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Solar and Heliospheric Plasma Physics

Proceedings of the
8th European Meeting on Solar Physics
Held at Halkidiki, Greece,
13–18 May 1996

Springer

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Cataloging-in-Publication Data applied for.
Die Deutsche Bibliothek - CIP-Einheitsaufnahme

Solar and heliospheric plasma physics : proceedings of the 8th European Meeting on Solar Physics, held at Halkidiki, near Thessaloniki, Greece, 13 - 18 May 1996 / G. M. Simnett ... (ed.). - Berlin ; Heidelberg ; New York ; Barcelona ; Budapest ; Hong Kong ; London ; Milan ; Paris ; Santa Clara ; Singapore Tokyo : Springer, 1997
(Lecture notes in physics ; Vol. 489)
ISBN 3-540-63072-4

ISSN 0075-8450
ISBN 3-540-63072-4 Springer-Verlag Berlin Heidelberg New York

Typesetting: Camera-ready by the authors/editors
Cover design: *design & production* GmbH, Heidelberg
SPIN: 10550811 55/3144-543210 - Printed on acid-free paper

Dynamics of Flux Tubes in the Solar Atmosphere: Theory

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Abstract. The modes of oscillation of a photospheric magnetic flux tube are reviewed, taking into account both linear and nonlinear aspects. Analytical and computational developments are discussed, beginning with the basic characteristics of linear wave propagation and progressing to a consideration of nonlinearity and the question of the generation of tube waves and the energy flux they transport.

1 Introduction

The concept of a magnetic flux tube, which goes back to Michael Faraday, has proved to be particularly fruitful for solar studies. In the Sun's photosphere observations show that isolated small-scale magnetic flux tubes occur in the downdraught lanes between supergranules; these tubes are magnetically strong, with fields of about 2 kG confined to radii of about 100 km (see the reviews by Stenflo 1989, 1994; Solanki 1990, 1993, 1997; Schüssler 1991). Larger tube-like concentrations occur in the form of pores and sunspots, though the detailed internal structure of a sunspot is presently uncertain: a sunspot may be described as a uniform plug of magnetic field (a monolith) or alternatively as a conglomeration of individual small-scale magnetic tubes, assembled to form a sunspot, that retain their separate identities in the layers below the visible surface (see the reviews in Thomas & Weiss 1992). Observations of the absorption of p-modes in sunspots are likely to shed light on this question in due course (see the review by Bogdan 1992).

There are no *isolated* flux tubes in the corona, which is completely filled with magnetic field. But flux tubes still exist, corresponding to magnetised plasma regions that are delineated from their surroundings, appearing in the form of coronal loops. A coronal flux tube is primarily a region of high plasma density, although temperature and magnetic field differences may also arise.

Magnetic flux tubes are communication channels, linking one part of the Sun's atmosphere or interior with another. Their essential one-dimensionality means that the linkage is likely to be efficient. As such they are important conduits for momentum and energy transport, carrying flows or waves from one site to another. Although there are a number of similarities between photospheric and coronal flux tubes, there is an important distinction: whereas a photospheric tube is a region of enhancement in Alfvén speed, a coronal flux tube is a region of low Alfvén speed. Finally, it is interesting to note that there are a number of similarities in the basic structure of waves in isolated photospheric magnetic tubes and the waves in extragalactic jets (Roberts 1987; Bodo et al. 1989; Hardee 1995).

In this review we concentrate on the photospheric flux tube. Waves in magnetic flux tubes have also been reviewed in Roberts (1980, 1981, 1985a, 1986, 1990a, b, 1991a, b, 1992a, b), Spruit (1981a, b), Spruit & Roberts (1983), Thomas (1985, 1990), Hollweg (1986, 1990a), Edwin & Roberts (1987), Ryutova (1990a, b), and Edwin (1991, 1992).

2 Basic Modes of Oscillations

The basic modes of oscillation of an isolated magnetic flux tube are now well understood. Geometrically, there are *sausage* modes, *kink* modes and *fluting* modes. These geometrical forms are defined in terms of the patterns that the boundary of the flux tube makes when it is disturbed (see equation (13) below). If the displaced tube remains a circle centred on the axis of symmetry of the undisturbed tube, then this is the sausage mode; if the displaced tube remains circular, but is no longer about the axis of symmetry, then this is the kink mode. The various higher order distortions of the tube boundary, ranging from elliptical to highly castellated, are the fluting modes. The modes may be classified further according to their nature in the radial coordinate r: disturbances that inside the tube are *oscillatory* in r are referred to as *body* waves, and those that are *exponential* (or evanescent) in form are the *surface* waves. Basically, in a wide tube the surface waves are confined to near the boundary of the tube and do not penetrate far into the centre of the tube, whereas body modes disturb the centre of a wide tube; however, in a thin tube, the centre is disturbed for both modes. In addition to the modes that are trapped within a tube, disturbing the tube's environment only slightly, there arise *leaky* waves. Leaky waves are generated by motions within the tube that result in an outflow of wave energy: the tube is a generator for waves in its environment. These classifications may be applied to both fast and slow magnetoacoustic waves, so the description becomes complicated. Additionally, a tube may support torsional Alfvén waves.

The characteristic speeds that govern wave propagation in a magnetic flux tube are readily established. Consider a plasma of density ρ_0 and pressure p_0 within which is embedded a magnetic field $\mathbf{B_0}$ of strength B_0. The sound speed c_S and Alfvén speed c_A are defined by

$$c_S = \left(\frac{\gamma p_0}{\rho_0}\right)^{1/2}, \quad c_A = \left(\frac{{B_0}^2}{\mu_0 \rho_0}\right)^{1/2}, \tag{1}$$

where μ_0 is the magnetic permeability and γ the adiabatic index of the gas (taken to be 5/3). From these two speeds we may construct the *fast* magnetoacoustic speed, c_f, and the *slow* (or tube) magnetoacoustic speed, c_T:

$$c_f^2 = c_S^2 + c_A^2, \quad c_T^{-2} = c_S^{-2} + c_A^{-2}. \tag{2}$$

The fast speed c_f is super-sonic and super-Alfvénic, and the slow speed c_T is sub-sonic and sub-Alfvénic; the slow speed is particularly significant for waves in a thin magnetic flux tube (Defouw 1976; Roberts & Webb 1978). We may illustrate these speeds as follows. In a photospheric flux tube with magnetic field strength $B_0 = 2$ kG and plasma density $\rho_0 = 2.2 \times 10^{-4}$ kg m^{-3}, the Alfvén speed is $c_A = 12$ km s^{-1}; for a sound speed of $c_S = 8$ km s^{-1} we obtain $c_T = 6.7$ km s^{-1} and $c_f = 14.4$ km s^{-1}.

A speed equivalent to c_T is in fact common to a variety of elastic tubes, with the role of the Alfvén speed being played by the appropriate elastic speed of the physical situation. For example, in the case of a blood vessel the speed equivalent to c_A is the elastic speed in the membrane of the blood vessel. In this case, the speed of sound c_S in blood is much larger than the elastic speed, so effectively $c_S \gg c_A$ giving $c_T \approx c_A$; wave propagation in a blood vessel proceeds with a speed that is close to the elastic speed of the membrane walls. For water in a pipe, the relative magnitudes of the two basic speeds depends upon the material of the pipe. In a metal pipe, the elastic speed of the metal membrane is much larger than the speed of sound in water, and so the effective propagation speed is close to the sound speed in water (about 1.4 km s^{-1}). In a plastic pipe, the orderings in the two speeds are reversed and the effective propagation speed is close to the elastic speed in plastic (about 10 m s^{-1}), lying far below the speed of sound in water.

The kink mode disturbs both the tube and its environment, and so the characteristic speed for this wave involves both the density ρ_0 of the gas inside the tube and the density ρ_e of the gas in the environment. The kink mode is principally a result of the magnetic tension force, B_0^2/μ_0, in the magnetic field and so its speed of propagation, c_k, is given by (Ryutov & Ryutova 1976; Parker 1979; Spruit 1982)

$$c_k^2 = \frac{B_0^2/\mu_0}{\rho_0 + \rho_e} = \left(\frac{\rho_0}{\rho_0 + \rho_e}\right) c_A^2. \tag{3}$$

This speed too is sub-Alfvénic; for $\rho_e = 2\rho_0$ (consistent with a sound speed in the tube's environment of 9.6 km s^{-1}) we obtain $c_k = 6.9$ km s^{-1}, roughly 60% of the Alfvén speed.

To progress further it is necessary to consider the linear equations of ideal magnetohydrodynamics in some detail. Consider an equilibrium magnetic field $\mathbf{B_0} = B_0(r)\hat{\mathbf{z}}$ aligned with the z-axis of a cylindrical polar coordinate system r, θ, z. To begin with we will ignore the effects of gravity (see Section 4). Then the equilibrium gas pressure $p_0(r)$ and density $\rho_0(r)$ are structured by the magnetic field so as to maintain total pressure balance: the sum of the gas pressure and the magnetic pressure is a constant,

$$\frac{d}{dr}\left(p_0(r) + \frac{B_0^2(r)}{2\mu_0}\right) = 0. \tag{4}$$

Small amplitude motions $\mathbf{v}$ about the equilibrium (4) satisfy the wave equation

$$\frac{\partial^2 \mathbf{v}}{\partial t^2} - c_A^2 \frac{\partial^2 \mathbf{v}}{\partial z^2} = -c_A^2 \hat{\mathbf{z}} \frac{\partial}{\partial z} \mathrm{div}\, \mathbf{v} - \frac{1}{\rho_0} \mathrm{grad} \left(\frac{\partial p_T}{\partial t} \right). \tag{5}$$

Equation (5) follows from the time derivative of the momentum equation combined with the magnetohydrodynamic induction equation for an ideal medium. The total pressure, $p_T(r, \theta, z)$, the sum of the gas pressure perturbation p and the perturbation in the magnetic pressure, $B_0 B_z/\mu_0$, satisfies the evolution equation

$$\frac{\partial p_T}{\partial t} = \rho_0 c_A^2 \frac{\partial v_z}{\partial z} - \rho_0 (c_S^2 + c_A^2) \mathrm{div}\, \mathbf{v}. \tag{6}$$

Equation (6) is a combination of the isentropic and induction equations.

For a flow $\mathbf{v}(r, \theta, z) = (v_r, v_\theta, v_z)$, the components of equation (5) give (Roberts 1986, 1992a)

$$\rho_0(r) \left(\frac{\partial^2}{\partial t^2} - c_A^2(r) \frac{\partial^2}{\partial z^2} \right) v_r + \frac{\partial^2 p_T}{\partial r \partial t} = 0, \tag{7}$$

$$\rho_0(r) \left(\frac{\partial^2}{\partial t^2} - c_A^2(r) \frac{\partial^2}{\partial z^2} \right) v_\theta + \frac{1}{r} \frac{\partial^2 p_T}{\partial \theta \partial t} = 0, \tag{8}$$

and

$$\left(\frac{\partial^2}{\partial t^2} - c_T^2(r) \frac{\partial^2}{\partial z^2} \right) v_z = - \left(\frac{c_S^2(r)}{c_f^2(r)} \right) \frac{1}{\rho_0(r)} \frac{\partial^2 p_T}{\partial z \partial t}, \tag{9}$$

with the evolution in $p_T(r, \theta, z)$ described by equation (6).

It is evident from the form of (7)–(9) that an inhomogeneous medium supports the phenomena of *phase-mixing* (Barston 1964; Heyvaerts & Priest 1983) and *resonant absorption* (Chen & Hasegawa 1974; Ionson 1978). Phase-mixing describes the process by which wave fronts become increasingly more corrugated as they propagate, in response to the fact that the Alfvén speed is different on different field lines. Resonant absorption occurs whenever the phase speed of an Alfvén wave matches the local Alfvén speed c_A or the phase speed of a slow wave matches the local slow speed c_T. Both processes produce small spatial scales *across* the magnetic field, the direction in which neither the Alfvén wave nor the slow wave is able to propagate in a uniform medium. Since both processes depend upon nonuniformity in the equilibrium state, they are are likely to occur preferentially where inhomogeneities are most pronounced, such as on the boundaries of isolated flux tubes.

Phase-mixing is evident when $\partial/\partial\theta = 0$, so that motions are symmetric about the axis of the tube. Then equation (8) decouples from (7)–(9) to give

$$\frac{\partial^2 v_\theta}{\partial t^2} = c_A^2(r) \frac{\partial^2 v_\theta}{\partial z^2}. \tag{10}$$

These are *torsional Alfvén waves* (e.g., Spruit 1982; Hollweg 1988, 1990b, 1991). Symmetric motions $v_\theta(r, z, t)$ take place independently of motions in the r and z-directions. The r-dependence in the torsional Alfvén wave satisfying (10) is arbitrary, being fixed by the means by which the wave is generated.

To illustrate the implications of the wave equation (10) consider the torsional oscillation

$$v_\theta = v_0 \sin[k_z z - k_z c_A(r) t]. \tag{11}$$

Initially (at $t = 0$), this gives a disturbance of amplitude v_0 and wavelength $2\pi/k_z$. According to (11), the initial shape propagates forward with speed c_A, but its radial gradient, given by

$$\frac{\partial v_\theta}{\partial r} = -k_z v_0 \left(\frac{dc_A}{dr} \right) t \cos[k_z z - k_z c_A(r) t], \tag{12}$$

grows secularly on a timescale of $(dc_A/dr)^{-1}$. This is phase-mixing; cross-field gradients are rapidly built-up, making any dissipative processes more efficient (Heyvaerts & Priest 1983; Ireland 1997). The phase-mixing timescale can be surprisingly short: for an isolated tube with c_A changing from about 10 km s^{-1} in the tube to zero outside it, over a distance of 10 km (one-tenth of a tube radius), it is 1 second. This suggests that any such modes, if generated, would rapidly be dissipated by non-adiabatic processes enhanced by phase-mixing (or resonant absorption), and so the edges of isolated flux tubes ought to be excessively hot, producing locally a bright ring around the tube. Much the same effect should operate on the edge of sunspots (Hollweg 1988; Roberts 1992a) or throughout the body of a spot, if spots are strongly inhomogeneous (Ryutova & Persson 1984). However, no such bright ring has been observed, either for thin tubes or for sunspots. Also, on theoretical grounds, we must note that unless there exists an ignorable coordinate (so that the assumption $\partial/\partial\theta = 0$ is valid), then the waves are coupled, through equations (7)–(9), and no simple phase-mixing occurs (Davila 1987, 1991; Parker 1991), though resonant absorption then takes place (eg., Goedbloed 1975, 1983; Rae & Roberts 1981; Lee & Roberts 1986; Poedts et al 1990). Resonant absorption would seem to be particularly important for coronal loops. The topic is reviewed by Hollweg (1990a, b) and Goossens (1991); see also Goossens, Ruderman & Hollweg (1995).

It is convenient to introduce the Fourier form of equations (7)-(8). Write

$$v_r(r, \theta, z, t) = v_r(r) \exp i(\omega t - n\theta - k_z z), \tag{13}$$

with a similar form for p_T. The integer $n (= 0, \pm 1, \pm 2, \cdots)$ describes the geometrical form of the perturbations. The case $n = 0$ gives the *sausage* mode, and corresponds to *symmetric* motions of the tube. These are compressional oscillations ($p_T \neq 0$), in addition to the torsional Alfvén waves. The *kink* mode of the tube is given by setting $n = 1$ (or $n = -1$); in such modes the instantaneous motion of the tube resembles a snake, with the boundary of the tube a displaced circle. Finally, waves with $|n| \geq 2$ are the *fluting* modes of the tube.

With the Fourier form (13) for v_r and other variables, equations (6) - (9) yield ordinary differential equations. It proves convenient to work in terms of the total pressure perturbation, p_T, which satisfies (Edwin & Roberts 1983)

$$\rho_0(r)(k_z^2 c_A^2(r) - \omega^2)\frac{1}{r}\frac{d}{dr}\left\{\frac{1}{\rho_0(r)(k_z^2 c_A^2(r) - \omega^2)} r \frac{dp_T}{dr}\right\} = \left(m^2(r) + \frac{n^2}{r^2}\right) p_T, \tag{14}$$

where

$$m^2(r) = \frac{(k_z^2 c_S^2(r) - \omega^2)(k_z^2 c_A^2(r) - \omega^2)}{(c_S^2(r) + c_A^2(r))(k_z^2 c_T^2(r) - \omega^2)}. \tag{15}$$

The radial velocity component follows from (7):

$$\rho_0(r)(k_z^2 c_A(r) - \omega^2) v_r = -i\omega \frac{dp_T}{dr}. \tag{16}$$

Equation (14) is singular at $\omega^2 = k_z^2 c_A(r)$ and at $\omega^2 = k_z^2 c_T(r)$ (Appert, Gruber & Vaclavik 1974; Goedbloed 1975, 1983); the first singularity is associated with the Alfvénic continuous spectrum and the second with the slow mode continuous spectrum. Both spectra have been investigated in detail for a number of equilibria (see the review by Goossens 1991), especially in connection with coronal heating (see the reviews by Narain & Ulmschneider (1990, 1996) and Browning (1991)).

Now we are interested in the solution of equation (14) appropriate for a magnetic flux tube. Consider, then, a uniform magnetic field $B_0\hat{\mathbf{z}}$ confined to a tube of radius $r = a$, embedded in a field-free environment:

$$B_0(r) = \begin{cases} B_0, & r < a, \\ 0, & r > a. \end{cases} \tag{17}$$

The interface $r = a$ is a current sheet across which conditions change discontinuously, while preserving total pressure balance with the external gas pressure p_e of the environment:

$$p_0 + \frac{B_0^2}{2\mu_0} = p_e. \tag{18}$$

Combined with the ideal gas law, pressure balance implies a connection between the density ρ_0, sound speed c_S and Alfvén speed c_A of the tube and the density ρ_e and sound speed c_{Se} in the environment:

$$\frac{\rho_e}{\rho_0} = \frac{c_S^2 + \frac{1}{2}\gamma c_A^2}{c_{Se}^2}. \tag{19}$$

The temperature structure of an isolated tube in the photosphere is complicated (Schüssler 1990; Solanki 1997), and temperature differences between the interior of a flux tube and its surroundings may arise (and in fact are

required to reproduce observed chromospheric canopy heights (Solanki & Steiner 1990), as suggested by carbon monoxide observations (Ayres et al. 1986; Solanki et al. 1994)). But it is evident from (19) that we may expect $\rho_0 < \rho_e$: the tube is partially evacuated by the magnetic field.

Now the media inside and outside the flux tube are *uniform*, and this affords us some simplification. In a uniform medium, equation (14) for p_T inside the tube reduces to

$$r^2 \frac{d^2 p_T}{dr^2} + r\frac{dp_T}{dr} - (m_0^2 r^2 + n^2) p_T = 0, \tag{20}$$

where m_0^2 is the value of m^2 inside the tube. (We have cancelled a factor $(k_z^2 c_A^2 - \omega^2)$ corresponding to Alfvén waves.) Equation (20) is a form of Bessel's differential equation, with has solutions in form of the modified Bessel functions $I_n(m_0 r)$ and $K_n(m_0 r)$. The solution $K_n(m_0 r)$ is singular at $r = 0$ and so is rejected. Accordingly, we take

$$p_T = A_0 I_n(m_0 r), \qquad r < a, \tag{21}$$

where A_0 is an arbitrary constant.

In the field-free environment of the flux tube, where the sound speed is c_{Se} and the Alfvén speed is zero, solutions $p_T \propto I_n(m_e r)$ and $K_n(m_e r)$ arise; here m_e^2 is the value of $m^2(r)$ outside the tube,

$$m_e^2 = k_z^2 - \frac{\omega^2}{c_{Se}^2}. \tag{22}$$

If now we assume that $m_e^2 > 0$, corresponding to selecting waves that decay (in r) outside the tube, then the solution that is bounded for $r \to \infty$ is

$$p_T(r) = A_e K_n(m_e r), \quad r > a, \tag{23}$$

where A_e is an arbitrary constant.

It remains to match p_T and v_r across $r = a$. The result is the dispersion relation (Roberts & Webb 1978, 1979; Spruit 1982; Edwin & Roberts 1983; Cally 1985, 1986; Evans & Roberts 1990)

$$\frac{1}{\rho_0(k_z^2 c_A^2 - \omega^2)} m_0 \frac{I_n'(m_0 a)}{I_n(m_0 a)} + \frac{1}{\rho_e \omega^2} m_e \frac{K_n'(m_e a)}{K_n(m_e a)} = 0, \tag{24}$$

where a prime (′) denotes the derivative of a modified Bessel function (e.g., $I_n'(m_0 a) \equiv dI_n(x)/dx$ evaluated at $x = m_0 a$, etc.). This is the dispersion relation governing magnetoacoustic waves in an isolated magnetic flux tube.

Equation (24) is valid whatever the nature of m_0, but it is written in a form that is particularly suitable for surface waves ($m_0^2 > 0$). For body waves ($m_0^2 < 0$) an alternative form is more convenient, obtained by writing $n_0^2 = -m_0^2$:

$$\frac{1}{\rho_0(k_z^2 c_A^2 - \omega^2)} n_0 \frac{J_n'(n_0 a)}{J_n(n_0 a)} + \frac{1}{\rho_e \omega^2} m_e \frac{K_n'(m_e a)}{K_n(m_e a)} = 0. \tag{25}$$

Here J_n denotes the Bessel function of order n. Both (24) and (25) are subject to the constraint that $m_e > 0$.

Solutions of the transcendental dispersion relations (24) and (25) are best obtained numerically (Edwin & Roberts 1983; Evans & Roberts 1990) or graphically, or analytically in certain limits. Their structure depends upon the ordering of the various speeds, c_S, c_{Se} and c_A. For an isolated photospheric tube we take $c_S < c_A$ and $c_{Se} < c_A$. Then there are slow body modes (sausage and kink) with phase-speeds ω/k_z that lie between c_T and c_S; these modes may be viewed as waves that are constrained within the tube, bouncing from side to side as they propagate along its interior. There are also *slow surface waves* with phase-speeds that are less than c_T. Finally, there are fast surface waves which have phase-speeds between c_k and c_{Se}.

Having established the basic structure of the linear modes of a flux tube in as simple a circumstance as may be envisaged, namely a uniform tube in the absence of gravity or flow, we turn now to a consideration of some of the complications that add to our picture.

3 Nonlinear Analytical Aspects

The case of most interest for photospheric tubes is the *long wavelength limit*, namely $k_z\mathrm{a} \ll 1$. This restriction corresponds to longitudinal wavelengths $2\pi/k_z \gg 2\pi\mathrm{a}$, which for a tube of radius a = 100 km means wavelengths much greater than 600 km. Granules have dimensions ranging from a few hundred km to 2000 km, with 1000 km being typical, and so tube waves generated by a typical granule just about satisfies this extreme. Supergranules, with scales of 3×10^4 km, certainly give long wavelength modes.

Consider, then, the long wavelength limit of the dispersion relation. For $c_{Se} = c_S$, the slow sausage mode ($n = 0$) gives (Roberts & Webb 1978, 1979; Edwin & Roberts 1983; Roberts 1985b)

$$\omega \sim k_z c_T \left[1 - \frac{1}{4}\left(\frac{\rho_e}{\rho_0}\right)\left(\frac{c_T}{c_A}\right)^4 k_z^2\mathrm{a}^2 K_0\left(\lambda|k_z|\mathrm{a}\right)\right] \tag{26}$$

where $\lambda = c_T/c_A$, and the kink mode ($n = 1$) gives (Edwin & Roberts 1983)

$$\omega \sim k_z c_k \left[1 + \frac{1}{2}\left(\frac{\rho_e}{\rho_e + \rho_0}\right)\hat{\mu}^2 k_z^2\mathrm{a}^2 K_0(\hat{\mu}|k_z|\mathrm{a})\right] \tag{27}$$

where $\hat{\mu} = (c_{Se}^2 - c_k^2)^{1/2}/c_{Se}$.

These approximate formulae for the wave speeds are important in nonlinear theories. Indeed, it has been shown that weakly nonlinear, weakly dispersive slow sausage surface waves have motions $v(z,t)$ along a thin tube which satisfy the nonlinear integrodifferential equation (Roberts 1985b)

$$\frac{\partial v}{\partial t} + c_T\frac{\partial v}{\partial z} + \beta_0 v\frac{\partial v}{\partial z} + \alpha_0\frac{\partial^3}{\partial z^3}\int_{-\infty}^{\infty}\frac{v(s,t)ds}{[\lambda^2\mathrm{a}^2 + (z-s)^2]^{1/2}} = 0. \tag{28}$$

The constant α_0 is directly connected with the dispersive correction in (26), and the coefficient β_0 of the nonlinear term is

$$\beta_0 = \frac{[(\gamma+1)c_A^2 + 3c_S^2]c_A^2}{2(c_S^2 + c_A^2)^2}.$$

Equation (28) is sometimes referred to as the Leibovich–Roberts equation; it has been solved numerically (Weisshaar 1989) and exhibits solutions that are soliton-like in character. However, no analytical solution of equation (28) is known, though a number of its properties have been found (Bogdan & Lerche 1988). The derivation of (28) given by Roberts (1985b) rests upon the thin tube approximation (see below), but an alternative approach, directly from the full set of magnetohydrodynamic equations, yields the same result (Molotovshchikov & Ruderman 1987).

An equation of similar form to (28) arises in a magnetic *slab* (Roberts & Mangeney 1982; Roberts 1985b; Merzljakov & Ruderman 1985):

$$\frac{\partial v}{\partial t} + c_T\frac{\partial v}{\partial z} + \beta_0 v\frac{\partial v}{\partial z} + \alpha_1\frac{\partial^2}{\partial z^2}\int_{-\infty}^{\infty}\frac{v(s,t)}{s-z}ds = 0. \tag{29}$$

This is the Benjamin–Ono equation; it has been studied extensively and has a soliton solution. The constant α_1 is a measure of dispersion.

Many other aspects of tube waves have recently been explored, in attempt to describe analytically the nonlinear behaviour of the rich spectrum of nonlinear waves that an isolated flux tube or slab may support; see Ferriz-Mas (1988), Ruderman (1993), Zhelyazkov et al. (1994) and Nakariakov, Zhugzhda & Ulmschneider (1996).

4 Gravitational Aspects

Stratification, so far ignored in our account, is in fact important in the photosphere. The pressure scale height at the temperature minimum falls to its lowest value, of about 100 km, and this is comparable with the radius of a tube. So stratification effects are important. (In the corona, where the scale height is large, such effects are of less consequence.) The addition of gravity to the description of modes given in Section 2 has not so far proved possible, mainly because the flux tube expands with height in a stratified atmosphere and this seriously complicates the description. Progress has been made by considering either a thin tube or a uniform unbounded field.

The case of a thin tube has been investigated by use of the so called *thin tube equations*. The sausage and kink modes are treated separately. Gravity renders an isolated flux tube non-uniform in height. The fall-off of the confining gas pressure in the environment of the tube forces the tube to expand outwards. Consider the *thin tube equations* for the *sausage* mode:

$$\frac{\partial}{\partial t}\rho A + \frac{\partial}{\partial z}\rho v A = 0, \tag{30}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g, \tag{31}$$

$$\frac{\partial p}{\partial t} + v\frac{\partial p}{\partial z} = \frac{\gamma p}{\rho}\left(\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial z}\right), \tag{32}$$

$$BA = \text{constant}, \tag{33}$$

$$p + \frac{B^2}{2\mu_0} = p_e. \tag{34}$$

In the above thin tube equations, $B(z,t)$ is the field-strength of a thin tube with cross-sectional area $A(z,t)$, $v(z,t)$ is the longitudinal flow speed within the tube, where the gas pressure and density are given by $p(z,t)$ and $\rho(z,t)$. The external gas pressure $p_e(z,t)$ is calculated on the boundary of the tube, with equation (34) embodying a boundary condition namely that the total pressure inside the tube is balanced at all times by the external pressure field p_e, which itself may vary in z and t in response to waves in the tube or to externally imposed motions. The z-axis is aligned with gravity $g\hat{\mathbf{z}}$, pointing downwards. A derivation of equations (30)–(34) has been given by Roberts & Webb (1978), based upon expanding all dependent variables in Taylor series about the central axis ($r = 0$) of the tube, assuming symmetry and no motions in the θ-direction. Special cases of these equations were written down on physical grounds by Parker (1974), for an incompressible fluid, and by Defouw (1976) for an isothermal gas.

In equilibrium ($v = 0, \partial/\partial t = 0$) the thin tube equations yield

$$p_0(z) = p_0(0)e^N, \qquad \rho_0(z) = \rho_0(0)\frac{\Lambda_0(0)}{\Lambda_0(z)}e^N,$$

$$A_0(z) = A_0(0)e^{N/2}, \qquad B_0(z) = B_0(0)e^{N/2}, \tag{35}$$

where

$$N(z) = \int_0^z \frac{dz}{\Lambda_0(z)}$$

is the integrated pressure scale-height $\Lambda_0(z)\,(= p_0(z)/g\rho_0(z))$ inside the tube. For simplicity, we have taken the temperature inside the tube to be equal to that in the environment, assumed to be in hydrostatic equilibrium.

The linear form of the thin tube equations is readily found for the equilibrium (35). Consider first the $g = 0$ case, for which the equilibrium state is a uniform tube ($N = 0$). The linearised form of (30)–(34) with $g = 0$ yields (Roberts 1981):

$$\frac{\partial^2 v}{\partial t^2} - c_T^2\frac{\partial^2 v}{\partial z^2} = -\frac{1}{\rho_0}\frac{c_T^2}{c_A^2}\frac{\partial^2 p_e}{\partial z \partial t}. \tag{36}$$

To progress further requires consideration of $p_e(z,t)$, the external pressure field on the boundary of the oscillating tube. The simplest assumption to make is that the external pressure remains at its equilibrium value, a constant $p_e(z)$, in which case the above equation immediately yields $\omega^2 = k_z^2 c_T^2$, giving

the slow sausage mode as expected. A more refined analysis, though, is to allow for disturbances in the environment of the tube, and then the external gas pressure satisfies the wave equation in r, z and t. By solving this equation for disturbances that decline exponentially as $r \to \infty$ we may recover the approximate dispersion relation (26), showing that *the dispersive correction is due to the tube wave disturbing the environment of the tube* (Roberts & Webb 1979; Roberts 1985b).

Returning to the stratified flux tube ($g \neq 0$), we progress by again assuming that p_e is simply the unperturbed external gas pressure, a function of z but not of time. Then, after some algebra, we obtain the Klein–Gordon equation (Roberts 1981; Rae & Roberts 1982; Roberts 1992b)

$$\frac{\partial^2 Q}{\partial t^2} - c_T^2(z)\frac{\partial^2 Q}{\partial z^2} + \Omega^2(z)Q = 0, \tag{37}$$

where $Q(z,t)$ is related to the flow $v(z,t)$ through

$$Q(z,t) = \left[\frac{\rho_0(z)A_0(z)c_T^2(z)}{\rho_0(0)A_0(0)c_T^2(0)}\right]^{1/2} v(z,t) \tag{38}$$

and Ω^2 is given by

$$\Omega^2(z) = \frac{c_T^2}{4\Lambda_0^2}\left[3\Lambda_0' + \frac{9}{4} - \frac{2}{\gamma} + \frac{4c_S^2}{\gamma c_A^2}\left(\frac{\gamma - 1}{\gamma} + \Lambda_0'\right)\right]. \tag{39}$$

Here a prime denotes differentiation with respect to depth z.

In an *isothermal* atmosphere we obtain some simplification in the above: Λ_0, c_T and Ω^2 are constants, with Ω^2 reducing to (Defouw 1976; Roberts & Webb 1978)

$$\Omega^2 = \Omega_T^2 \equiv \frac{c_T^2}{4\Lambda_0^2}\left[\left(\frac{9}{4} - \frac{2}{\gamma}\right) + \frac{4c_S^2}{\gamma c_A^2}\left(1 - \frac{1}{\gamma}\right)\right]. \tag{40}$$

The presence of constant coefficients in the Klein–Gordon equation (37) applied to an isothermal atmosphere leads to a familiar dispersion relation,

$$\omega^2 = k_z^2 c^2 + \Omega_T^2, \tag{41}$$

where here the propagation speed c is the slow speed c_T. Equation (41) shows that Ω, given by (40), is the *cutoff frequency* for sausage modes in a thin tube. Much the same equation arises in the vertical propagation of sound waves in the absence of a magnetic field, where (41) applies with the c being the sound speed c_S and the cutoff frequency being $c_S/2\Lambda_0$. For the sausage mode, Ω^2 may be viewed as made up of two contributions, the first (corresponding to the first term on the right-handside of (40)) arising from the *geometrical shape* of the tube and the second (corresponding to the second term on the right of (40)) being determined by the tube's *elasticity*. A *rigid tube* ($c_A \gg c_S$), with exponential cross-sectional area determined according to the equilibrium (35) with Λ_0 constant, has cutoff frequency $(9/4 - 2/\gamma)^{1/2} c_S/2\Lambda_0$. A straight and vertical rigid tube has cutoff $c_S/2\Lambda_0$, the same as a vertically propagating sound wave. See also the discussion in Campos (1986).

In general the cutoff frequency for the sausage mode in a tube is less than the cutoff frequency of a rigid tube. In accordance with (41), an impulsively generated sausage wave results in a wave-front propagating with the tube speed c_T, trailing an oscillating wake which rises and falls with the frequency Ω (Rae & Roberts 1982).

The Klein–Gordon equation also describes the *kink* mode (Spruit & Roberts 1983; Roberts 1986). The linearised form of Spruit's (1981) thin tube equations for the kink mode lead to the wave equation

$$\frac{\partial^2 \xi}{\partial t^2} = c_k^2(z)\frac{\partial^2 \xi}{\partial z^2} + g\left(\frac{\rho_0 - \rho_e}{\rho_0 + \rho_e}\right)\frac{\partial \xi}{\partial z} \tag{42}$$

for transverse displacement $\xi(z,t)$. It is easy to cast equation (42) into an equation of the form (37), for suitable Q related to ξ; such a procedure allows us to compare acoustic waves, sausage tube waves and kink tube waves all in the one frame-work (Rae & Roberts 1982; Roberts 1986). With (42) cast in the form of (37), the speed c becomes the kink speed c_k and the square of the cutoff frequency becomes

$$\Omega^2 = \frac{c_k^2}{4\Lambda_0^2}\left(\frac{1}{4} + \Lambda_0'\right). \tag{43}$$

We can now compare the sausage and kink modes, and also a vertically propagating sound wave, by reference to the Klein–Gordon equation in an isothermal atmosphere, simply by noting their differing propagation speeds, cutoff frequencies and e-folding distances. Suppose that $c_A = 12$ km s^{-1}, $c_S = 8$ km s^{-1}, $\Lambda_0 = 140$ km. The sound wave has propagation speed 8 km s^{-1} and cyclic cutoff frequency, $(c_S/2\Lambda_0)/2\pi$, of 4.5 mHz (period 220 s), and the wave e-folds once in 280 km (two scale heights). By contrast, both the sausage and kink modes take 560 km to e-fold once (four scale heights), the sausage wave propagating with a speed $c_T = 6.7$ km s^{-1} and the kink wave with a speed $c_k = 6.9$ km s^{-1}. As noted by Spruit (1981b), the two waves have entirely different cutoff frequencies: the cyclic cutoff frequency $(\Omega/2\pi)$ for the sausage wave is 4.2 mHz (period 240 s), close to the sound wave, whereas the kink mode has a cyclic cutoff frequency of 2 mHz (period 500 s), about a factor of two different.

5 Nonlinear Numerical Computations

Using Eqs. (30) to (33), nonlinear time-dependent solutions for longitudinal wave propagation in thin magnetic flux tubes may be obtained numerically,

through the method of characteristics. Radiation effects may also be included. For this, Eq. (32) is written in an entropy conservation form,

$$\frac{dS}{dt} = \frac{\partial S}{\partial t} + v\frac{\partial S}{\partial z} = \left.\frac{dS}{dt}\right|_{Rad}, \tag{44}$$

where $(dS/dt)|_{Rad}$ is the radiative damping function. Eq. (33) follows from (44) on assuming $dS/dt|_{Rad} = 0$ and using the thermodynamic relations

$$\frac{d\rho}{\rho} = \frac{2}{\gamma - 1}\frac{dc_S}{c_S} - \frac{\mu}{\Re}dS\,, \quad \frac{dp}{p} = \frac{2\gamma}{\gamma - 1}\frac{dc_S}{c_S} - \frac{\mu}{\Re}dS\,. \tag{45}$$

Here $\Re$ is the gas constant and μ the mean molecular weight. For one-dimensional computations it is convenient to use a Lagrangian frame (which follows the motion of a parcel of fluid) as this, when replacing differential equations by difference equations, permits us to more easily ensure mass and energy conservation in the numerical scheme. In place of the Eulerian scheme, which considers a fixed location, namely the height z, one now uses the Lagrangian height a as independent variable; a is the Eulerian height at time $t = 0$, the start of the calculation. Consider a mass element contained initially in the height interval da at height a which at a later time moves to the Eulerian height $z(a,t)$ and expands to the size dz. If $A_0(a)$ and $\rho_0(a)$ are, respectively, the cross-section and density of the element at time $t = 0$, while $A(a,t)$ and $\rho(a,t)$ are the same quantities at a later time, then mass conservation in a tube gives

$$\rho_0(a)A_0(a)da = \rho(a,t)A(a,t)dz\,, \tag{46}$$

from which one obtains the scale factor

$$l_a = \left(\frac{\partial z}{\partial a}\right)_t = \frac{\rho_0 A_0}{\rho A}\,. \tag{47}$$

Using the transformation equations between the Eulerian and Lagrangian frames,

$$\left(\frac{\partial f}{\partial t}\right)_a = \left(\frac{\partial f}{\partial t}\right)_z + v\left(\frac{\partial f}{\partial z}\right)_t, \quad \left(\frac{\partial f}{\partial a}\right)_t = l_a\left(\frac{\partial f}{\partial z}\right)_t, \tag{48}$$

Eqs. (30) and (31), with the help of (34), are written in the forms

$$\frac{1}{l_a}\left(\frac{\partial v}{\partial a}\right)_t + \frac{2c_S}{\gamma - 1}\frac{1}{c_T^2}\left(\frac{\partial c_S}{\partial t}\right)_a - \frac{\mu}{\gamma\Re}\left(\frac{c_S^2}{c_A^2} + \gamma\right)\left(\frac{\partial S}{\partial t}\right)_a - \frac{v}{\rho c_A^2}\frac{dp_e}{dz} = 0 \tag{49}$$

$$\left(\frac{\partial v}{\partial t}\right)_a + \frac{1}{l_a}\left[\frac{2c_S}{\gamma - 1}\left(\frac{\partial c_S}{\partial a}\right)_t - \frac{c_S^2\mu}{\gamma\Re}\left(\frac{\partial S}{\partial a}\right)_t\right] + g = 0. \tag{50}$$

We have used Eq. (45) to eliminate the thermodynamic variables p and ρ in favour of the sound speed c_S and entropy S. Suitably combining Eqs. (49) and (50) with help of (44) brings these equations into characteristic form

$$dv \pm \frac{2}{\gamma-1}\frac{c_S}{c_T}dc_S \mp \frac{\mu c_S^2}{\gamma \Re c_T}dS \mp \left[\frac{\mu c_T}{\gamma \Re}(\gamma-1)\left.\frac{dS}{dt}\right|_{Rad} + \frac{v c_T}{\rho c_A^2}\frac{dp_e}{dz}\right] dt + g dt = 0, \tag{51}$$

along the two characteristics C^+, C^- given by

$$\left(\frac{da}{dt}\right)_{\pm} = \pm\frac{c_T}{l_a}, \tag{52}$$

where the upper sign in these two equations is for the C^+ characteristic and the lower sign for the C^- characteristic. Instead of the two partial differential Eqs. (49) and (50) we now have four ordinary differential equations (51) and (52).

The numerical solution proceeds as follows. From a height point P at the new time level $t + \Delta t$, where one assumes preliminary values of the three unknowns c_S(P), S(P) and v(P), one constructs the characteristics (52) in the t -a plane and their intersection points with the old time level t at which the solution is assumed to be given everywhere. From these intersection points using the finite difference forms of Eqs. (51) along the two C^+ and C^- characteristics, two of the three unknowns can be computed. The third unknown is obtained by integrating Eq. (44) along the fluid path $a = const.$ using $dS = (dS/dt)|_{Rad}dt$. In this way new estimates of c_S(P), S(P) and v(P) are obtained and the process can be repeated until convergence.

This procedure is very accurate and stable and allows also one to treat the formation and propagation of shocks. Extensive longitudinal wave propagation calculations using this procedure have been performed by Herbold et al. (1985) and Rammacher & Ulmschneider (1989). In these calculations H^- continuum radiation was included in the optically thin approximation, while Mg II k and Ca II K line radiation were treated as optically thick. For these non-grey radiation treatments the NLTE statistical rate equations where solved together with the radiative transfer equations.

For the problem of coupled transverse and longitudinal tube waves the set of equations (30) to (34) have to be augmented by the transverse components. The tube is now considered to be inclined to the vertical. Let $\hat{\mathbf{e}}_1$ be the unit vector along the tube, with direction cosines l_x, l_y and l_z with respect to the coordinate axes; the direction cosines are related by

$$l_x^2 + l_y^2 + l_z^2 = 1\,. \tag{53}$$

Then the longitudinal Eqs. (30) and (31) together with (34) can be written

$$\frac{\hat{\mathbf{e}}_1}{l_a}\cdot\left(\frac{\partial \mathbf{v}}{\partial a}\right)_t + \frac{2c_S}{\gamma-1}\frac{1}{c_T^2}\left(\frac{\partial c_S}{\partial t}\right)_a - \frac{\mu}{\gamma\Re}\left(\frac{c_S^2}{c_A^2}+\gamma\right)\left(\frac{\partial S}{\partial t}\right)_a - \frac{v_z}{\rho c_A^2}\frac{dp_e}{dz} = 0\,, \tag{54}$$

$$\hat{\mathbf{e}}_1\cdot\left(\frac{\partial \mathbf{v}}{\partial t}\right)_a + \frac{1}{l_a}\left[\frac{2c_S}{\gamma-1}\left(\frac{\partial c_S}{\partial a}\right)_t - \frac{c_S^2\mu}{\gamma\Re}\left(\frac{\partial S}{\partial a}\right)_t\right] + g l_z = 0\,. \tag{55}$$

These equations should be compared with (49) and (50). Combining the two equations together with (44) into characteristic form one obtains

$$\hat{\mathbf{e}}_1\cdot d\mathbf{v} \pm \frac{2}{\gamma-1}\frac{c_S}{c_T}dc_S \mp \frac{\mu c_S^2}{\gamma\Re c_T}dS \mp \left[\frac{\mu c_T}{\gamma\Re}(\gamma-1)\left.\frac{dS}{dt}\right|_{Rad} + \frac{v_z c_T}{\rho c_A^2}\frac{dp_e}{dz}\right]dt + g l_z dt = 0 \tag{56}$$

along the two characteristics C_1^+, C_1^- given by

$$\left(\frac{da}{dt}\right)_{\pm} = \pm\frac{c_T}{l_a}\,, \tag{57}$$

where the top sign in the last two equations is for the C_1^+ and the bottom sign for the C_1^- characteristic. Note that for purely vertical propagation, when $l_z = 1$, Eq. (56) reduces to (51).

The system (56) and (57) describes the longitudinal wave, where the signal propagation speed, as observed from the moving mass element, is $dz/dt = l_a da/dt = \pm c_T$.

For the kink mode the transverse components of the equation of motion and of the combined induction and continuity equations must be considered. From the equation of motion

$$\rho\frac{d\mathbf{v}}{dt} = -\nabla p - \frac{1}{\mu_0}\mathbf{B}\times(\nabla\times\mathbf{B}) + \rho\mathbf{g}\,, \tag{58}$$

we take the transverse component

$$\rho\left(\frac{d\mathbf{v}}{dt}\right)^{\perp} = \frac{B^2}{\mu_0}\kappa\hat{\mathbf{e}}_2 + \hat{\mathbf{e}}_1\times\left[\rho\mathbf{g} - \nabla\left(p + \frac{B^2}{2\mu_0}\right)\right]\times\hat{\mathbf{e}}_1\,. \tag{59}$$

Here $\hat{\mathbf{e}}_2$ is the unit vector in direction to the local center of curvature and $\kappa\hat{\mathbf{e}}_2 = (\partial\hat{\mathbf{e}}_1/l_a\partial a)_t$. Assuming horizontal pressure balance, and outside the tube hydrostatic equilibrium ($\nabla p_e = \rho_e\mathbf{g}$), Eq. (59) is written

$$\rho\left(\frac{d\mathbf{v}}{dt}\right)^{\perp} = \rho c_A^2\kappa\hat{\mathbf{e}}_2 + (\rho-\rho_e)\mathbf{g}^{\perp}\,. \tag{60}$$

This equation does not include forces due to the backreaction of the external medium. The complete Eq. (60) thus reads

$$\rho\left(\frac{d\mathbf{v}}{dt}\right)^{\perp} = \rho c_A^2\kappa\hat{\mathbf{e}}_2 + (\rho-\rho_e)\mathbf{g}^{\perp} + \mathbf{f}_{\mathbf{ext}}\,. \tag{61}$$

In recent years there has been an extensive discussion about the correct form of the back-reaction force term $\mathbf{f}_{\mathbf{ext}}$ (Spruit 1981; Choudhuri 1990;

Cheng 1992; Fan, Fisher & McClymont 1994; Moreno-Insertis, Ferriz-Mas & Schüssler 1994; Moreno-Insertis, Schüssler & Ferriz-Mas 1996). Osin, Volin & Ulmschneider (1996) take

$$\mathbf{f_{ext}} = -\rho_e \left(\frac{d_\perp \mathbf{v}^\perp}{dt}\right)^\perp . \tag{62}$$

Then, with Eqs. (61) and (62), the transverse component of the equation of motion can be written (Osin et al. 1996)

$$\left(\frac{\partial \mathbf{v}}{\partial t}\right)_a - \hat{\mathbf{e}}_1 \cdot \left(\frac{\partial \mathbf{v}}{\partial t}\right)_a \hat{\mathbf{e}}_1 - \frac{(\rho c_A^2 - \rho_e v^{\|2})}{(\rho+\rho_e) l_a}\left(\frac{\partial \hat{\mathbf{e}}_1}{\partial a}\right)_t - \frac{2\rho_e v^\|}{(\rho+\rho_e)}\left(\frac{\partial \hat{\mathbf{e}}_1}{\partial t}\right)_a - \frac{(\rho-\rho_e)}{(\rho+\rho_e)}\mathbf{g}^\perp = 0 , \tag{63}$$

where $v^\| = \hat{\mathbf{e}}_1 \cdot \mathbf{v}$.

The transverse component of the combined induction and continuity equation (30) is given by

$$\left(\frac{\partial \hat{\mathbf{e}}_1}{\partial t}\right)_a = \frac{1}{l_a}\left[\left(\frac{\partial \mathbf{v}}{\partial a}\right)_t - \hat{\mathbf{e}}_1 \cdot \left(\frac{\partial \mathbf{v}}{\partial a}\right)_t \hat{\mathbf{e}}_1\right] . \tag{64}$$

Combining Eqs. (63) and (64) into characteristic form one finds (after some algebra) the two equations

$$\begin{aligned} &(1-l_x^2)\, dv_x - l_x l_y dv_y - l_x l_z dv_z - c_k^\pm dl_x \\ &\qquad - \frac{\rho-\rho_e}{\rho+\rho_e} g l_x l_z dt = 0 , \\ &(1-l_y^2)\, dv_y - l_x l_y dv_x - l_y l_z dv_z - c_k^\pm dl_y \\ &\qquad - \frac{\rho-\rho_e}{\rho+\rho_e} g l_y l_z dt = 0 , \end{aligned} \tag{65}$$

both along the characteristics C_2^+, C_2^- given by

$$\left(\frac{da}{dt}\right)_\pm = \frac{c_k^\pm}{l_a} . \tag{66}$$

Here the kink speed $c_k^\pm$, the propagation speed of the pure transverse wave mode, is given by

$$c_k^\pm = -\frac{\rho_e}{\rho+\rho_e} v^\| \pm \sqrt{\left(\frac{\rho_e}{\rho+\rho_e}\right)^2 v^{\|2} + \frac{\rho c_A^2 - \rho_e v^{\|2}}{\rho+\rho_e}} . \tag{67}$$

Note that the pure longitudinal wave propagates with the tube speed c_T and in the above approximation is not affected by the back-reaction of the external medium, whereas the propagation of the transverse mode is

strongly affected by the back-reaction of the external fluid. Propagation of the transverse mode is in general asymmetric, $c_k^- \neq -c_k^+$. Symmetry is restored when the longitudinal flow speed $v^\|$ is negligible, and then

$$c_k^\pm = \pm c_k , \tag{68}$$

the kink speed introduced in Eq. (3). The square root in Eq. (67) points to the possibility of hyperbolicity violation and hence to the development of a wave instability. This will happen when the longitudinal fluid velocity $v^\|$ is large enough:

$$v^\| > \sqrt{\frac{\rho+\rho_e}{\rho_e}} c_A , \tag{69}$$

defining the threshold of the *fire-hose instability*, so-called because the tube experiences a kink-like disturbance driven by the flow (much as for water in a fire-hose).

In contrast with the case of purely longitudinal wave propagation, one now has 8 unknowns c_S, S , $\mathbf{v}$, $\hat{\mathbf{e}}_1$ in the coupled longitudinal-transverse tube wave problem. To solve for these unknowns we have the two relations (56) along the C_1^+, C_1^- characteristics given by Eq. (57) and two relations (65) each along the C_2^+, C_2^- characteristics given by Eq. (66), the entropy conservation relation (44) along the fluid path and Eq. (53). Using this procedure, longitudinal-transverse wave computations have been performed by Ulmschneider & Zähringer (1989), Ulmschneider, Zähringer & Musielak (1991) and Osin et al. (1996). Moreover, as shown by Zhugzhda, Bromm & Ulmschneider (1995), the above procedure also allows to compute shock formation and propagation.

Although presently a procedure which incorporates all three coupled modes, the longitudinal, transverse and torsional wave propagation using the thin flux tube approximation, has not yet been described, the problem of a combined time-dependent longitudinal-torsional wave propagation has been investigated numerically by Hollweg, Jackson & Galloway (1982) and Ferriz-Mas, Schüssler & Anton (1989).

An attractive feature of the method of characteristics is that it automatically leads to the wave propagation speeds, and allows one to derive relations between the amplitudes of the various fluctuating physical quantities in the waves. Similar to the well known relations

$$\frac{v}{c_S} = \frac{\rho'}{\rho} = \frac{p'}{\gamma p} = \frac{2}{\gamma-1}\frac{c_S'}{c_S} = \frac{1}{\gamma-1}\frac{T'}{T} \tag{70}$$

for acoustic waves, Eqs. (45) and (56) allow us to derive the amplitude relations for longitudinal tube waves:

$$\frac{v}{c_S} = \frac{c_S}{c_T}\frac{\rho'}{\rho} = \frac{c_S}{c_T}\frac{p'}{\gamma p} = \frac{2}{\gamma-1}\frac{c_S}{c_T}\frac{c_S'}{c_S} = \frac{1}{\gamma-1}\frac{c_S}{c_T}\frac{T'}{T} = \frac{c_A^2}{c_S c_T}\frac{B'}{B} . \tag{71}$$

Here a prime indicates perturbations, considered small.

6 Magnetohydrodynamic Wave Generation

6.1 Analytical methods based on Lighthill's approach

The generation of different types of Alfvén waves and magnetoacoustic waves in the solar atmosphere has been studied primarily by using analytical methods based on the theory of sound generation by Lighthill (1952). In terrestrial applications for acoustic sound generation, Lighthill's theory is in excellent agreement with observations. Kulsrud (1955) and Osterbrock (1961) extended Lighthill's theory by including magnetic field effects, and Musielak & Rosner (1987, 1988) improved it by accommodating the presence of stratification and an embedded uniform magnetic field in the wave generation region (see also Rosner & Musielak 1989). More recently, Collins (1989a, b, 1992) has modified this type of wave generation theory to explore the excitation of MHD waves by periodic velocity fields in diverging magnetic flux tubes. The common feature of these studies is that they look at the magnetic field in a non-local way to obtain mean generated wave fluxes.

A further advance occurred when a detailed local flux tube geometry was considered. Musielak, Rosner & Ulmschneider (1989) and Musielak et al. (1995) have investigated the interaction between turbulent motions in the solar convection zone and thin magnetic flux tubes. They have considered vertically oriented magnetic flux tubes and restricted their approach to the linear regime. For a magnetic flux tube in the solar convection zone the external pressure can be written as

$$p_{ext} = p_e + p_{turb}, \tag{72}$$

where p_e is the external gas pressure and

$$p_{turb} \equiv \rho_e \left(v_x(\mathbf{r},\mathbf{t})^2 + v_y(\mathbf{r},\mathbf{t})^2 + v_z(\mathbf{r},\mathbf{t})^2\right) \tag{73}$$

the external turbulent pressure. Here v_x, v_y, v_z are the turbulent velocities in x, y, z-directions, functions of position $\mathbf{r}$ and time. Upon time averaging one gets

$$u_{xt} = \sqrt{\overline{v_x(\mathbf{r},\mathbf{t})^2}},\ u_{yt} = \sqrt{\overline{v_y(\mathbf{r},\mathbf{t})^2}},\ u_{zt} = \sqrt{\overline{v_z(\mathbf{r},\mathbf{t})^2}}\ . \tag{74}$$

For the case of homogenous isotropic turbulence there is no longer a dependence on $\mathbf{r}$ due to the assumed homogeneity, and for the three spatial components isotropy implies that

$$u_t \equiv u_{xt} = u_{yt} = u_{zt}. \tag{75}$$

Here u_t is the rms velocity amplitude in one spatial direction, taken to be the same in the x, y and z-directions; it is independent of space and time. Note that here u_t is defined differently from in Musielak et al. (1995). From the above equations one has a time-averaged turbulent pressure

$$\overline{p_{turb}} = 3\rho_e u_t^2\ , \tag{76}$$

and a fluctuating turbulent pressure

$$p'_{turb} = 3\rho_e {u'_t}^2 = 3\rho_e(v_x^2 - u_t^2)\ . \tag{77}$$

Here one has used the fact that the velocity fluctuations in the three spatial directions are uncorrelated. For the generation of longitudinal tube waves external pressure fluctuations have to be translated into fluctuations inside the tube. From the horizontal pressure balance (34) one has for the internal pressure perturbations

$$p' + \frac{B_0 B'}{\mu_0} = p'_{turb}\ . \tag{78}$$

The Lighthill approach starts with an inhomogeneous wave equation

$$\left[\frac{\partial^2}{\partial t^2} - c_T^2\frac{\partial^2}{\partial z^2} + \Omega_T^2\right] p_1(z,t) = S_t(z,t)\ , \tag{79}$$

where $p_1 = p'/\sqrt{\rho_o B_o}$, B_0 the undisturbed field strength of the tube and Ω_T the tube (Defouw) frequency defined by Eq. (40). The source function is given by

$$S_t(z,t) = \frac{3\rho_e}{2\sqrt{\rho_o B_o}}\frac{c_T^2}{c_A^2}\left(\frac{\partial^2}{\partial t^2} + \Omega_{BV}^2\right){u'_t}^2\ , \tag{80}$$

where Ω_{BV} is the Brunt-Väisälä frequency. For the time-averaged longitudinal tube wave flux one has

$$\overline{F(z,t)} = \overline{-B_o\,(1+\delta)\,p_1\left(\frac{\partial}{\partial t}\right)^{-1}\left(\frac{\partial}{\partial z} + k_h\right)\left[1 + \Omega_{BV}^2\left(\frac{\partial}{\partial t}\right)^{-2}\right]^{-2} p_1^*}, \tag{81}$$

where $\delta = 5c_S^2/2\gamma c_A^2$, $k_h = (4-3\gamma)/4\gamma\Lambda_0$ for pressure scale height Λ_0, and complex conjugate p_1^* of p_1. By using Fourier transforms Eqs. (79) to (81) can be solved and the mean wave energy generation rate for a given flux tube can be written in the form (expressed in cgs units, erg cm^{-3} s^{-1} Hz^{-1}, the choice of the original authors):

$$\frac{\partial^2}{\partial z_o \partial\omega}\overline{F(z_o,\omega)}_{z_o,t_o} = \frac{\pi}{32}\left(\frac{\rho_e}{\rho_o}\right)^2\left(\frac{V_t}{V_a}\right)^2\frac{\rho_o\omega(1+\delta)}{V_t^3}\sqrt{\omega^2 - \Omega_{BV}^2}\,J_c(k_o,\omega), \tag{82}$$

where the convolution integral J_c is given by

$$J_c(k_0,\omega) = \frac{1}{2\pi^2}\int_{-\infty}^{+\infty} dr \int_{-\infty}^{+\infty} d\tau \Big(2R_{xx}^2(r,\tau) + R_{zz}^2(r,\tau)\Big)\exp^{-i(k_0 r - \omega\tau)}\ . \tag{83}$$

The averages extend over a suitably large height z_o and time t_o interval. Here the correlation tensors $R_{xx}(r,\tau)$ and $R_{zz}(r,\tau)$ can be expressed in terms of second order correlations averaged over z_o and t_o. These correlation tensors can be written in terms of the turbulent energy spectrum $E(k,\omega)$,

$$R_{xx}(r,\tau) = R_{yy}(r,\tau) =$$

$$\int_0^\infty d\omega \cos\omega\tau \int_0^\infty dk\ E(k,\omega)\left(\frac{\sin kr}{kr}+\frac{\cos kr}{k^2r^2}-\frac{\sin kr}{k^3r^3}\right) \tag{84}$$

and

$$R_{zz}(r,\tau) = 2\int_0^\infty d\omega \cos\omega\tau \int_0^\infty dk\ E(k,\omega)\left(\frac{\sin kr}{k^3r^3}-\frac{\cos kr}{k^2r^2}\right) \tag{85}$$

where

$$E(k,\omega) = E(k)\Delta\left(\frac{\omega}{ku_k}\right)\ ; \tag{86}$$

the mean velocity of a turbulent eddy with wave number k is given by

$$u_k = \left[\int_k^{2k} E(k')dk'\right]^{1/2} . \tag{87}$$

The computation of the longitudinal tube wave flux thus reduces to the specification of the turbulent energy spectrum. The turbulent energy spectrum appropriate for the solar convection zone has been discussed in detail by Musielak et al. (1994). These authors argue on the basis of observations and numerical convection calculations that a realistic turbulent energy spectrum should be reasonably well described by an extended Kolmogorov spectrum, $E(k)$, and a modified Gaussian frequency factor, $\Delta(\omega/ku_k)$. The extended Kolmogorov spatial component is given by

$$E(k) = \begin{cases} 0, & 0 < k < 0.2k_t, \\ 0.758\frac{u_t^2}{k_t}\left(\frac{k}{k_t}\right), & 0.2k_t \le k < k_t, \\ 0.758\frac{u_t^2}{k_t}\left(\frac{k}{k_t}\right)^{-5/3}, & k_t \le k \le k_d, \end{cases} \tag{88}$$

where $k_t = 2\pi/\Lambda_0$, and the modified Gaussian frequency factor by

$$\Delta\left(\frac{\omega}{ku_k}\right) = \frac{4}{\sqrt{\pi}}\frac{\omega^2}{|ku_k|^3}e^{-\left(\frac{\omega}{ku_k}\right)^2} . \tag{89}$$

Using the above, Musielak et al. (1995) have computed longitudinal tube wave fluxes. Their results indicate fluxes of the order of several times 10^7 erg cm^{-2}s^{-1}, which seem relatively low to account for the observed enhanced heating in the chromospheric network. In a similar treatment for transverse tube waves, Musielak, Rosner & Ulmschneider (1997) show that the wave energy flux carried by these waves can be of the order of 10^8 erg cm^{-2}s^{-1}.

6.2 Numerical methods based on the direct perturbation of flux tubes

There are also methods of MHD wave generation which are not based on the Lighthill approach. In these methods solar magnetic flux tube models are perturbed from the outside with velocity or pressure fluctuations which in magnitude and spectral shape are consistent with observations.

To perturb the flux tube one assumes that v_x can be written as a spectrum of N partial waves

$$v_x = \sum_{n=1}^{N} u_n \sin(\omega_n t + \varphi_n)\ , \tag{90}$$

where $\varphi_n = 2\pi r_n$ is an arbitrary but constant phase angle, r_n a random number in the interval [0, 1]. The partial wave amplitude u_n is determined by the turbulent energy spectrum as follows. Time averaging v_x^2 one finds (Huang et al. 1995)

$$\overline{v_x^2} = u_t^2 = \frac{1}{2}\sum_{n=1}^{N} u_n^2 . \tag{91}$$

As is customary (e.g. Musielak et al. 1995) the turbulent energy spectrum is normalized to

$$\frac{3}{2}u_t^2 = \int_0^\infty d\omega \int_0^\infty dk\ E(k)\Delta\left(\frac{\omega}{ku_k}\right) = \int_0^\infty E'(\omega)d\omega . \tag{92}$$

From this one obtains

$$\frac{3}{2}u_t^2 = \frac{3}{4}\sum_{n=1}^{N} u_n^2 = \int_0^\infty E'(\omega)d\omega = \sum_{n=1}^{N} E'(\omega_n)\Delta\omega, \tag{93}$$

which allows to determine u_n

$$u_n = \sqrt{\frac{4}{3}E'(\omega_n)\Delta\omega}\ , \tag{94}$$

where

$$E'(\omega_n) = \int_0^\infty E(k)\Delta\left(\frac{\omega_n}{ku_k}\right)dk\ ; \tag{95}$$

for $E(k)$ and $\Delta(\omega/ku_k)$, the extended turbulent energy spectrum and the modified Gaussian frequency factor of Eqs. (88) and (89) are taken.

For a transverse wave generation calculation the velocity v_x can be directly applied as a boundary condition, representing the horizontal shaking velocity acting on the magnetic flux tube. A similar uncorrelated shaking arises from v_y. For values of u_t = 1.0 to 2.0 km s^{-1} and shaking at various heights, Huang et al. (1995) with a correction described by Ulmschneider & Musielak (1996) obtain total transverse wave fluxes (shaking in both horizontal directions) of $6 \cdot 10^9$ to $3 \cdot 10^{10}$ erg cm^{-2}s^{-1}.

In the process of longitudinal tube wave excitation, the tube is compressed symmetrically by the external turbulent pressure. This turbulent pressure consists of the time averaged term $\overline{p_{turb}}$ which augments the external gas pressure p_e and the fluctuating term which gives rise to longitudinal tube waves (cf. Eqs. (76) to (78)). For the generation of longitudinal tube waves these external pressure fluctuations have to be translated into fluctuations inside the tube. From the horizontal pressure balance one has gas pressure and magnetic field variations inside the tube (cf. Eq. (78)). Using the relations among the amplitudes for longitudinal tube waves one has

$$\frac{B'}{B_0} = \frac{c_S^2}{c_A^2}\frac{p'}{\gamma p_0}, \tag{96}$$

where p_0 the undisturbed gas pressure in the tube. From this one obtains

$$p' = \frac{1}{2}p'_{turb}, \tag{97}$$

which shows that the external pressure fluctuations are divided equally into an internal gas pressure fluctuation and a magnetic pressure fluctuation. As the tube wave code normally uses a velocity boundary condition one translates the gas pressure fluctuations into longitudinal velocity fluctuations $v_\parallel$ by using the amplitude relations for longitudinal tube waves, finding

$$v_\parallel = \frac{c_S^2}{c_T}\frac{p'}{\gamma p_0} = \frac{p'_{turb}}{2\rho_0 c_T}, \tag{98}$$

with ρ_0 the undisturbed density in the tube.

Finally, the normalized flux of longitudinal tube waves is

$$\overline{F} = \overline{\frac{A}{A_0}\rho_0 v_\parallel^2 c_T} \tag{99}$$

while that of transverse tube waves is

$$\overline{F} = -\overline{\frac{A}{A_0}\frac{B_0}{\mu_0}B' v_x},$$

where v_x is the transverse velocity and B' the transverse magnetic field perturbation.

Using this procedure Ulmschneider & Musielak (1996) have computed various longitudinal wave energy fluxes and find values of the order of several times 10^8 erg cm^{-2}s^{-1}. Note that due to the nonlinear treatment, which uses the partial wave synthesis after Eq. (90), a very spiky velocity perturbation results which leads to much larger fluxes both for the transverse and the longitudinal wave fluxes as compared to the linear results. We have the suspicion that due to the lack of cancellations when shaking simultaneously at several

heights, the time-dependent nonlinear treatments tends to overestimate wave fluxes, while the linear analytical treatment tends to underestimate the fluxes.

Finally, we note that detailed studies of the various waves excited by footpoint motions of flux tubes and slabs have also been recently carried out; see, e.g., Murawski & Roberts (1993), Berghmans & De Bruyne (1995), Murawski et al. (1996), and Cargill, Spicer & Zalesak (1996). But so far such studies are aimed more at coronal conditions and as such lie outside the scope of the present review.

7 Concluding Remarks

It is evident from the above discussion that many aspects of flux tubes in the solar atmosphere are now understood. However, a number of important theoretical questions remain unanswered. For example, we note that the basic set of thin tube equations for the sausage mode presented earlier were derived under the assumption that there were no motions in the θ-direction. This has the effect of losing the torsional Alfvén waves given for a finite tube by equation (10). As recently stressed by Zhugzhda (1996), it is clearly important to extend our understanding so as to include the coupling of motions. Zhugzhda (1996) has considered such an extension of the thin tube equations to include v_θ terms, using the Taylor series expansion about $r = 0$ as originally employed by Roberts & Webb (1978). (The expansion approach has also been adopted by Ferriz-Mas, Schüssler & Anton (1990).) All extensions of this form are in principle capable of being tested against the known linear results for a finite radius tube, and thereby to examine how successful the approach is in describing the *dispersive* behaviour of the waves.

The standard derivation of thin tube equations for the kink mode makes use of the assumption that the kink mode displaces an equal proportion of fluid in the environment of the tube as is contained by the tube (Parker 1979; Spruit 1981). This has the effect of augmenting the gas density ρ_0 with ρ_e, so that the characteristic speed of the kink mode is c_k (Eq. (3)). However, to describe properly the motions in a nonlinear theory it is necessary to describe also the dispersive influence of the tube's surroundings, as given in linear theory by the long wavelength result (27). As discussed earlier, there have been a number of attempts (Choudhuri 1990; Cheng 1992, 1994; Fan, Fisher & McClymont 1994; Moreno-Insertis, Schüssler & Ferriz-Mas 1996; Osin et al. 1996) to extend Spruit's (1981) thin tube theory of the kink mode, to take proper account of inertial effects, but the matter is currently unresolved. Again, we note that it is a test of any approximate description that it is able to describe properly the *dispersive* influence of the tube's environment, evident in (26) and (27); this proved possible in the derivation of the Leibovich–Roberts equation for the sausage mode, but the kink mode remains to be fully treated.

We should note too that the presence of flows in the regions externally bordering isolated photospheric tubes has an effect on the waves a tube supports. Nakariakov & Roberts (1995) have shown that if an external steady flow of magnitude U_e exists outside a uniform magnetic slab, then the fast surface wave propagating against the direction of the flow becomes leaky if $U_e > c_{Se} - c_S$ and the slow body waves leak if $U_e > c_{Se} - c_T$. Such flows may be met in the photosphere: from our earlier illustration of speeds, we see that $c_{Se} - c_S = 1.6$ km s^{-1} and $c_{Se} - c_T = 2.9$ km s^{-1}, so downdraughts of 3 km s^{-1} might cause strong leakage from the tube.

In summary, then, while many aspects of flux tube dynamics are now understood, there remain a number of important fundamental questions for theoretical investigation. But above all, it is to be hoped that observational evidence of flux tube behaviour will point out the most important features of flux tube dynamics that require our greater study and understanding.

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