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On the generation of nonlinear magnetic tube waves in the solar atmosphere

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Abstract. The nonlinear time-dependent response to purely transverse shaking of a thin vertical magnetic flux tube embedded in the solar atmosphere is investigated numerically. The shaking is imposed on the tube at different heights in the solar atmosphere and the resulting magnetic wave energy fluxes are calculated for the observationally established range of velocity amplitudes and tube magnetic fields. The obtained results clearly demonstrate that typical wave energy fluxes carried by nonlinear transverse tube waves are of the order of 10^9 erg/cm^2s. This, in contrast to previous analytical studies, seems to indicate that there is enough wave energy to account for the enhanced heating observed in the chromospheric network, and that magnetic tube waves may also play some role in the heating of other regions of the solar atmosphere.

Key words: methods: numerical – sun: chromosphere – sun: corona – sun: magnetic fields – waves

1. Introduction

The generation of different types of magnetohydrodynamic (MHD) waves in the solar atmosphere has been studied primarily by using analytical methods based on the theory of sound generation by Lighthill (1952). Kulsrud (1955) and Osterbrock (1961) extended this theory by including magnetic field effects, and Musielak & Rosner (1987) improved it by accommodating the presence of stratification and an embedded uniform magnetic field in the wave generation region (see also Rosner & Musielak 1989). More recently, Collins (1989a, 1989b, 1992) has modified this theory of wave generation to explore the excitation of MHD waves by periodic velocity fields in diverging magnetic flux tubes. The common feature of these studies is that they look at the magnetic field in a non-local way to get mean generated wave fluxes. Further advances occurred when the detailed local field geometry was considered.

It has been known for a number of years that the distribution of magnetic fields on the solar surface is highly inhomogeneous and that magnetic inhomogeneities outside sunspots form flux tube structures (e.g., Stenflo 1978; Zwaan 1978). Individual magnetic flux tubes are regions of intense magnetic fields that rapidly diverge in the solar chromosphere. It has been suggested that these tubes may become "windows" through which the wave energy generated in the solar convection zone is carried by various types of waves (longitudinal, transverse and torsional - see Spruit 1982) to the overlying chromosphere and corona (e.g., Spruit & Roberts 1983).

This has motivated Musielak et al. (1989) to investigate the interaction between turbulent motions in the solar convection zone and thin magnetic flux tubes. To separate longitudinal and transverse magnetic tube waves, they have considered only vertically oriented magnetic flux tubes and restricted their approach to the linear regime. The obtained results indicate that the wave energy flux carried by longitudinal tube waves along a single magnetic flux tube can be of the order of 10^7 erg/cm^2s or less, which seems to be too little to account for the observed enhanced heating in the chromospheric network. In recent work by Musielak et al. (1994a), it has been shown that the flux can be considerably higher if a more refined treatment of generation of longitudinal tube waves is considered. In a similar treatment for transverse tube waves, Musielak et al. (1994b) have shown that the wave energy flux carried by these waves can be of the order of 10^8 erg/cm^2s. Finally as an investigation based on the Lighthill theory and concentrating on the local field geometry the work of Lee (1993) should be mentioned who studies the generation of MHD waves inside sunspot magnetic fields.

There are also methods of MHD wave generation which are not based on the Lighthill approach. These methods apply velocity fluctuations of an observed magnitude to detailed magnetic flux tube models. Such an approach has recently been taken by Choudhuri et al. (1993a) as well as by Choudhuri et al. (1993b) who have investigated the generation of magnetic kink waves by rapid foot point motions of the magnetic flux tube. They argue that occasional rapid motions can account for the entire energy flux needed to heat the quiet corona. They find that pulses are much more efficient than continuous excitation to get wave energy into the corona and that the energy flux from pulses actually increases when there is a transition layer temperature jump in the atmosphere.

The basis for such an approach was prompted by recent observations of the proper motions of footpoints of magnetic flux tubes at the photospheric level (Muller 1989; Muller et al. 1994). These observations clearly show that horizontal velocities as large as 3 km/s occur in the solar photosphere. Velocities of this magnitude and larger have also been reported by Title (1994, private communication). These authors also recognized that the interaction between the large velocity motions and magnetic flux tubes may become an efficient source of magnetic tube waves which can propagate along the tubes and carry energy to the chromosphere and corona. A rough estimate of the generated wave energy fluxes by Muller et al. (1994) clearly demonstrates that the amount of wave energy available for heating is sufficient to sustain the mean level of the observed radiative losses from both the solar chromosphere and corona.

These observations of large velocity fluctuations affecting foot points of magnetic flux tubes are also found to be in good agreement with recent theoretical advances. Time-dependent numerical simulations of the solar convection zone performed by several different groups (e.g., Nordlund & Dravins 1990; Nordlund & Stein 1991; Cattaneo et al. 1991; Steffen 1993) all suggest the presence of motions with horizontal velocities larger than 2 km/s near the top of the solar convection zone. In these numerical simulations, the presence of horizontally propagating shocks near the top of the solar convection zone has been detected (Cattaneo et al. 1991; Steffen 1993; Steffen et al. 1994).

Thus in principle the approach to compute the MHD wave energy generation from known velocity fluctuations is very promising. Unfortunately at the present time the exact magnitude and time-dependence of these fluctuations are not well known. This is particularly true for the frequency of the sudden large foot point displacements. Another shortcoming in all the above mentioned methods of wave generation is that analytical methods were used which only treat the production of linear waves. The results obtained must therefore be taken only as lower bounds for realistic wave energy fluxes that may be carried by magnetic tube waves (as well as by MHD waves) to the upper layers of the solar atmosphere.

The main goal of this paper is twofold. First we want to calculate the wave energy fluxes carried by transverse tube waves in a nonlinear regime and second we try to take into account the continuous excitation of the flux tube by the turbulent flow field of the convection zone. To represent this turbulence realistically we specify the rms velocity amplitude and employ an extended Kolmogorov spectrum with a modified Gaussian frequency factor which presently is the best guess for the largely unresolved velocity fluctuations (Musielak et al. 1994c). This approach has the advantage of incorporating important features of both the Lighthill and Choudhuri et al. methods. Yet it will not include the occasional sudden foot point motions (shocks) and thus will constitute only a lower bound for the total wave flux. But our approach will provide an estimate of the contribution from the continuous wave excitation which will eventually have to be augmented by the contribution due to sudden events when their magnitude and frequency are better known. For the excitation of vertical flux tubes by large horizontal velocity pulses see also the recent time-dependent simulations by Zhugzhda et al. (1994) which show the generation of kink shocks in the generated transverse tube waves and the very large mode conversion to longitudinal tube wave pulses in such events. The treatment of nonlinear wave generation by intense pulses will be described in a future paper.

In the present work, we use a numerical approach to calculate the efficiency of the generation of nonlinear transverse tube waves. Our approach is based a one-dimensional, time-dependent, nonlinear MHD code originally developed by Ulmschneider et al. (1991, hereafter called UZM) to study the propagation of longitudinal and transverse waves in thin magnetic flux tubes. We modify this code to investigate the problem of generation of transverse magnetic tube waves by continuous large amplitude motions in the external medium.

We consider a thin, vertically oriented magnetic flux tube embedded in the solar atmosphere and excite the waves by shaking the tube perpendicular to its axis at four different heights which correspond to optical depths τ_{5000} = 100, 10, 1 and 0.1 outside the tube. The shaking velocity is a superposition of partial waves with random phase derived from the local turbulent flow field. The maximum velocity of the turbulent motions is taken from a range of observed velocities on the solar surface (e.g., Muller 1989; Muller et al. 1994) as well as from numerical convection zone calculations by Cattaneo et al. (1991) and Steffen (1993). In addition, we assume that the strength of the magnetic field inside the tube ranges from 1000 to 1500 G. We calculate the resulting tube wave energy fluxes and compare them with previous work.

As in the case considered in this paper the excited transverse tube waves are nonlinear, some portion of their energy will be transferred to longitudinal tube waves via the process of nonlinear mode-coupling as shown by UZM. In general, this process may lead to damping of transverse waves during their propagation along magnetic flux tubes and to heating of the local medium. Here, we consider only the generated transverse wave energy flux at the local region of excitation and disregard the subsequent wave propagation and mode conversion.

The main problem addressed in this paper is whether the wave energy fluxes generated by turbulent motions in the solar convection zone are sufficient to sustain the mean level of radiative losses observed from active regions in the solar chromosphere (i.e. the chromospheric network) where the enhanced heating is observed. A first step to this is to see whether enough wave energy is available. It is the purpose of this work to estimate the likely transverse wave energy fluxes. The paper is organized as follows: Sect. 2 describes briefly the UZM approach, discusses the shaking and the evaluation of the tube wave energy flux. Our results are presented in Sect. 3, while Sect. 4 gives our conclusions.

2. Method

To calculate the efficiency of excitation of nonlinear magnetic tube waves, we follow the time-dependent nonlinear approach developed by UZM who solved the basic ideal MHD equations for thin, vertically oriented magnetic flux tubes by using the method of characteristics. Choudhuri (1990) and Cheng (1992) have criticized that in the equations used in this approach there are centrifugal and Coriolis terms missing (for a discussion of this see Zhugzhda et al. 1994). However these terms apply only at greater heights where considerable longitudinal flows occur in the tube. We feel that although these terms have to be included eventually, we do not have to worry about them in our present application. In the following, we briefly describe the UZM approach.

2.1. The magnetohydrodynamic equations

The standard set of ideal MHD equations can be written in the form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \, , \tag{1}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p - \frac{1}{4\pi} \mathbf{B} \times (\nabla \times \mathbf{B}) + \rho \mathbf{g} \, , \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \, , \tag{3}$$

with

$$\nabla \cdot \mathbf{B} = 0 \, , \tag{4}$$

and

$$\frac{\partial S}{\partial t} + \mathbf{v} \cdot \nabla S = 0 \, , \tag{5}$$

where ρ is the density, $\mathbf{v}$ the velocity, p the gas pressure, $\mathbf{B}$ the magnetic field strength, $\mathbf{g}$ the gravitational acceleration, S the entropy and t represents time. To solve the above system, the relationship between p, ρ, T and S must be known. UZM use the ideal gas law for a nonionizing gas

$$p = \rho \frac{\mathscr{R}}{\mu} T \, , \tag{6}$$

and the thermodynamic relation

$$\frac{\rho}{\rho_o} = \left(\frac{c_S}{c_{S_o}} \right)^{\frac{2}{\gamma - 1}} e^{-\mu (S - S_o)/\mathscr{R}} \, , \tag{7}$$

where $\mathscr{R}$ is the universal gas constant, μ the mean molecular weight, γ the ratio of specific heats and $c_S = \gamma p / \rho$ the sound speed. Subscript o denotes a reference state. With Eq. (5) our approach is restricted to adiabatic waves.

To account for nonlinear wave motions propagating along a thin magnetic flux tube embedded in a field-free atmosphere, this system of equations is modified by assuming that the physical variables in the tube are described sufficiently well by their values on the tube axis, and that they vary in a one-dimensional way along the tube. This so called "thin flux tube approximation" allows to separate the transverse (perpendicular to the tube axis) and longitudinal (parallel to the tube axis) components of the wave motions in the above system of equations and to derive a set of ordinary differential equations which describe the motions of the gas elements in the tube in three dimensions. By combining different equations from this set, UZM derived six basic equations that can be solved by using the method of characteristics. The longitudinal components give two equations that can be written in the following form:

$$\mathbf{l} \cdot d\mathbf{v} \pm \frac{2}{\gamma - 1} \frac{c_S}{c_T} dc_S \mp \frac{\mu c_S^2}{\gamma \mathscr{R} c_T} dS \mp \left[\frac{v_z c_T}{\rho c_A^2} \left(\frac{dp_e}{dz} \right) \mp g l_z \right] dt = 0 \, , \tag{8}$$

along the two characteristics C_1^+ and C_1^- given by

$$\left(\frac{da}{dt} \right)_{\pm} = \pm \frac{c_T}{l_a} \, , \tag{9}$$

while the transverse components lead to four equations

$$(1 - l_x^2) dv_x - l_x l_y dv_y - l_x l_z dv_z \mp c_k dl_x - \frac{\rho - \rho_e}{\rho + \rho_e} g l_x l_z dt = 0 \, , \tag{10}$$

$$(1 - l_y^2) dv_y - l_x l_y dv_x - l_y l_z dv_z \mp c_k dl_y - \frac{\rho - \rho_e}{\rho + \rho_e} g l_y l_z dt = 0 \, , \tag{11}$$

along the two characteristics C_2^+ and C_2^- given by

$$\left(\frac{da}{dt} \right)_{\pm} = \pm \frac{c_k}{l_a} \, . \tag{12}$$

Here the top and bottom signs in Eqs. (8) to (12) correspond to the C^+ and C^- characteristics, respectively, and the tube speeds c_T of the purely longitudinal and c_k of the purely transverse wave are given by

$$c_T = \sqrt{\frac{c_S^2 c_A^2}{c_S^2 + c_A^2}} \, , \tag{13}$$

and

$$c_k = c_A \sqrt{\frac{\rho}{\rho + \rho_e}} \, , \tag{14}$$

where c_A is the Alfvén velocity, p_e is the external gas pressure and l_a is the scale factor. In addition, we have

$$l_x^2 + l_y^2 + l_z^2 = 1 \, , \tag{15}$$

where l_x, l_y and l_z are components of the unit arc length vector $\mathbf{l}$. All physical variables are functions of time t and Lagrange height a. Finally, we consider only adiabatic waves with

$$dS = \left(\frac{\partial S}{\partial t} \right)_a dt = 0 \, , \tag{16}$$

which is valid along the fluid path $a = const.$

There are 8 unknown variables $\mathbf{v}, \mathbf{l}, c_S$ and S which are fully determined by 8 equations (Eqs. 8, 10, 11, 15 and 16) along the five characteristics given by Eqs. (9), (12), and $a = const.$ In order to solve the above equations, we prescribe boundary conditions and adopt a model for the external atmosphere. We use the so-called "open" or "transmitting" boundary conditions described in detail by UZM.

2.2. Testing the code

Before we describe our method of tube wave generation and present the results, we want to check our code by trying to reproduce analytical results recently obtained by Choudhuri et al. (1993a). As already mentioned in the Introduction, these authors have studied the response of a thin vertical flux tube to rapid footpoint motions imposed at the photospheric level. They have assumed that the footpoint is shaken by a sudden event with a velocity which varies exponentially with time. Choudhuri et al. (their Eq. 8) take

$$v_x = v_o \exp(-bt^2) , \tag{17}$$

where v_x is the shaking velocity, v_o is the maximum velocity and $b = 1/t_W^2$, with t_W being the effective duration time of the shaking event. The authors have shown that the total generated energy, E_{tot}, in the tube can be calculated analytically. The result (see their Eq. 28) is

$$E_{tot}(\tau) = \frac{4\rho_o A_o v_o^2 H}{\lambda^2} F(\lambda, \tau) , \tag{18}$$

where ρ_o and A_o are the density and the cross-sectional area of the tube at the bottom, respectively, H is the density scale height and $\lambda = v_o/(\omega_c L)$ is the dimensionless velocity amplitude, with $L = v_o\sqrt{\pi/b}$ being the total displacement of the tube. Here

$$\omega_c^2 = \frac{g}{8H}\frac{1}{2\beta + 1} \quad , \tag{19}$$

is the cutoff frequency (often called Spruit frequency) of the kink tube mode and $\beta = 8\pi p/B^2$ is a constant. $\tau = \omega_c t$ is a dimensionless time measured in units of the cutoff period. The explicit expression for $F(\lambda, \tau)$, which repesents the total (dimensionless) amount of kinetic energy put into the tube, is given by Eq. (29) in Choudhuri et al. (1993a) and is plotted versus τ for different values of λ in their Fig. 4. The results presented in this figure show that for large values of τ the amount of wave energy generated in the tube approaches an asymptotic value which is taken by the authors to be the total kinetic energy transported to the corona due to the footpoint motion event. These analytical results can be directly used to test our code.

To compare our numerical results with the analytical values of Choudhuri et al. (1993a), we have computed E_{tot} as a function of τ for different values of λ and plotted $F(\lambda, \tau)$ in Fig. 1. The results shown in this figure should be directly compared with those given in Fig. 4 of Choudhuri et al. (1993a). The comparison clearly shows that the shape of the curves and the

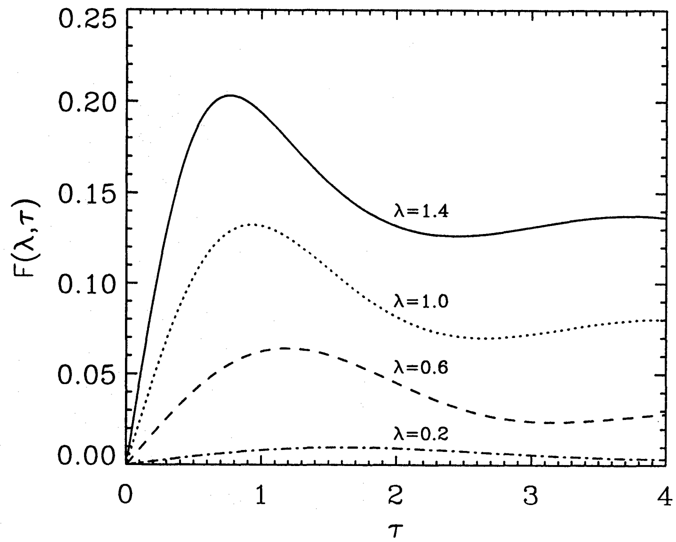

Fig. 1. The function $F(\lambda, \tau)$, which represents the total dimensionless kinetic energy of the tube, is plotted versus dimensionless time τ for different values of the maximum dimensionless shaking velocity amplitude λ. The results are obtained for $v_o = 1.0 \times 10^4\ cm/s$ and should be compared to those of Fig. 4 of Choudhuri et al. (1993a)

asymptotic behavior of $F(\lambda, \tau)$ for large values of τ is very similar. The difference is only in absolute values of $F(\lambda, \tau)$, namely, the numerically calculated $F(\lambda, \tau)$ is approximately two times lower than that found analytically. However, this discrepancy can easily be explained by the fact that the τ-interval $(0, +\infty)$ for the numerical computation is by a factor of two shorter than that used $(-\infty, +\infty)$ in the analytical derivation. The favourable comparison between the numerical and analytical results thus give us confidence that our numerical code is working properly and that it can be used to study the generation of transverse tube waves.

2.3. Shaking velocities and the tube model

To calculate the transverse wave energy fluxes generated in a magnetic flux tube we must prescribe the shaking of the tube. For this we picture the tube as a thin vertically oriented highly concentrated magnetic field region which sits in the non-magnetic turbulent flow field of the solar convection zone that in a random fashion displaces the tube horizontally. The main difference between our approach presented in this paper and those previously considered (see Sect. 1 for references) is that we are allowing for the excitation of transverse tube waves in a very nonlinear way. The large horizontal motions of magnetic foot points observed by Muller (1989) and Muller et al. (1994) clearly indicate that the horizontal displacements of the tube are much larger than the tube diameter. This has been suggested long ago (e.g. Parker 1981, Fig. 2) and is also a result of numerical convection zone simulations. The time-dependent numerical convection zone calculations by Cattaneo et al. (1991), Nordlund & Dravins (1990) and Steffen (1993) all show highly concentrated cyclonic type downflow regions in the center of which the magnetic flux tubes are assumed to reside.

The model presented in this paper is fairly simple but even so it does have all necessary components to make our results trustworthy. We assume that the excitation of the flux tube occurs at a single specific height along the length of the tube. By varying this excitation height in a series of computational runs, we hope to get a rough estimate of the effect of a more realistic excitation of the tube along its entire length. Exciting the tube at the given height with a typical flow velocity amplitude derived from the turbulent spectrum of the convection zone, we also get a crude estimate of the effect of a nonlinear excitation. However, the model considered here does not account for the correlation effects which result from the fact that the flux tube is a connected structure and its elements cannot move independently; we discuss this problem in some detail in Sect. 3.

For the magnetic tube model we assume that in the solar atmosphere described by model C of Vernazza et al. (1981) we have a vertically oriented flux tube which at optical depth $\tau_{5000} = 1$ at 5000 Å has a radius of 50 km and a field strength of $B_0 = 1500\ G$. To show the dependence on the field strength we also consider tubes with $B_0 = 1000$ and 1250 G. The tubes are assumed to spread exponentially with height in accordance to horizontal pressure balance and magnetic flux conservation. As the maximum of the convective velocities both in the mixing-length models and in numerical convection zone models occur considerably deeper than $\tau_{5000} = 1$, we select excitation heights at $\tau_{5000} = 1$, 10 and 100 in optical depth measured outside the tube. Because the Vernazza et al. model extends only to $\tau_{5000} \approx 10$, we extended that model for a few points by fitting a convection zone model of Bohn (1981, 1984). There is also a fourth excitation point at $\tau_{5000} = 0.1$. This is because, different to the mixing-length models, which let the convective velocities decrease to zero near $\tau_{5000} = 1$, observations and numerical convection zone calculations show that considerable overshooting occurs into the convectively stable photosphere.

We now discuss the magnitude of the shaking velocity. To reproduce the fluctuations of the shaking velocity of the turbulent flow which result from the turbulent energy spectrum of the convection zone we employ a procedure discussed in the next subsection. Here we only discuss the maximum rms convective flow velocities u_t which we assume to occur at the four chosen excitation heights. High resolution white light observations by Muller (1989) show that tiny bright points in the dark intergranular lanes, which have been interpreted as foot points of magnetic flux tubes, move with velocities as large as 2 km/s. More recent observations of the proper motion of network bright points show that 50% of these points move faster than 1 km/s, 25% faster than 1.5 km/s, 15% faster than 2.5 km/s and only a few percent faster than 3 km/s (Muller et al. 1994).

In a comparison with the mixing-length theory, Steffen (1993) in his time-dependent two-dimensional numerical convection zone simulations found that maximum rms convective velocities of 2.5 km/s were reached; these results are similar to those obtained by using the mixing-length theory with a mixing-length parameter $\alpha = 1.7$. This maximum occurs much deeper (at $\tau_{5000} \approx 100$) than the velocity maximum in the mixing-length model which occurs near $\tau_{5000} = 10$. In addition, while the mixing-length velocities decrease to very low values near $\tau_{5000} = 1$, the numerical simulations have 2.2 km/s at $\tau_{5000} = 10$, 1.7 km/s at $\tau_{5000} = 1$ and 1.6 km/s at $\tau_{5000} = 0.1$. Similar values have been earlier found by Cattaneo et al. (1991) and Nordlund & Dravins (1990).

The observational and numerical results discussed above clearly show that the rms velocities $u_t = 1.0$ to 2.0 km/s are reasonable shaking amplitudes at the surface of the Sun. However, to adopt the same velocity u_t at all four shaking heights would not be appropriate as the convection zone calculations show a considerable height dependence of u_t. On the other hand it is also not a good idea to use the height dependent velocities u_t from the convection zone computations. This latter usage would underestimate the total wave flux which comes from other layers than the shaking height. We thus make a compromise and scale the value of u_t at the other heights relative to the value of u_t at $\tau_{5000} = 10$ by using $\rho u_t^3 = const$ in accordance to the behaviour in an efficient convection zone. Values of the rms shaking u_t found in such a way are given in Table 1.

2.4. The shaking spectrum

At a given height we shake the tube horizontally with a flow which derives from a turbulent spectrum appropriate for the convection zone at that height. As we are only interested in those tube waves which propagate towards the solar surface, we take the shaking height to be the bottom of our flux tube model. For the shaking velocity we assume a spectrum of $N = 20$ partial waves

$$v_x = \sum_{n=1}^{N} u_n \sin(\omega_n t + \varphi_n) , \tag{20}$$

where $\varphi_n = 2\pi r_n$ is an arbitrary but constant phase angle, r_n a random number in the interval [0, 1] and u_n is determined by the turbulent energy spectrum as follows. Using Eq. (20) we have for the time average

$$\overline{v_x^2} = \frac{1}{T}\int_0^T \left[\sum_{n=1}^{N} u_n\big(\sin(\omega_n t)\cos\varphi_n + \cos(\omega_n t)\sin\varphi_n\big)\right]^2 dt, \tag{21}$$

where in the square bracket there will be terms like $\sin(\omega_n t)$ $\sin(\omega_l t)\cos\varphi_n\cos\varphi_l$. Using the orthogonality relation

$$\frac{1}{T}\int_0^T \sin(\omega_n t)\sin(\omega_l t)\cos\varphi_n\cos\varphi_l dt = \delta_{nl}\cos\varphi_n^2 , \tag{22}$$

and a similar one where cos and sin are exchanged, noting that mixed expressions drop out upon integration, we get

$$\overline{v_x^2} = \frac{1}{T}\int_0^T \sum_{n=1}^{N} u_n^2\big(\sin^2(\omega_n t)\cos^2\varphi_n + \cos^2(\omega_n t)\sin^2\varphi_n\big)dt$$

$$= \frac{1}{T}\sum_{n=1}^{N} u_n^2 (\cos^2\varphi_n \int_0^T \sin^2(\omega_n t)dt + \sin^2\varphi_n \int_0^T \cos^2(\omega_n t)dt)$$

$$= \frac{1}{2}\sum_{n=1}^{N} u_n^2 (\cos^2\varphi_n + \sin^2\varphi_n)$$

$$= \frac{1}{2}\sum_{n=1}^{N} u_n^2 \quad . \tag{23}$$

The turbulent energy spectrum is normalized to

$$\frac{3}{2}u_t^2 = \int_0^\infty d\omega \int_0^\infty dk\, E(k)\Delta\left(\frac{\omega}{ku_k}\right) = \int_0^\infty E'(\omega)d\omega \,. \tag{24}$$

We assume now that the flux tube is shaken only in one horizontal direction. The additional shaking in the other horizontal direction would give us a factor of roughly two if the correlation effects were again neglected. We thus write

$$\frac{3}{2}u_t^2 = \frac{3}{2}\sum_{n=1}^{N} u_n^2 = \int_0^\infty E'(\omega)d\omega = \sum_{n=1}^{N} E'(\omega_n)\Delta\omega \,, \tag{25}$$

from which we have

$$u_n = \sqrt{\frac{2}{3}E'(\omega_n)\Delta\omega} \,, \tag{26}$$

with

$$E'(\omega_n) = \int_0^\infty E(k)\Delta\left(\frac{\omega_n}{ku_k}\right) dk \quad . \tag{27}$$

For the turbulent energy spectra appropriate for the solar convection zone we follow Musielak et al. (1994c). These authors show that the turbulent energy spectra can very likely be described using an extended Kolmogorov spectrum $E(k)$ and a modified Gaussian frequency factor $\Delta(\frac{\omega}{ku_k})$. The extended Kolmogorov spatial component can be written as

$$E(k) = \begin{cases} 0 & 0 < k < 0.2k_t \\ a\frac{u_t^2}{k_t}\left(\frac{k}{k_t}\right) & 0.2k_t \le k < k_t \\ a\frac{u_t^2}{k_t}\left(\frac{k}{k_t}\right)^{-5/3} & k_t \le k \le k_d \end{cases} \tag{28}$$

where the factor $a = 0.758$ is determined by the normalization condition

$$\int_0^\infty E(k)dk = \frac{3}{2}u_t^2 \,. \tag{29}$$

The modified Gaussian frequency factor is given by

$$\Delta\left(\frac{\omega}{ku_k}\right) = \frac{4}{\sqrt{\pi}}\frac{\omega^2}{|ku_k|^3}e^{-(\frac{\omega}{ku_k})^2} \tag{30}$$

where u_k is computed from

$$u_k = \left[\int_k^{2k} E(k')dk'\right]^{1/2} \quad . \tag{31}$$

Here u_t is the prescribed rms shaking velocity discussed in the last subsection and $k_t = 2\pi/H$, with H being the density scale height.

2.5. *Transverse wave energy fluxes*

The flux of transverse waves at height z can be computed (e.g., Hollweg 1978) by using

$$F(z,t) = -\frac{B}{4\pi}b_x v_x \,, \tag{32}$$

where B is the background magnetic field strength, b_x the perturbation of this field, and v_x the transverse velocity perturbation. Since we want to compare wave energy fluxes generated at different heights in the tube it is appropriate to normalize the fluxes to the solar surface, that is, to the height for which outside the tube we have $\tau_{5000} = 1$. At this height the tube radius has a cross-section A_0, while at other heights it has the cross-section A. Upon averaging the normalized flux can then be written as

$$\overline{F} = -\overline{\frac{A}{A_0}\frac{B}{4\pi}b_x v_x} \,, \tag{33}$$

where the bar indicates time averaging. The wave energy spectrum is calculated by taking the Fourier transform of F(z,t). Due to the nonlinear wave coupling, this spectrum will change with height, but this effect will not be investigated in this paper.

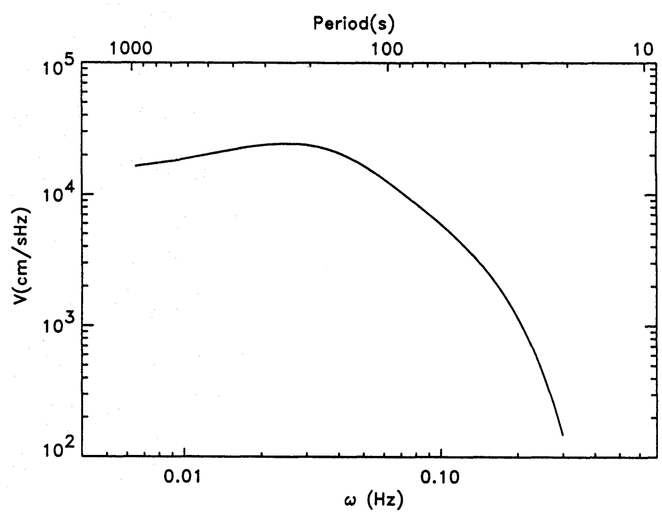

Fig. 2. Input spectrum of the fluctuating horizontal shaking velocity v_x computed from Eq. (20) for a prescribed rms shaking velocity of $u_t = 1\ km/s$. The spectrum is shown as function of circular frequency ω and period

Table 1. Transverse wave energy fluxes $\overline{F}$ (erg/cm^2s) generated in a tube of magnetic field strength B_0 (G), shaken at optical depth τ, by an rms convective velocity u_t (km/s). The area factor A/A_0 is the ratio of the tube cross-section at depth τ to that at τ = 1

case	B_0	τ	A/A_0	u_t	$\overline{F}$
1	1500	0.1	1.7	1.4	$1.4 \cdot 10^9$
2	1500	1	1.0	1.1	$1.1 \cdot 10^9$
3	1500	10	.81	1.0	$1.2 \cdot 10^9$
4	1500	100	.68	.94	$7.3 \cdot 10^8$
5	1000	10	.81	1.0	$1.2 \cdot 10^9$
6	1250	10	.81	1.0	$1.4 \cdot 10^9$
7	1500	10	.81	1.5	$3.4 \cdot 10^9$
8	1500	10	.81	2.0	$5.2 \cdot 10^9$
9	1500	0.1	1.7	1.0	$5.9 \cdot 10^8$
10	1500	1	1.0	1.0	$1.0 \cdot 10^9$
11	1500	100	.81	1.0	$8.0 \cdot 10^8$

3. Results and discussion

For specified rms shaking velocities in the range of u_t = 1 to 2 km/s at the four different optical depths (τ_{5000} = 0.1, 1, 10, 100) we have computed time-averaged transverse wave energy fluxes $\overline{F}$ in magnetic flux tubes with field strengths B_0 = 1000, 1250 and 1500 G where B_0 is specified at τ_{5000} = 1. The tube was excited by a fluctuating horizontal flow field with velocity v_x given by Eq. (20) where the u_n are computed from Eq. (26). As illustration, Fig. 2 shows a typical (smoothed) spectrum of the fluctuating input velocity v_x for a tube with B_0 = 1500 G and an rms shaking velocity u_t = 1 km/s at τ_{5000} = 10. This spectrum was obtained by temporal Fourier analysis of the first 2000 s of the velocity $v_x(t)$ at the shaking point. The velocity was sampled every sec. The lower limit of the spectrum is due to the total sampling time.

The resulting normalized wave energy fluxes are shown in Table 1. The results presented in Table 1 clearly demonstrate (cases 3, 7, 8) that the fluxes depend on the magnitude of the shaking velocity roughly as

$$\overline{F} \approx 9.8 \cdot 10^{-4} u_t^{2.44} \quad , \tag{34}$$

with a much weaker dependence on the magnetic field strength B_0 in the sense that with larger B_0 one gets larger $\overline{F}$ (cases 3, 5, 6). It is also seen that the wave fluxes do not depend strongly on the height of the shaking point (cases 1 to 4) up to τ = 10 and decrease markedly for larger τ. The rms shaking velocities in cases 1 to 4 have been scaled using $\rho u_t^3 = const$ from the value of u_t = 1 km/s at τ = 10. But even if the rms velocity u_t is the same at all four shaking heights (cases 9, 10, 3, 11), this wave flux behavior remains essentially the same, $\overline{F}$ increases with depth up to a maximum at τ = 10 and decreases with larger shaking depth. In addition, Table 1 shows that for typically expected shaking velocities, the normalized transverse tube wave energy fluxes $\overline{F}$ are in the range of a few times 10^9 $erg\, cm^{-2} s^{-1}$.

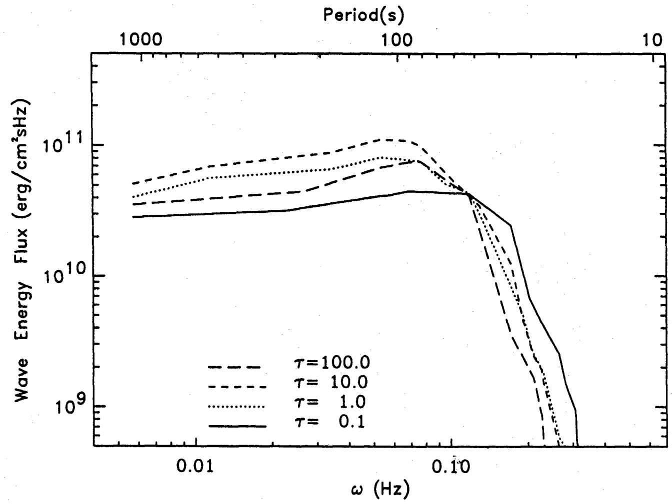

Fig. 3. Transverse tube wave energy spectra for excitation at different shaking heights τ = 0.1, 1, 10, 100 for a magnetic flux tube of field strength B = 1500 G (cases 1-4 in Table 1). The spectra are shown as function of circular frequency ω and period

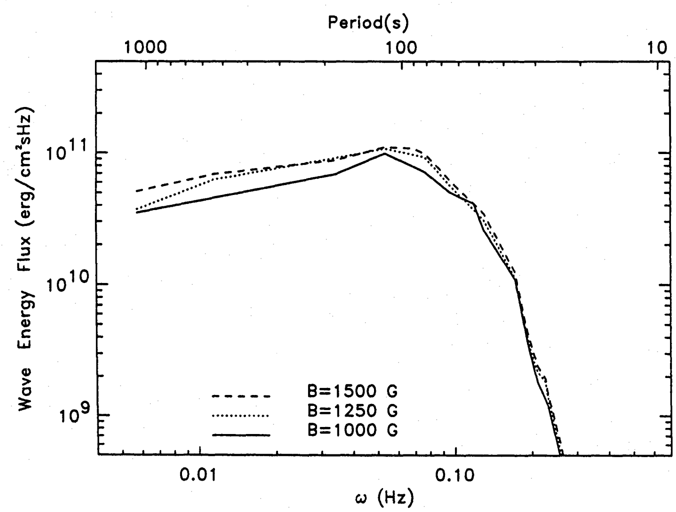

Fig. 4. Transverse tube wave energy spectra for excitation at shaking height τ = 10 for a flux tubes of different field strength B = 1000, 1250, 1500 G (cases 5,6,3 in Table 1). The spectra are shown as function of circular frequency ω and period

The spectra of the transverse wave energy flux for the tube B_0 = 1500 G and four different velocities at the different shaking heights (cases 1 to 4 of Table 1) are shown in Fig. 3. These spectra were obtained in a similar way as that of v_x by Fourier analyzing the wave energy fluxes at the respective shaking heights. It is seen that the wave energy spectra do not differ much with shaking height, in accordance to the frequency integrated total fluxes in Table 1, although there is a tendency that shaking at greater height produces a higher frequency spectrum.

Finally, Fig. 4 shows the dependence of the wave energy spectrum on the flux tube model for the excitation at a common depth τ_{5000} = 10. It is seen that the spectra in tubes with B_0 = 1000 to 1500 G are almost identical.

All our results presented in Figs. 3 and 4 are obtained for just one form of the input spectrum given in Fig. 2. To see whether the calculated wave energy spectra are sensitive to a different form of the input spectrum, we have considered an input spectrum which has a maximum at high frequencies and reaches zero at low frequencies; our new input spectrum is simply obtained by reversing the input spectrum presented in Fig. 2. For this new input spectrum, we have found an increase in the wave energy flux by approximately 20%, which is consistent with the results obtained analytically by Choudhuri et al. (1993a).

After presenting and discussing our results, we now compared them with those previously obtained. Musielak et al. (1994b) have found that typical energy fluxes carried by transverse tube waves are of the order of 10^8 erg/cm^2s, which is roughly one order of smaller than the fluxes obtained here. The result is not surprising because it is expected that the nonlinear effects taken into account in the present calculations should lead to a higher efficiency of transverse wave excitation. However, some caution is required when the comparison between the previous analytical and the present numerical results is made. Namely, an important caveat regarding the results presented in this paper is that the shaking of the flux tube takes place only at one local height point and that it does not include the correlation effects which occur when the tube is shaken over its entire length. It is clear that the flux tube is a connected structure which does not allow tube elements on adjacent heights to behave fully independently and, as a result, the wave generation in the tube will be considerably influenced by such correlation effects; the latter may lead to a decrease or an increase in the generated wave energy fluxes. In contrast to our numerical approach presented in the present paper, the analytical calculations described by Musielak et al. (1994b) include some correlation effects because the tube is shaken along a significant portion of its length.

Our computed wave fluxes are also in some discepancy with those obtained by Choudhuri et al. (1993a, 1993b) who found that the transverse wave generation by slow excitation does not seem sufficient to heat the quiet solar corona. We attribute this discrepancy primarily to the different turbulent velocity amplitudes used in the shaking of the tube.

Our wave energy fluxes can also be compared with those estimated by Muller et al. (1994). The approach presented by these authors is very simple, its advantage is that the estimated fluxes are directly related to the velocities observed on the solar surface. The authors do not dwell upon the process of interaction of magnetic flux tubes with the external motions but instead measure the proper motion of network bright points and then use the velocity of the observed motion to estimate the transverse wave energy flux. This gives the flux of the order of 10^{10} erg/cm^2s, which is roughly one order of magnitude higher than the flux obtained here. The difference is probably caused by a number of assumptions underlying the above estimate (for example, the group velocity of transverse tube waves is assumed to be the same as the Alfvén velocity, which is actually not the case) and by the fact the horizontal motions observed in the velocity histograms are identified with the wave transverse motions; this does not have to be true if the interaction between the tube and the observed motions is properly taken into account.

Finally, it must be noted that the discussed wave energy fluxes are obtained for a single magnetic flux tube and, therefore, to estimate the total amount of the wave energy flux available for heating the filling factor (representing a portion of the solar surface covered by flux tubes) has to be taken into account. It is presently well-known that the filling factor varies over the solar cycle and that its value ranges from approximately 10^{-3} to 10^{-2} (e.g., Title et al. 1992; Muller et al. 1994). Applying the latter value to our wave energy fluxes given in Table 1, one sees that the generated wave energy fluxes are high enough to sustain the mean level of heating required in the solar chromospheric network; however, this is not true when the smaller value of the filling factor is used. An interesting result is that for both values of the filling factor the wave energy fluxes obtained by Muller et al. (1994) can be important in the chromospheric and coronal heating if the wave energy can be effectively transferred to these upper layers of the solar atmosphere.

4. Conclusions

From our studies of nonlinear generation of magnetic transverse waves in solar magnetic flux tubes excited by shaking at various heights, we find that

1. The wave energy fluxes $\overline{F}$ increase for a fixed shaking velocity when the optical depth of the shaking position increases. The fluxes $\overline{F}$ reach a maximum near depth $\tau = 10$ and decrease with greater shaking depth.
2. The wave energy fluxes for depths $\tau < 10$ are roughly equal if the shaking velocities u_t are scaled by $\rho u_t^3 = const$ with the density ρ, but decrease for larger depth τ.
3. There is a slight increase of the wave energy flux with increasing magnetic field strength.
4. Typical transverse wave energy fluxes of the order of a few times $10^9 erg/cm^2s$ are found. As a function of the shaking velocity these fluxes can be roughly estimated using Eq. (26). The magnitude of these fluxes strengthens the belief that transverse waves might be candidates for the heating of the chromosphere and corona in magnetic structures.
5. The obtained wave energy fluxes are roughly one order of magnitude higher than those found by using analytical methods (Musielak et al. 1994b), and approximately one order of magnitude lower than the fluxes estimated from observations of the proper motion of network bright points (Muller et al. 1994). Our fluxes are also considerably higher than those computed by Choudhuri et al. (1993a, 1993b).

As a note of warning it must be emphasized that the computation of the wave energy flux in this study is highly idealized and simplified. Reflection and wave energy lost through nonlinear wave coupling or by atmospheric damping is not taken into account. Also this study describes the shaking only at a local height point and does not include the correlation effects which occur when the tube is shaken over its entire length. It also does not take into account additional heating by occasional sudden disturbances which have been estimated to contribute significantly to the coronal heating (Choudhuri et al. 1993a, 1993b).

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