The Generation of Longitudinal Tube Waves in Late Type Stars

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Abstract.

The aim of the present work is to compute the generated nonlinear tube wave energy fluxes carried by longitudinal waves as a result of the interaction between a vertically directed thin magnetic flux tube and the turbulent medium in the stellar convection zone of late type stars. The computations are based on work by Ulmschneider and Musielak (1998). The current computations are for stars of gravities $log\ g = 3, 4, 5$ and temperature range from $T_{eff} = 3500$ to 7000 K.

Key Words: MHD waves - methods: numerical - stars: convection zone - stars: chromoaphere - stars: magnetic fields

1 Introduction

The magnetic flux tubes have their roots in the very turbulent medium of the stellar convection zone, where pressure fluctuations generated by the unsteady turbulence interact with the tubes in different forms, e.g. compression and shaking. This interaction has been suggested as a source of the generation of different types of magnetohydrodynamic waves; e.g. longitudinal and transverse tube waves. These waves can propagate through the flux tubes up to the chromosphere and dissipate energy in form of shocks which lead to local heating. Recently the generation of longitudinal and transverse tube waves have been studied analytically (Musielak et al. 1995, 1999) and numerically (Huang et al. 1995; Ulmschneider & Musielak 1998). In the current computations we use the approach developed by Ulmschneider & Musielak (1998), to calculate the longitudinal wave energy fluxes for late-type stars as a result of squeezing a vertical magnetic flux tube by external pressure fluctuations produced by turbulent motions. Figure 1 shows the interaction between the magnetic flux tube and the external turbulent medium.

2 Formulation

2.1 Magnetic flux tube models

A thin, vertically oriented magnetic flux tube is embedded in the field free stellar atmosphere of stars with effective temperatures ranging from $T_{eff} = 3500$ to 7000 K and with surface gravities ranging from $log\ g = 3 - 5$. Three different field strengths were considered at the stellar surfaces, $B/B_{eq} = 0.75, 0.85, 0.95$, where $B_{eq} = \sqrt{8\pi P_e}$ is the equipartition field strength, P_e is the external gas pressure .

2.2 Turbulence

Turbulence is usually described by a turbulent energy spectrum. We assume that the horizontal turbulent velocity fluctuations in the x-direction can be represented by a linear combination of $N = 500$ partial waves

$$v_x(t) = \sum_{n=1}^{N} u_n \sin(\omega_n t + \varphi_n) \quad , \tag{1}$$

where u_n is the velocity amplitude of the partial waves, which is determined from the turbulent energy spectrum, ω_n is the wave frequency, and $\varphi_n = 2\pi r_n$ an arbitrary but constant phase angle with r_n being a random number in the interval [0, 1]. In the current computations the spatial and temporal external turbulence components are represented by an extended Kolmogorov spectrum given by

$$E(k) = \begin{cases} 0 & 0 < k < 0.2k_t \\ a\frac{u_t^2}{k_t}\left(\frac{k}{k_t}\right) & 0.2k_t \leq k < k_t \\ a\frac{u_t^2}{k_t}\left(\frac{k}{k_t}\right)^{-5/3} & k_t \leq k \leq k_d \end{cases} \quad , \tag{2}$$

here $a = 0.758$, u_t is the rms velocity amplitude of the external turbulent motions, $k_t = 2\pi/H$ with H being the scale height, and k_d is the wave number at which the turbulent cascade ends. The modified Gaussian frequency factor is given by

$$G\left(\frac{\omega}{ku_k}\right) = \frac{4}{\sqrt{\pi}}\frac{\omega^2}{|ku_k|^3}e^{-\left(\frac{\omega}{ku_k}\right)^2} \quad , \tag{3}$$

where u_k is computed from

$$u_k = \left[\int_k^{2k} E(k')dk'\right]^{1/2} \quad . \tag{4}$$

The velocity amplitude, u_n, is evaluated from

$$u_n = \sqrt{\frac{4}{3}E'(\omega_n)\Delta\omega} \quad , \tag{5}$$

where

$$E'(\omega_n) = \int_0^\infty E(k)G\left(\frac{\omega_n}{ku_k}\right)dk \quad . \tag{6}$$

The calculated velocity $v_x(t)$ may now be used to define the turbulent pressure fluctuations p'_{turb} that are responsible for the squeezing of the flux tube. As shown by Ulmschneider & Musielak (1998), these fluctuations consist of a time averaged term $\overline{p_{turb}} = 3\rho_e u_t^2$, which augments the external gas pressure p_e, and a fluctuating term $p_{turb} = 3\rho_e v_x^2(t)$, which gives rise to longitudinal tube waves; note that ρ_e is the gas density in the external medium. Combining these two terms, we get

$$p'_{turb} = 3\rho_e(v_x^2 - u_t^2) \quad . \tag{7}$$

These turbulent pressure fluctuations lead to the gas pressure perturbations p' inside the tube

$$p' = \frac{\beta}{2/\gamma + \beta}p'_{turb} \quad , \tag{8}$$

where $\beta = 8\pi p_0/B_0^2$ is the plasma β. Finally, the gas pressure fluctuations inside the tube can be translated into internal longitudinal velocity fluctuations

$$v_\parallel = \frac{c_S^2}{c_T}\frac{p'}{\gamma p_0} \quad , \tag{9}$$

where c_S is the sound speed, $c_T = (1/c_S^2 + 1/c_A^2)^{-1/2}$ the tube speed, c_A the Alfvén velocity, γ the ratio of specific heats, and p_0 the gas pressure inside the tube. The velocity $v_\parallel$ can also be expressed in terms of $v_x(t)$ by using Eqs. (7) through (9) resulting in

$$v_\parallel = \frac{\beta}{2/\gamma + \beta}\frac{3\rho_e(v_x^2 - u_t^2)}{\rho_0 c_T} \quad . \tag{10}$$

The derived $v_\parallel$ can now be used as the velocity boundary condition in our MHD wave code. Using the instantaneous internal velocity $v_\parallel(z,t)$ and the internal pressure perturbations $p'(z,t)$, we then calculate the instantaneous wave energy flux, $F(z,t) = v_\parallel(z,t)p'(z,t)$, and the time-averaged wave energy flux, $\overline{F(z,t)}$. If $A(z)$

represents changes of the tube cross-section with height, we then can write $A(z)\overline{F(z,t)} = A(z=0)\overline{F(z=0,t)}$, which gives

$$F_L \equiv \overline{F(z=0,t)} = \frac{A(z)}{A_0}\overline{v_{\parallel(z,t)}p'(z,t)} \quad , \tag{11}$$

which is the final expression used for the wave energy fluxes carried by longitudinal tube waves in stellar atmospheres. In the computation of the wave generation, the phases between the different physical variables must be taken into account, so a time-dependent magneto-hydrodynamic wave code has to be used.

2.2.1 The root mean square of the turbulent velocity

Based on hydrodynamic simulations of the convection zone of the Sun (Steffen, 1992), it has been found that the velocities of the eddies in the overshooting regions have values roughly equal to half the value of the maximum convective velocity. Since the magnetic flux tubes sit at the stellar surface, in regions which have overshooting from the convection zone, we expect the *rms* value of the turbulent velocity is in the range of the velocities of the overshooting region. A value of $u_t = 0.5\ u_{mcv}$, where u_{mcv} is the maximum convective velocity, is a good choice for the value, v_{rms}. Stars with low surface gravities have much more extended convection zones compared with the stars of higher surface gravities, and the maximum convective velocities are higher. For stars of a given surface gravity, hotter stars have higher convective velocities than cool stars. The cases which are considered here are all subsonic convective velocity cases, supersonic cases are excluded from the computations. Figure 2 shows the root mean square of the turbulent velocities for stars of surface gravities $log\ g = 3, 4, 5$ and effective temperature range from $T_{eff} = 3500\ -\ 7000$ K.

2.3 Separating the propagating and non-propagating waves

The generated longitudinal tube waves include both propagating and non-propagating waves. Applying spectral analysis techniques enables us to separate the propagating and non-propagating parts of the waves. By means of a Fourier transformation, the time sequence of the velocity and pressure perturbations are represented in the frequency domain. Then a high pass filter is applied to permit only waves of frequencies above the Defouw cut-off frequency (propagating waves) and to cut out all waves of frequencies below that frequency (non-propagating waves). Figure 3 shows the velocity power spectrum of the propagating waves for a star of surface gravity $g = 10^5\ cm/s^2$ and effective temperature $T_{eff} = 6500\ K$ and magnetic field strength $B = 0.85 B_{eq}$, while Fig. 4 shows the phase difference spectra between the velocity and pressure fluctuations for the same star, indicating the propagation nature of the waves.

3 Results

The distribution of the longitudinal wave energy fluxes (LTW) for stars of different surface gravities and effective temperatures are shown in Fig. 5, while Fig. 6 shows the same distribution but for propagating waves. The generated energy fluxes are higher for hot stars because of the higher turbulent velocities in comparison with cool stars, the decrease of the wave fluxes is due to the lower turbulent velocities and the sharp increase in the magnetic field strengths, which results in increasing the stiffness of the magnetic tube. The computed fluxes can be used as an input to our models to study the propagation and dissipation of longitudinal tube waves (see: Fawzy, Ulmschneider, & Cuntz, 1998).

References

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Figure Caption

Fig. 1. - The interaction between the magnetic flux tube and the external turbulent medium

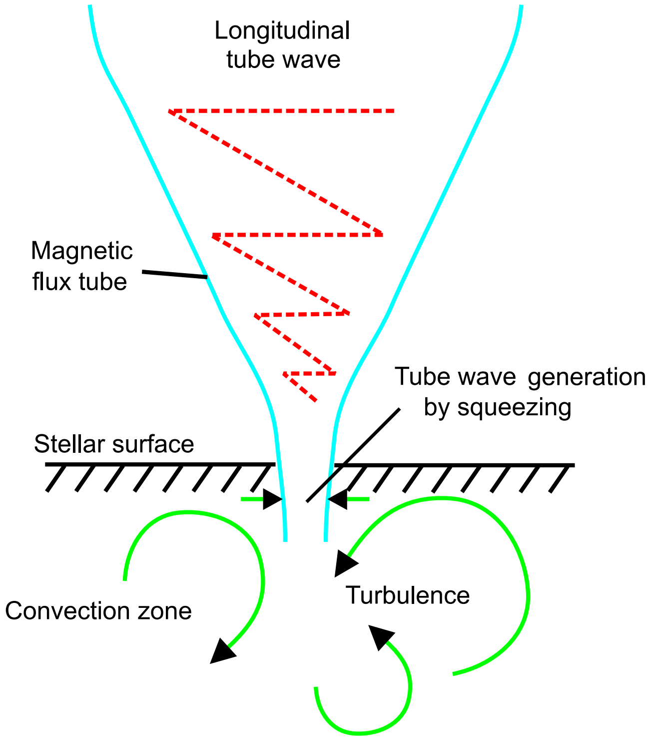
Fig. 2. - The root mean square of the turbulent velocities for stars of surface gravities $log\ g = 3,\ 4,\ 5$ and effective temperature range from $T_{eff} = 3500\ -\ 7000\ K$

Fig. 3. - The velocity power spectrum for a star of surface gravity $g = 10^5\ cm/s^2$, with effective temperature $T_{eff} = 6500\ K$, and a magnetic field strength $B = 0.85B_{eq}$

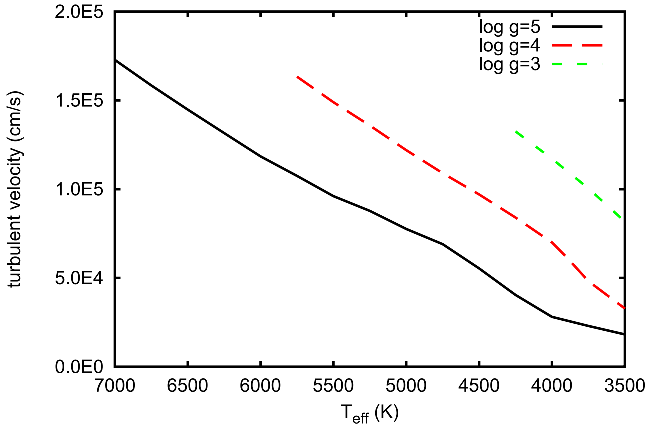
Fig. 4. - The phase difference spectra between the velocity and pressure fluctuations for a star of surface gravity $g = 10^5\ cm/s^2$, with effective temperature $T_{eff} = 6500\ K$, and a magnetic field strength $B = 0.85B_{eq}$

Fig. 5. - The longitudinal tube waves versus effective temperatures for different values of surface gravities and magnetic field strengths

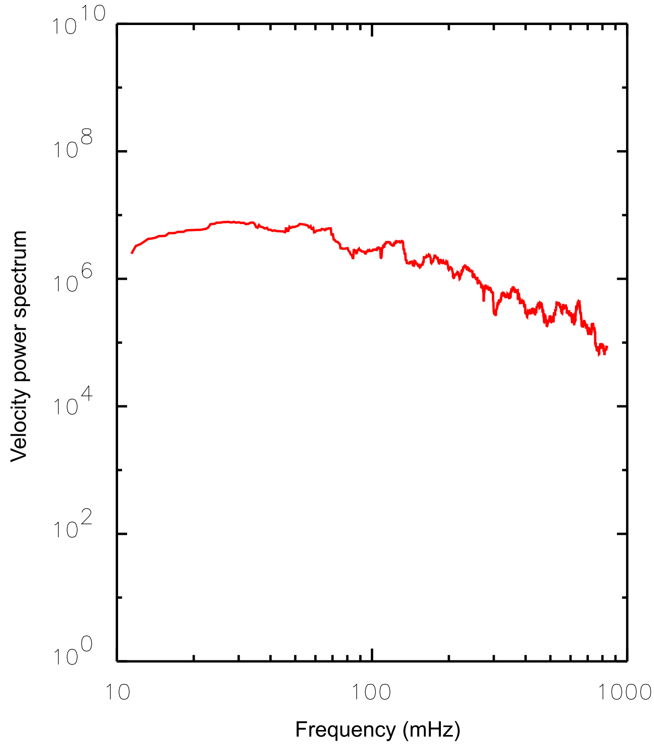
Fig. 6. - The same as Fig. 4 but for propagating waves

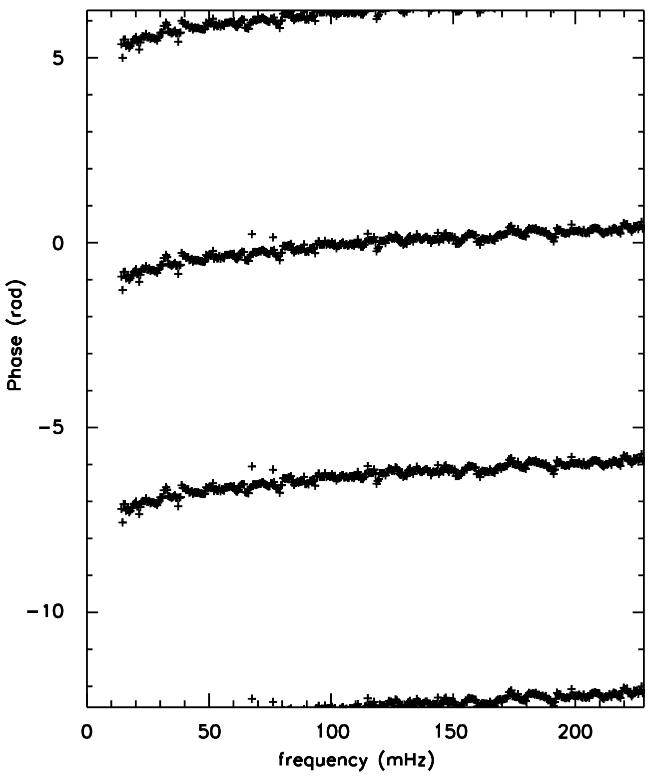


Longitudinal tube wave
Magnetic flux tube
Tube wave generation by squeezing
Stellar surface
Convection zone
Turbulence

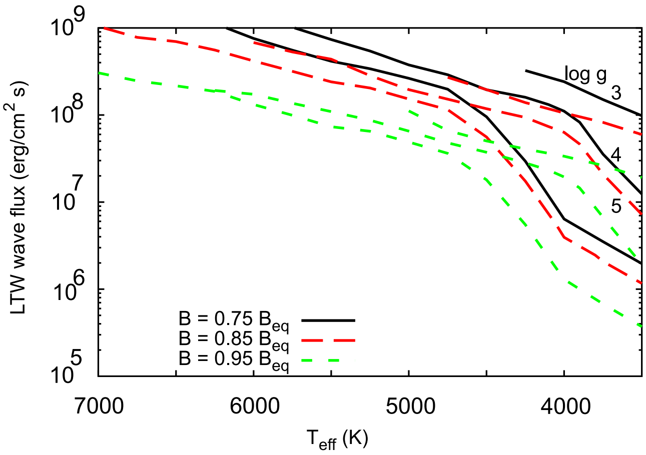


turbulent velocity (cm/s)
2.0E5
1.5E5
1.0E5
5.0E4
0.0E0
7000
6500
6000
5500
5000
4500
4000
3500
T_eff (K)
log g=5
log g=4
log g=3

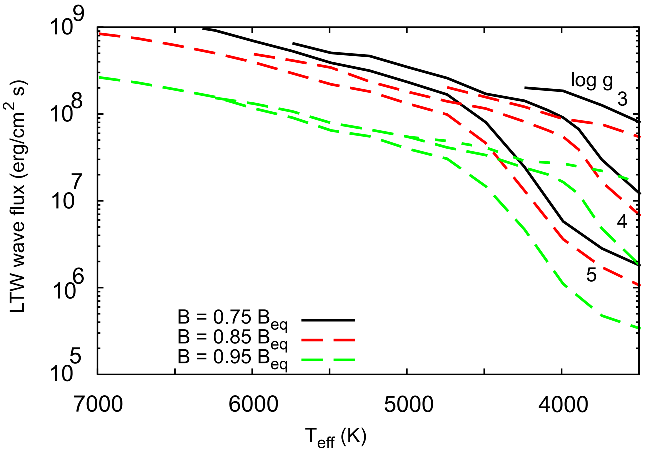

Velocity power spectrum
Frequency (mHz)
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10
100
1000



5
0
−5
−10
Phase (rad)
0
50
100
150
200
frequency (mHz)



LTW wave flux (erg/cm2 s)
T_eff (K)
log g
3
4
5
B = 0.75 B_eq
B = 0.85 B_eq
B = 0.95 B_eq



LTW wave flux (erg/cm^2 s)
10^9
10^8
10^7
10^6
10^5
7000
6000
5000
4000
T_{eff} (K)
log g
3
4
5
B = 0.75 B_{eq}
B = 0.85 B_{eq}
B = 0.95 B_{eq}