

# LONGITUDINAL-TRANSVERSE MAGNETIC TUBE WAVES IN THE SOLAR ATMOSPHERE

P. ULMSCHNEIDER

Institut für Theoretische Astrophysik,  
Im Neuenheimer Feld 561,  
D-6900 Heidelberg, Federal Republic of Germany

and Z. MUSIELAK

Center for Space Plasma and Aeronomic Research,  
University of Alabama,  
Huntsville, AL 35899, USA

**ABSTRACT** The propagation of nonlinear adiabatic magnetohydrodynamic waves in a thin magnetic flux tube is studied. The waves are excited by purely transverse shaking. Due to nonlinear coupling there is a significant energy transfer to the longitudinal wave. This transfer is largest for long period waves and increases with the shaking amplitude. We find lifting of the tube mass due to the increased swaying with height.

## INTRODUCTION

Spruit (1982) has shown that in thin magnetic fluxtubes there are three possible wave modes: a *torsional mode*, a *transverse mode* and a *longitudinal mode*. The nonlinear behaviour of the torsional mode has been studied by Hollweg et al. (1982) and of the longitudinal mode by Herbold et al. (1985). The latter authors found that longitudinal waves behave essentially like acoustic tube waves. Hollweg et al. (1982) have shown that due to nonlinear mode-coupling considerable longitudinal wave power developed when starting with a purely torsional excitation. Preliminary studies of the nonlinear behaviour of the transverse mode were made by Zähringer and Ulmschneider (1987, henceforth called ZU). In the present paper we want to extend these transverse wave mode studies to investigate how the magnetic flux tube reacts to shaking with different frequencies and amplitudes. In particular we are interested in how this shaking affects the nonlinear coupling between transverse and longitudinal waves. This nonlinear mode-coupling could be an important heating mechanism in the higher chromosphere and corona.

## METHOD

We assume a vertically oriented magnetic flux tube of circular cross-section embedded in an otherwise non-magnetic gravitational atmosphere. A fluid element in the tube can be uniquely identified by the Lagrange height,  $a$ . At a later time,  $t$ , if the excitation

is only in the  $z - x$  plane, the position of the fluid is given by the coordinates  $z(a, t)$  and  $x(a, t)$ . For the initial physical state of the tube see ZU. As shown by ZU the time-dependent magnetohydrodynamic equations can be simplified for the thin flux tube situation envisioned here and results in 7 partial differential equations. This is possible by making two important assumptions. The validity of these assumptions is presently unclear and can only be ascertained when detailed 3D calculations become available. The first assumption is that of horizontal pressure balance, where the external gas pressure is assumed to be independent of time. The second is that the back reaction of the external gas on the swaying tube can be taken into account by an increased tube mass. Both assumptions act to reduce the 3D nature of the swaying tube problem to a one-dimensional and thus easily solveable problem. For the details of the solution of the 7 mhd equations see ZU.

## RESULTS

We have computed adiabatic waves of periods  $P = 45, 90, 180, 300$  s by purely transverse shaking at the bottom with velocity amplitudes of  $v_0 = 0.2, 0.4, 0.8, 1.6$  km/s. As boundary conditions, both at the bottom and at the top, three velocities each must be specified. For the top we assume all three velocities to be transmitting. Different to ZU we assume for the bottom  $v_{\perp x} = -v_0 \sin(2\pi t/P)$ ,  $v_{\perp y} = 0$ ,  $v_{\parallel z} = \text{transm.}$  Fig. 1 shows a snapshot of a wave computations with period  $P = 90$  s and  $v_0 = 0.4$  km/s at time  $t = 600$  s. With the transverse wave speed  $c_F$  roughly constant over most of the tube the horizontal velocity amplitude  $v_x$  behaves roughly like  $\rho^{-1/2}$  and thus grows similar in the four wave calculations. The horizontal displacement  $x$  has maxima at the nodes of  $v_x$  and oscillates around a value  $x_0 = v_0 P / (2\pi)$ . The profile  $x$  versus height gives the actual geometrically distorted shape of the tube axis. The maximum swaying amplitude increases with the wave period. At time  $t = 600$  s we find for the waves maximum horizontal swaying amplitudes of  $\Delta x = 22, 50, 88, 108$  km at periods  $P = 45, 90, 180, 300$  s, respectively, which are only a fraction of the tube diameter. In addition to the horizontal velocities, *vertical velocity components*  $v_z$  appear which have roughly twice the frequency of the transverse wave. The reason for the appearance of these longitudinal (compressional) components is the action of the curvature forces which are always perpendicular to the local tube direction. These forces have horizontal and vertical components. In one wavelength of the  $\Delta x$  profile the vertical force components change sign twice, while the horizontal force component only once. The action of the vertical force components is to compress and expand the tube gas which leads to longitudinal waves.

In the long period waves the more extensive swaying excursions  $\Delta x$  lead to greater gas pressure fluctuations. This results in a greater longitudinal wave energy generation for long period waves: The ratio of the velocity maxima  $v_x/v_z$  is 12, 5.5, 3.5, 2.3 for the waves with  $P = 45, 90, 180, 300$  s, respectively. This ratio decreases with time in a given wave computation, which points to resonance effects. The swaying amplitude  $\Delta x$  and the horizontal velocity  $v_x$  increase slowly with time. In addition, the  $v_x/v_z$  ratio decreases with increasing excitation amplitude  $v_0$ . For the  $P = 90$  s wave,  $v_x/v_z$  is 7.3, 5.5, 3.0 if  $v_0 = 0.2, 0.4, 0.8$  km/s, respectively. Thus the ratio of longitudinal to transverse wave energy increases with increasing swaying amplitude. Another important effect found in our calculations is the lifting of the gas column in the tube due to the centrifugal forces associated with the increased swaying with height. Fig. 1 shows the lifting  $z - a$  of the fluid element originally at height  $a$  as function of height. This has

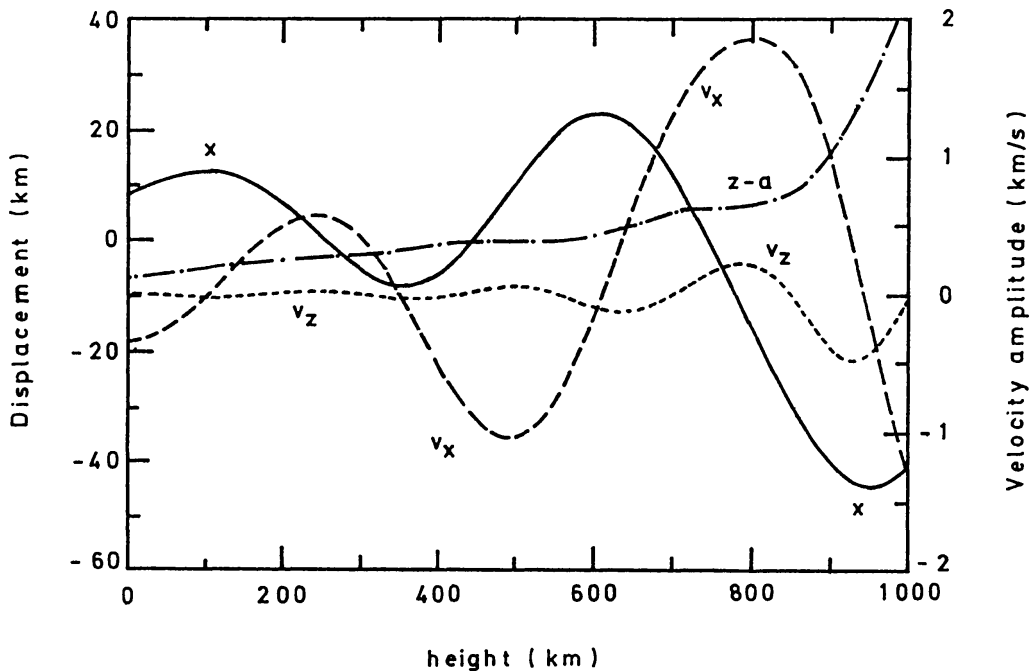
also been found by ZU, where due to a different bottom boundary condition a much larger effect resulted. The topmost point in of or wave calculations at  $t = 600$  s is lifted between 31 and 47 km roughly independent of the wave period. The bottom of the tube moved into the sun by between 4 and 7 km.

## CONCLUSIONS

We have shown that foot point shaking of magnetic flux tubes results in transverse waves which have horizontal swaying velocities and amplitudes which increase rapidly with height. The swaying velocities are similar for waves of different period but result in larger horizontal tube excursions in long period waves. Due to stretching and compressive actions of the vertical components of the curvature forces, longitudinal (compressional) waves are generated. The generation efficiency of these waves is greatest for long period waves and increases with the magnitude of the shaking. The increased swaying with height leads to lifting of the gas column in the tube due to centrifugal forces, which in these adiabatic calculations results in expansion cooling of the upper tube.

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*Fig. 1. Snapshot of a longitudinal-transverse wave of period  $P = 90$  s at time 600 s after pure transverse sinusoidal shaking at the bottom has started with a velocity amplitude of  $v_0 = 0.4$  km/s.*