotation correl. for late-type stars

Tab. 1 Success of the acoustic-, as well as the Alfven-, slow-, and torsional mhd waves to explain the observational constraints

highly simplistic and the details need to be worked out. For instance it is clear that the transverse Alfven waves which should be generated very efficiently have not been investigated sufficiently and their role in the high chromosphere has not been clarified enough. The same can be said about the torsional waves.

Yet the above picture of wave heating lacks an important observational fact. The wave heating picture tells where energy is generated and where it is transported. It can also explain the tight correlation between chromospheric, transition-layer and coronal heating /2/ because the generation of waves is accomplished by similar mechanisms, as is the distribution of wave energy. So, if coronae were heated by Alfven waves (e.g. through mode-coupling to slow and fast waves) and chromospheres by slow-mode waves we would still find a tight correlation between chromospheric, transition layer and coronal emission because e.g. the same convection zone has produced both waves and the same flux tube has transported both wave fluxes. The missing aspect of the above picture is the observational fact that new magnetic flux appears and reconnection along current sheets is observed /7/,/8/. The wave heating picture works well for a static magnetic field distribution. For dynamic phenomena flares and microflares are important heating mechanisms /7/,/8/. It is presently not known how much of the chromospheric emission is due to these latter processes. As however both reconnection and mhd-waves are tied to the magnetic field, the chromospheric-coronal emission correlation appears to be valid even when non-wave heating mechanisms are considered. Thus the only way to finally identify the chromospheric (and coronal) heating processes is to develop all the possible mechanisms and bring ever more refined observational tests to bear on them.

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