Research Note

Radiation damping in acoustically heated atmospheres of late-type stars

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Summary. It is shown that radiation damping of acoustic waves propagating through a stellar photosphere is much larger in giant stars than in dwarfs. This could remove a major argument against acoustic waves as possible heating mechanism for the chromospheres of slowly rotating stars.

Key words: radiation damping – acoustic energy generation – stellar chromospheres – chromospheric line emission

1. Introduction

It has recently been suggested that very slowly rotating stars show a basal chromospheric emission which is not correlated with magnetic activity and thus may be purely acoustic in origin (Schrijver, 1987). However, one of the strongest arguments against an acoustic heating mechanism for stellar chromospheres is the fact that the observations of chromospheric emission line fluxes show no gravity dependence (e.g. Linsky and Ayres, 1978; Basri and Linsky, 1979; Weller and Oegerle, 1979; Rutten et al., 1988) while acoustic wave generation calculations by Renzini et al. (1977), Fontaine et al. (1981), and Bohn (1984) all show a large gravity dependence. The theoretical gravity dependence of the acoustic flux generation is in the sense that giant stars produce much more acoustic energy than dwarfs of the same effective temperature $T_{\text{eff}}$. However, such a direct comparison between acoustic fluxes from the top of the convection zone with emission fluxes from chromospheric heights is incorrect, because the influence of the intermediate photospheric layers must be taken into account. In the present work I want to show that these photospheric layers indeed greatly modify the acoustic wave energy flux due to radiation damping. Radiation damping works differently in giants and in dwarfs. It is shown that the more enhanced acoustic energy generation in giants is opposed by more efficient radiation damping in these stars. This effect works towards removal of the above mentioned discrepancy.

2. Generation of acoustic waves

In late-type stars with efficient convection zones it is expected that acoustic wave energy is amply generated by the turbulent convection. Calculations by Renzini et al. (1977) as well as Fontaine et al. (1981) based on the Lighthill formula for quadrupole sound generation show that the acoustic flux $F_{\text{ac}}$ is essentially proportional to $g^{-1}$, where $g$ is the gravitational acceleration at the stellar surface. A more detailed computation by Bohn (1984) who took into account improved opacities, a better treatment of molecules and the stellar density stratification after Stein (1967) decreased this gravity dependence as now additional contributions from dipole and monopole sound emission were considered. He found that $F_{\text{ac}}$ is proportional to $g^{-0.5}$. All this work was recently criticized by Goldreich (1987, private communication) who claims that forced turbulence was not properly taken into account. However, it is clear from very general arguments that the acoustic energy generation will show a considerable gravity dependence. This can be seen from the high power of the velocity dependence of the monopole ($v^4$), dipole ($v^6$) and quadrupole ($v^8$) terms of the acoustic energy generation. It is well known that in order to transport the total stellar flux $\sigma T_{\text{eff}}^4$ in an effective convection zone much higher velocities are found in the low density surface layers of giants and supergiants than in those of dwarfs.

3. Radiative damping of acoustic waves

The rather crude treatment of radiation damping of acoustic wave energy by Ulmschneider (1971; see also Spiegel, 1957) has been improved by Schmitz (1988) who finds for the radiative relaxation time

$$t_{\text{rad}} = \frac{2.5 c_p}{16 \sigma \kappa T^3}.$$  

(1)

Here $c_p$ is the specific heat at constant volume, $\sigma$ the radiation constant, $\kappa$ the opacity (cm$^2$ g$^{-1}$) and $T$ the temperature. To assess the importance of radiation damping, $t_{\text{rad}}$ has to be compared with the wave period $P$. A wave with $t_{\text{rad}} \ll P$ will suffer from extensive radiation damping, while a wave with $t_{\text{rad}} \gg P$ will conserve energy. The fact that stars have extensive radiation damping zones where $t_{\text{rad}} < P$ has been discussed elsewhere (Ulmschneider et al., 1979; Schmitz and Ulmschneider, 1981). As waves which efficiently heat the chromosphere must be propagating, the wave period $P$ has to be considerably smaller than the acoustic cut-off period $P_4$. From Bohn's (1984) acoustic energy spectra one finds that

$$P = \frac{1}{10} P_4 = \frac{1}{10} \frac{4 \pi c}{\gamma g}$$  

(2)

is a good estimate for the wave period at the peak of the acoustic spectrum. Here $c$ is the sound speed and $\gamma$ is the ratio of specific
Table 1. Wave period $P$ and radiative relaxation time $t_{\text{rad}}$ at the top of the convection zone, effective temperature $T_{\text{eff}}$ and gravity $g$ for stars of different luminosity class LC and spectral type ST

<table>
<thead>
<tr>
<th>LC</th>
<th>I</th>
<th>III</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST</td>
<td>$T_{\text{eff}}$ (K)</td>
<td>$\log g/g_0$</td>
<td>$t_{\text{rad}}$ (s)</td>
</tr>
<tr>
<td>$F_0$</td>
<td>7700</td>
<td>-2.7</td>
<td>7.9</td>
</tr>
<tr>
<td>$G_0$</td>
<td>5550</td>
<td>-3.1</td>
<td>80</td>
</tr>
<tr>
<td>$K_0$</td>
<td>4420</td>
<td>-3.5</td>
<td>450</td>
</tr>
<tr>
<td>$M_0$</td>
<td>3650</td>
<td>-4.3</td>
<td>3D3</td>
</tr>
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heats. Using a simple opacity formula valid for regions where the H$^-$ opacity is dominant (Ulmschneider et al., 1978)

$$\kappa = 1.376 \times 10^{-22} \rho^{0.738} T^8 \text{(cm}^2 \text{g}^{-1}),$$

(3)

the pressure $p$ at optical depth $\tau_{5000} = 1$ can be estimated following Renzini et al. (1977)

$$p \simeq 1.96 \times 10^{13} \rho^{0.575} T_{\text{eff}}^{-2.88},$$

(4)

From Eqs. (1) to (4) using a mean molecular weight $\mu = 1.3$ one finds approximately

$$t_{\text{rad}} \simeq 2.98 \times 10^{24} \rho^{-0.425} T_{\text{eff}}^{-5.58},$$

(5)

$$\frac{P}{t_{\text{rad}}} \simeq 2.61 \times 10^{-21} \rho^{-0.575} T_{\text{eff}}^{0.38}.$$  

(6)

For stars of different spectral type and luminosity class a comparison of wave periods $P$ and radiative relaxation times $t_{\text{rad}}$ at the top of the convection zone is shown in Table 1. The values of $T_{\text{eff}}$ and $g$ were taken from Schmidt-Kaler (1982).

Radiation damping decreases the acoustic flux like $F_M \sim \exp \left(-\int \rho/t_{\text{rad}} \, dn\right)$ where $n$ is the number of wave periods and where the integration must be carried over a similar number of wave periods for different stars. This results from the fact that the integration should extend over a similar range of optical depth. From Eq. (3) it can be shown following Renzini et al. (1977) that

$$\frac{\partial \tau}{\tau} = -1.7 \frac{dx}{H},$$

(7)

where $\tau$ is the optical depth, $H = c^2/\rho g$ the scale height and $x$ the geometrical height. The wavelength $\lambda = cp$ after Eq. (2) likewise scales with $H$. Thus a given optical depth range corresponds to a similar number of wave periods in stars of different gravity.

Table 1 and Eq. (6) show quite generally that acoustic waves in supergiants and giants suffer much more from radiation damping than in dwarfs. In addition, for the same luminosity class, radiation damping in earlier type stars is more severe than in late type stars. A consequence of this effect is, that the acoustic flux reaching chromospheric heights in supergiants and giants is greatly reduced from its initially enhanced value. That radiation damping of acoustic waves affects giants more than dwarfs has already been noted in the calculations of Schmitz and Ulmschneider (1981, their Table 1). The calculations of those authors were based on the much lower Renzini et al. (1977) fluxes employing a grey LTE approximation. Using Eq. (6) directly to infer acoustic fluxes at chromospheric heights is not very accurate as radiation damping depends non-linearly on the wave amplitude and as $t_{\text{rad}}$ changes quickly with height. For a more realistic evaluation of the behaviour of the acoustic flux in stellar photospheres improved non-linear time-dependent wave computations have to be made.

4. Conclusions

It has been found that a considerable gravity dependence of the acoustic wave generation process must be expected which leads to a significantly enhanced production of acoustic energy in giant and supergiant stars. However, this wave energy flux by travelling through the photospheric layers will be strongly reduced by radiation damping. It has been shown that radiation damping has a significant gravity dependence such that waves in giants and especially in supergiants suffer much more radiation damping than in dwarfs. Thus the acoustic fluxes reaching the chromosphere in supergiants, in giants and in dwarfs will be more similar, decreasing the gravity dependence of the acoustic flux at chromospheric heights. Whether the absence of the gravity dependence in the observed chromospheric emission line fluxes can be completely explained by this process must await more detailed time-dependent computations.

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References