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Excitation of transverse magnetic tube waves in stellar convection zones

II. Wave energy spectra and fluxes

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Abstract. The wave energy spectra and fluxes for transverse magnetic tube waves generated in stellar convection zones are computed by using the analytical method developed in the previous paper of this series. The main physical process responsible for the generation of these waves is shaking of a thin and vertically oriented magnetic flux tube by the external turbulent convection. The approach includes the correlation effects, which occur when the tube is shaken over a significant fraction of its length, but is limited to linear waves. The calculations are performed for population I stars with effective temperatures ranging from $T_{\rm eff} = 2000$ K to 10 000 K, and with gravities log $g = 3$–5. It is shown that the fluxes carried by linear transverse waves along a single flux tube are approximately one order of magnitude higher than those carried by linear longitudinal tube waves. The obtained results can be used to construct theoretical models of stellar chromospheres and winds.

Key words. methods: analytical – stars: chromospheres – stars: coronae – stars: magnetic fields – MHD – waves

1. Introduction

To identify the basic physical processes that are responsible for the observed stellar chromospheric and wind activity (e.g., Linsky 1980; Schrijver 1987; Rutten et al. 1991; Linsky 1991; Baliunas et al. 1996; Jordan 1997), we have constructed theoretical and time-dependent models of stellar chromospheres (Cuntz et al. 1998, 1999; Ulmschneider et al. 2001a) and winds (Ong et al. 1997). These models require the initial wave energy fluxes to be specified at the atmospheric height where the calculations begin: the bottom of the chromosphere or the wind base. We have calculated the energy fluxes carried by acoustic waves (Musielak et al. 1994; Ulmschneider et al. 1996, 1999) and longitudinal tube waves (Musielak et al. 1989, 1995; Ulmschneider & Musielak 1998; Musielak et al. 2000; Ulmschneider et al. 2001b) and used them to construct our stellar chromospheric models. The amount of energy carried by transverse tube waves has been added ad hoc in these models (see Ulmschneider et al. 2001a). Similarly, the amount of energy carried by transverse waves at the wind base has also been assumed in our wind models (see also MacGregor & Charbonneau 1994; Lau & Siregar 1996; and Ofman & Davila 1998). Clearly, detailed calculations of the efficiency of the generation of transverse tube waves in stellar convection zones are required and this is the main goal of the present paper.

Some attempts have been made to estimate the amount of energy carried by transverse tube waves in the solar atmosphere. Some authors have based their estimates on observational data (Muller et al. 1994), others have used analytical (Choudhuri et al. 1993a,b) or numerical (Huang et al. 1995) methods. The main conclusion is that transverse waves are generated 30 or even 40 times more efficiently than longitudinal waves; the fact that it is much easier to generate transverse tube waves than longitudinal ones has already been recognized by Spruit & Roberts (1983). Hence, there is a lot of extra wave energy that is not formally accounted for in our current theoretical models (see, however, Ulmschneider et al. 2001a).

On the other hand, it is currently unknown how efficiently this energy can be deposited in the solar chromosphere. The main physical process responsible for dissipation of these waves is their nonlinear coupling to

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longitudinal tube waves. This process may become important in higher atmospheric layers, where the wave amplitudes are large enough for the nonlinear effect to take place, and its efficiency has been studied by Ulmschneider et al. (1991). In addition, the work done by Huang (1996), Wu et al. (1996), Ziegler & Ulmschneider (1997a,b) and Huang et al. (1999) has shown that some fraction of the energy carried by transverse tube waves is always lost due to the energy leakage to the external medium. Note that this leakage depends on the filling factor. Higher up in the chromosphere, there is little or no external atmosphere left and leakage is counterbalanced by gains from neighboring flux tubes. Clearly, detailed three dimensional calculations of the propagation of these waves are needed, however, this is beyond the scope of the present paper.

In the previous paper of this series (Musielak & Ulmschneider 2001, hereafter called Paper I), we have developed a general analytical approach that describes the interaction between a single magnetic flux tube and the external turbulent motions. The tube is embedded in a non-magnetic medium and is assumed to be thin and oriented vertically. Its interaction with the external turbulence, which is considered to be subsonic and known a priori, is assumed to be weak enough so that only linear transverse tube waves are generated. The most efficient excitation of these waves takes place in the upper layers of stellar convection zones, where the existing turbulent motions are the most vigorous. The waves leave the region of generation as freely propagating and purely transverse waves that are not coupled to longitudinal tube waves.

In Paper I, we have derived general formulae for the wave energy spectra and fluxes carried by transverse tube waves by using a 3-D description of turbulence originally introduced by Musielak et al. (1994). In this description, the spatial component of the turbulent convection is represented by an extended Kolmogorov turbulent energy spectrum and its temporal component by a modified Gaussian frequency factor. We now use the results of Paper I to compute the transverse tube wave energy spectra and fluxes generated in convection zones of population I stars with effective temperatures ranging from $T_{\rm eff} = 2000$ K to 10 000 K, and with gravities in the range: log g = 3–5. While performing the calculations, we have carefully identified those regimes where our current model cannot be applied because of the failure of the small Mach number approximation (see Ulmschneider et al. 2001b and their Fig. 1). The obtained spectra and fluxes can be used to explain the enhanced heating observed in magnetized regions of stellar atmospheres (Narain & Ulmschneider 1990, 1996) and, as already mentioned above, to construct theoretical models of stellar chromospheres and winds.

To perform our calculations, we must assume that magnetic flux tubes on other late-type stars are similar to those observed on the Sun (e.g., Solanki 1993, and references therein). This seems to be a reasonable assumption based on the fact that our Sun is not unique among the stars of its category. The problem is, however, that the physical properties of stellar magnetic flux tubes cannot currently be determined by observations (Saar 1996; Rüedi et al. 1997), which means that we have to use solar observations to establish, for example, the range of field strengths in stellar magnetic flux tubes (see Sect. 2).

Solar observations have to be also used to determine the detailed structure of magnetic flux tubes, especially their rate of expansion in the upper atmospheric layers. The latter is of great importance for the propagation and dissipation of the wave energy in the solar and stellar atmospheres (e.g., Herbold et al. 1985; Ulmschneider et al. 1991; Fawzy et al. 1998; Cuntz et al. 1998; Huang et al. 1999) but does not effect the efficiency of the wave generation in stellar convection zones (see Sect. 3). Our paper is organized as follows: a brief summary of the theoretical results obtained in Paper I is given in Sect. 2; the wave energy spectra and fluxes calculated for late-type stars are presented and discussed in Sect. 3; our conclusions are given in Sect. 4.

2. Basic formulae

In this section, we give a very brief summary of the most important results of Paper I and present the basic formula used in this paper to compute the wave energy spectra and fluxes for late-type stars.

We consider a magnetic flux tube embedded in a non-magnetized stellar convection zone, and assume that this tube is thin (its diameter is approximately equal to the local scale height) and circular, and that there are no longitudinal flows inside the tube. To describe the interaction of this tube with the external turbulent convection, we use the set of linearized MHD equations and the horizontal pressure balance across the interface separating the magnetized and non-magnetized medium. The inhomogeneous wave equation that describes the transverse oscillations of the tube resulting from the external turbulent motions can be written in the following form:

$$\left[\frac{\partial^2}{\partial t^2} - c_k^2 \frac{\partial^2}{\partial z^2} + \Omega_k^2\right] v_1(z,t) = \rho_{\rm o}^{1/4} S_x(z,t), \tag{1}$$

where the characteristic wave speed is

$$c_k = \frac{B_{\rm o}}{\sqrt{4\pi(\rho_{\rm e} + \rho_{\rm o})}}, \tag{2}$$

$B_{\rm o}$ is the field strength in the magnetic tube, and $\rho_{\rm e}$ and $\rho_{\rm o}$ are the gas density outside and inside the tube, respectively. The cutoff frequency, Ω_k, for transverse tube waves (see Spruit 1981) is defined as

$$\Omega_k = \frac{c_k}{4H}, \tag{3}$$

where H is the scale height. The velocity, v_1, is related to the tube wave velocity, v_x, by the following relationship:

$$v_1(z,t) = \rho_o^{1/4} v_x(z,t). \tag{4}$$

Defining

$$S_t(z,t) \equiv \rho_o^{1/4} S_x(z,t), \tag{5}$$

we write the source function, $S_{\rm t}(z,t)$, as

$$S_{\rm t}(z,t) = \rho_o^{1/4} \frac{\rho_{\rm e}}{\rho_{\rm o}+\rho_{\rm e}} \left(\frac{u_z}{H} - \frac{\partial u_z}{\partial z}\right) \times \left(\frac{\partial}{\partial t}\right)^{-1} \left(\frac{\partial^2 u_x}{\partial t^2} + g\frac{\partial u_x}{\partial z}\right), \tag{6}$$

where u_x and u_z are components of the turbulent velocity.

Note that there are three small printing errors in Paper I: on the RHS of Eqs. (15) and (19) the gas density ρ is without the subscript "o" and, in addition, the power of $\rho_{\rm o}$ in Eq. (15) should be $+1/4$; these errors have been corrected in this paper – see Eqs. (1) and (4).

The obtained inhomogeneous wave equation is solved by performing the Fourier transforms in time and space, and then evaluating asymptotic values of these transforms. The final expression for the mean wave energy generation rate per unit frequency [in units of erg cm^{-2} s^{-1} Hz^{-1}] can be written in the following form:

$$<< F(\omega) >_{t_{\rm o}} >_{z_{\rm o}} = \pi \int_{\rm o}^{z_{\rm turb}} {\rm d}z\ \rho_{\rm e}\eta_{\rm e} \frac{\omega}{k_o} \frac{\Omega_k^2}{c_k^4} \left(1 + \frac{\omega_{\rm g}^4}{\omega^4}\right) \times (1 + k_{\rm o}^2 H^2)\ J_{\rm c}(k_{\rm o},\omega), \tag{7}$$

where $z_{\rm turb}$ is the thickness of the turbulent region in the stellar convection zone, $\eta_{\rm e} = \rho_{\rm e}/(\rho_{\rm e}+\rho_{\rm o})$, $\omega_{\rm g} = \sqrt{gk_{\rm o}}$ with $k_{\rm o} = \sqrt{(\omega^2 - \Omega_k^2)/c_k^2}$, and $J_{\rm c}$ is the so-called convolution integral. To evaluate this integral (see Appendix C of Paper I, for details), we must specify the turbulent energy spectrum, $E(k,\omega)$. We follow Stein (1967) and write

$$E(k,\omega) = E(k)\Delta\left(\frac{\omega}{ku_k}\right), \tag{8}$$

where the mean velocity of the eddy with wave number k is given by

$$u_k = \left[\int_k^{2k} E(k'){\rm d}k'\right]^{1/2}. \tag{9}$$

For $E(k)$, we take an extended Kolmogorov turbulent energy spectrum (Musielak et al. 1994) given by

$$E(k) = \begin{cases} 0 & 0 < k < 0.2k_{\rm t} \\ a\frac{u_{\rm t}^2}{k_{\rm t}}\left(\frac{k}{k_{\rm t}}\right) & 0.2k_{\rm t} \le k < k_{\rm t} \\ a\frac{u_{\rm t}^2}{k_{\rm t}}\left(\frac{k}{k_{\rm t}}\right)^{-5/3} & k_{\rm t} \le k \le k_{\rm d} \end{cases}, \tag{10}$$

where the factor $a = 0.758$ is determined by the normalization condition

$$\int_0^\infty E(k){\rm d}k = \frac{3}{2}u_{\rm t}^2, \tag{11}$$

and $u_{\rm t}$ is the rms turbulent velocity defined as

$$u_{\rm t} = \sqrt{\overline{u_x(r,t)^2}} = \sqrt{\overline{u_z(r,t)^2}}. \tag{12}$$

We consider $\Delta(\omega/ku_k)$ to be of a Gaussian form (see Musielak et al. 1994 for justification)

$$\Delta\left(\frac{\omega}{ku_k}\right) = \frac{4}{\sqrt{\pi}} \frac{\omega^2}{|ku_k|^3} {\rm e}^{-(\frac{\omega}{ku_k})^2}, \tag{13}$$

and call it the modified Gaussian frequency factor.

All wave energy spectra calculated in this paper are obtained by using Eq. (7) with the extended Kolmogorov energy spectrum (Eq. (10)) and the modified Gaussian frequency factor (Eq. (13)). The wave energy fluxes, $F_{\rm T}$ [in units of erg cm^{-2} s^{-1}], are computed by integrating the spectra over the wave frequency domain.

3. Results and discussion

In this paper, we present the wave energy spectra and fluxes for a single magnetic flux tube embedded in the convection zone of late-type stars of different effective temperatures ($T_{\rm eff}$) and gravities (g). This means that the filling factor, representing the number of magnetic flux tubes per unit area of the stellar surface, will not be discussed here.

3.1. Magnetic flux tubes in stellar convection zones

The basic formula describing the efficiency of the generation of transverse tube waves in stellar convection zones is given by Eq. (7). We need to know the structure of the background medium outside and inside the tube embedded in an atmosphere of a star of given $T_{\rm eff}$ and g. To calculate the structure of the external non-magnetized medium that surrounds the tube, we use a version of stellar envelope computer code described by Bohn (1984) and later modified by Theurer (1993) and Ulmschneider et al. (1996). The code is based on the mixing-length description of convection, takes into account the formation of hydrogen molecules, and treats radiation transport in the gray LTE approximation; for discussion of the validity of this approximation see Musielak et al. (1994) and Ulmschneider et al. (1999).

To run the code, we must specify $T_{\rm eff}$, g, and the mixing-length parameter $\alpha = l_{\rm mix}/H$, where $l_{\rm mix}$ is the mixing-length. A value of $\alpha = 2$ seems to be indicated by time-dependent hydrodynamic simulations of stellar convection (see, for example, Steffen 1992; Trampedach et al. 1997) as well as by a careful fitting of evolutionary tracks of the Sun with its present luminosity, effective temperature and age (Hünsch & Schröder 1997; Schröder & Eggleton 1996). Most results presented in this paper are obtained for $\alpha = 2$ but we have also calculated the wave energy fluxes for $\alpha = 1$ and 1.5 for comparison (see Sect. 3.2).

For a given value of α, the code gives a model of a stellar convection zone, and convective velocity in this model is identified with the turbulence velocity $u_{\rm t}$ used in Eq. (10). As already shown by Musielak et al. (2000, see their Fig. 1), there is a small range of $T_{\rm eff}$ for which the convective velocity becomes comparable (for stars with

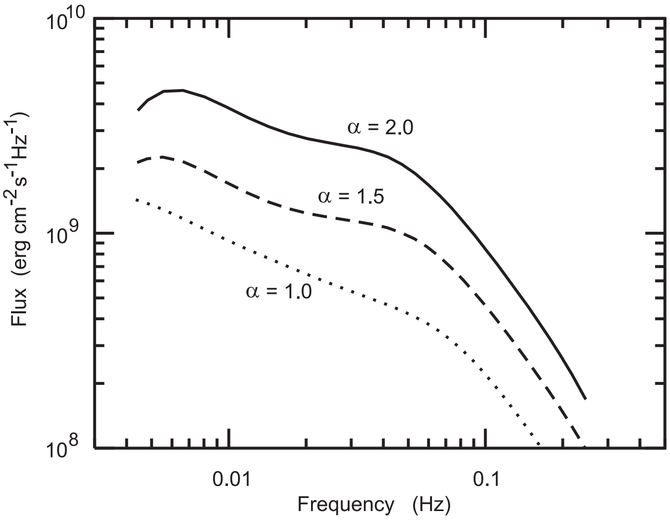

Fig. 1. Transverse wave energy spectra are plotted for three different stellar convection zones models obtained with the mixing-length parameter $\alpha = 1$, 1.5 and 2. The spectra were computed for a star with $T_{\rm eff} = 6000$ K and log $g = 4$, and in all these calculations the magnetic field strength at $\tau_{5000} = 1$ was assumed to be $B_{\rm o}/B_{\rm eq} = 0.85$.

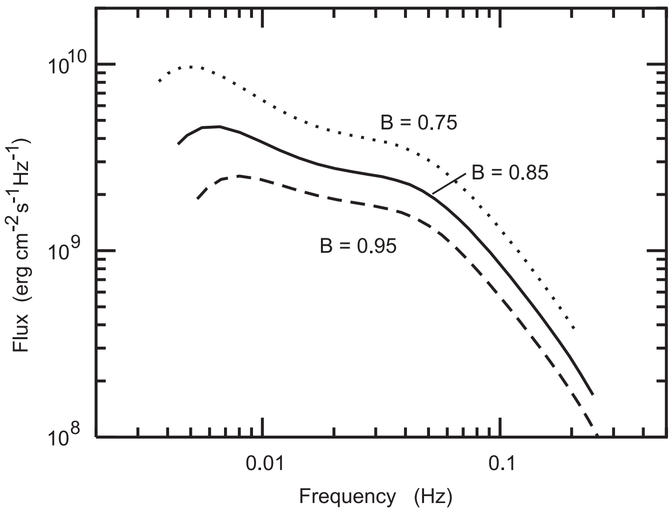

Fig. 2. Transverse wave energy spectra are plotted for three different values of the magnetic field strength at $\tau_{5000} = 1$, namely, $B = B_{\rm o}/B_{\rm eq} = 0.75$, 0.85 and 0.95. The spectra were computed for a star with $T_{\rm eff} = 6000$ K and log $g = 4$, and in all these calculations the stellar convection zone model was obtained by using the mixing-length parameter $\alpha = 2$.

log $g = 4$) to, or even higher (for stars with log $g = 3$) than, the local sound speed. Note that our analytical method of the generation of transverse tube waves can only be used with confidence when the convective velocities are lower than the local sound speed. We give special attention to this problem in Sect. 3.3.

To calculate the distribution of physical parameters with depth inside the tube, we need to specify the strength of the magnetic field of the tube. The results obtained in Paper I showed that the efficiency of the generation of transverse tube waves is sensitive to the magnetic field strength in the tube and increases when the field strength decreases (see Fig. 3 in Paper I). Hence, the problem of choosing the correct magnetic field strength of the tube is important and it is unfortunate that its value can neither be determined from observations nor from numerical computations. However, based on the solar observations it is known that the field strength inside magnetic flux tubes is $B_{\rm o} = 1500$ G (e.g., Solanki 1993), which corresponds to $B_{\rm o} = 0.85\ B_{\rm eq}$ with $B_{\rm eq}$ being the equipartition field. For late-type stars considered in this paper, we take three different values of the magnetic field strength, namely, $B_{\rm o}/B_{\rm eq} = 0.75$, 0.85 and 0.95, and calculate both the wave energy spectra and the corresponding fluxes for all these values. Note that the field strength inside stellar magnetic flux tubes is specified at the atmospheric depth corresponding to $\tau_{5000} = 1$, and its increase with depth is determined by the horizontal pressure balance (see Paper I).

3.2. Dependence on physical parameters

The wave energy spectra for transverse tube waves generated in stellar convection zones are calculated by using Eq. (7) and the convection zone models described in the previous subsection. Since this equation is only valid for the propagating waves, we assume that the wave frequency domain extends from 1.002 Ω_k to 57 Ω_k. To compute the wave energy fluxes, we perform the Laguerre integration of Eq. (7) over ω by using 32 points within the frequency domain. We begin our presentation of the obtained results by showing the dependence of the calculated wave energy spectra and fluxes on the parameter α and the magnetic field strength $B_{\rm o}$ for a chosen star.

The dependence of the wave energy spectra on the mixing-length parameter α is shown in Fig. 1. The presented results were obtained for a star with $T_{\rm eff} = 6000$ K, log $g = 4$ and $B_{\rm o} = 0.85\ B_{\rm eq}$, and for $\alpha = 1$, 1.5 and 2. Since the convective velocities increase with α, the amount of wave energy generated for a given frequency also increases. As seen in Fig. 1, the shape of the calculated spectrum does not change much with α. The maximum of the spectrum computed with $\alpha = 1$ is near the cutoff frequency for these waves, and for higher values of α shifts toward higher frequencies.

Table 1. Average transverse tube wave fluxes, $F_{\rm T}$ (erg cm^{-2} s^{-1}) for a star with $T_{\rm eff} = 6000$ K and log $g = 4$.

α	$B_{\rm o}/B_{\rm eq}$	$F_{\rm T}$
1.0	0.85	6.2×10^7
1.5	0.85	1.3×10^8
2.0	0.85	2.6×10^8
2.0	0.75	4.1×10^8
2.0	0.95	1.7×10^8

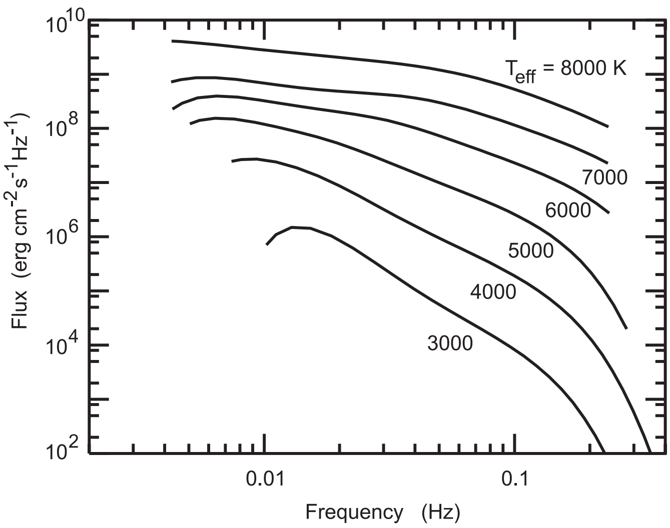

Fig. 3. Transverse wave energy spectra are plotted for stars of different spectral types and log $g = 5$. The presented results were obtained by taking the mixing-length parameter $\alpha = 2$ and by assuming that $B_{\rm o}/B_{\rm eq} = 0.85$ at $\tau_{5000} = 1$.

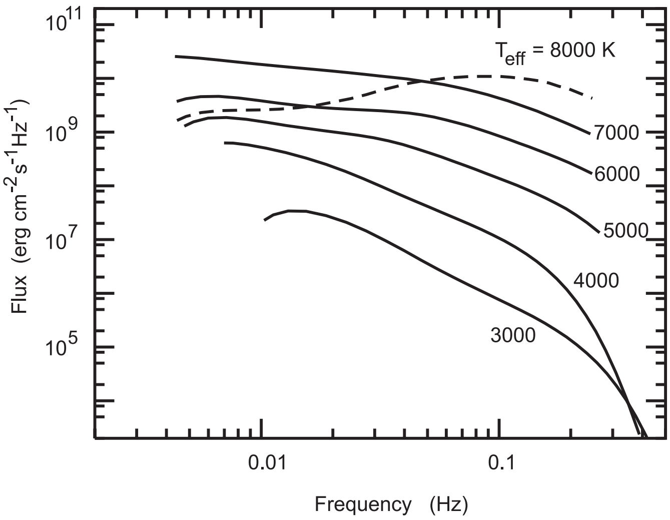

Fig. 4. Same as Fig. 3 but for log $g = 4$.

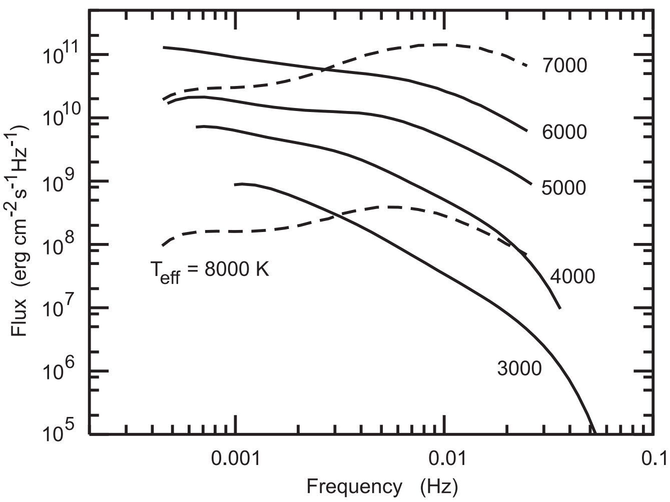

Fig. 5. Same as Fig. 3 but for log $g = 3$.

We also computed the wave energy spectra for the same star with $\alpha = 2$ but considered three different cases: $B_{\rm o}/B_{\rm eq} = 0.75$, 0.85 and 0.95. The obtained results are plotted in Fig. 2. It is seen that all spectra have similar shapes but their maxima are shifted toward higher frequencies for stronger magnetic field strength; this effect is caused by the dependence of the cutoff frequency on the magnetic field strength (see Eq. (3)). This decrease of the total wave energy flux with increasing field strength $B_{\rm o}/B_{\rm eq}$ is caused by increasing stiffness of the tube against external perturbations; a more detailed discussion of this effect is given in Paper I as well as in our two previous papers (Musielak et al. 2000; Ulmschneider et al. 2001b).

The dependence of the computed wave energy fluxes on the mixing-length parameter, α, is given in Table 1. Based on these results, we find that the dependence of $F_{\rm T}$ on α can be approximated by

$$F_{\rm T} \approx 6.2 \times 10^{7} \alpha^{2.1} \text{ erg cm}^{-2} \text{ s}^{-1}. \tag{14}$$

The derived α-dependence is similar to that found for the rate of the transverse wave generation in the Sun (see Paper I), however, it is higher than the value of 1.8 found for *longitudinal tube waves* by Musielak et al. (1995) and much lower than the value of 3.8 obtained by Musielak et al. (1994) for the *acoustic wave generation.*

Table 1 also shows that the total wave energy flux F_T decreases with increasing magnetic field strength $B = B_{\rm o}/B_{\rm eq}$. The dependence of F_T on B can approximately be fitted by the following expression:

$$F_{\rm T} \approx 1.5 \times 10^{8} B^{-3.5} \text{ erg cm}^{-2} \text{ s}^{-1}. \tag{15}$$

This dependence is practically identical to that found for the Sun in Paper I; this means that the dependence of the transverse wave generation on the magnetic field strength does not change when $T_{\rm eff}$ increases and it is also not very sensitive to changes in gravity. It is interesting to compare this dependence to that obtained for *longitudinal tube waves.* The analytical results give the value of -9.4 for the exponent (see Musielak et al. 2000), however, the value of this exponent for the numerical results is -7.2 (Ulmschneider et al. 2001b); these authors also showed that the dependence of longitudinal wave energy fluxes on the magnetic field is very similar for stars of different gravities. In both cases, it is seen that the process of excitation of transverse tube waves does not depend as strongly on the magnetic field strength as the process of generation of longitudinal tube waves.

3.3. Wave energy spectra for late-type stars

Typical stellar wave energy spectra calculated for stars of different spectral types but similar gravity are shown in Figs. 3–5. All spectra were computed by taking $\alpha = 2$ and $B_{\rm o}/B_{\rm eq} = 0.85$. By comparing the spectra obtained for

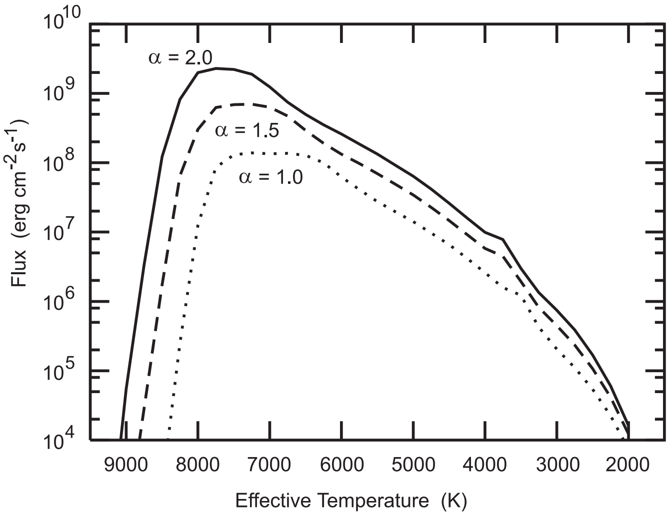

Fig. 6. Transverse wave energy fluxes computed for stars of different spectral types are plotted for three different values of the mixing-length parameter $\alpha = 1$, 1.5 and 2. The fluxes were obtained for stars with log $g = 4$ and for the magnetic field strength $B_{\rm o}/B_{\rm eq} = 0.85$ at $\tau_{5000} = 1$.

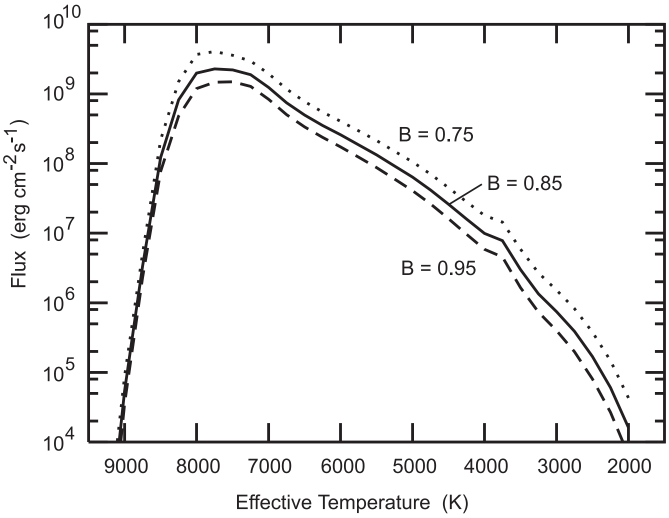

Fig. 7. Transverse wave energy fluxes computed for stars of different spectral types are plotted for three different values of the magnetic field strength: $B = B_{\rm o}/B_{\rm eq} = 0.75$, 0.85 and 0.95. The results were obtained for stars with log $g = 4$ and for the mixing-length parameter $\alpha = 2$.

stars with different gravities, one may draw four general conclusions. First, for a given gravity the amount of energy carried by transverse tube waves of the same frequency is much higher for hot stars than for cool ones; this can easily be explained by the higher convective velocities in hot than in cool stars. Second, the shapes of the computed spectra seem to be similar for stars of different gravity with the exception of three cases plotted as dashed lines in Figs. 4 and 5. These cases represent spectra obtained with the turbulent velocities comparable to (or even higher than) the local sound speed, which means that they are not realistic because our theory is only valid for subsonic turbulent motions. Third, all spectra peak at the low frequency limit. While being relatively flat at high $T_{\rm eff}$, they become much steeper towards lower $T_{\rm eff}$. The effect can be explained by the dependence of the cutoff frequency, Ω_k and the factor $(1-\Omega_k^2/\omega^2)^2$ on the tube magnetic field and the scale height in different stars; this factor was introduced in Paper I to remove non-propagating waves from the computed wave energy spectra. Fourth, the transverse wave energy spectra extend roughly over one to two orders of magnitude above the cut-off frequency. Finally, it is interesting to note that all above conclusions (expect the last one) are very similar to those reached by Musielak et al. (2000) for the generation of longitudinal tube waves.

3.4. Wave energy fluxes for late-type stars

The dependence of the calculated fluxes on the mixing-length parameter α and the magnetic field strength, $B_{\rm o}$ for late-type stars of different $T_{\rm eff}$ and log $g = 4$ is shown in Figs. 6 and 7, respectively. Figure 6 shows that the wave energy flux increases with increasing α and that the increase is especially prominent for stars with $T_{\rm eff} \geq$ 6000 K. For $T_{\rm eff} <$ 7500 K the dependence of the wave energy fluxes on α increases with increasing $T_{\rm eff}$. These effects are due to the increase of the convective velocity with α and $T_{\rm eff}$. In Fig. 7 it is seen that the wave energy flux increases with decreasing magnetic field strength and stars of different $T_{\rm eff}$ are equally affected by this effect. As noted above this is due to the decreasing stiffness of the tube for smaller $B_{\rm o}/B_{\rm eq}$.

Figure 8 shows the total wave energy fluxes computed for late-type stars of different gravities and effective temperatures; all results presented in this figure were obtained by taking $\alpha = 2$ and $B_{\rm o}/B_{\rm eq} = 0.85$. The rising part of the flux curves at the left-hand side of this figure indicates that the efficiency of stellar convection strongly increases in these hot stars. For lower gravity this rise shifts to lower $T_{\rm eff}$ because giants have lower gas pressures at their surfaces, which permits the onset of hydrogen ionization and thus the efficient convection occurs at lower temperatures. Except for these parts of the diagram, it is seen that the stars with lower gravities but identical $T_{\rm eff}$ generate more transverse wave energy. In addition for given gravity, the transverse wave energy fluxes are seen to increase rapidly with greater $T_{\rm eff}$. The maximum wave fluxes are similar for stars of different gravities and occur for the hottest stars (A to early F-stars) just before stellar convection becomes inefficient towards higher effective temperature. Some selected total transverse wave energy fluxes for late-type stars are also given in Table 2.

3.5. Comparison with previous work

According to Muller (1989) and Muller et al. (1994), the amount of wave energy generated by the observed proper motions of footpoints of magnetic flux tubes in the solar photosphere is large enough to account for the observed

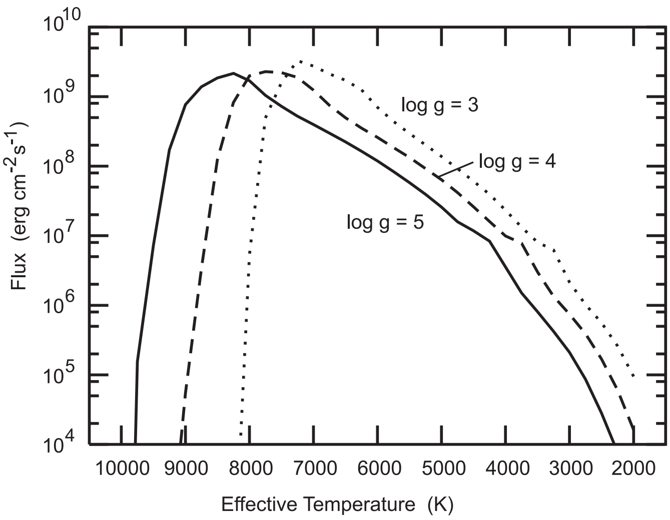

Fig. 8. Transverse wave energy fluxes computed for stars of different spectral types and different gravities. The fluxes were obtained for the mixing-length parameter $\alpha = 2$ and for the tube magnetic field $B_{\rm o}/B_{\rm eq} = 0.85$ at $\tau_{5000} = 1$.

Table 2. Average transverse tube wave fluxes, $F_{\rm T}$ (erg cm^{-2} s^{-1}) for selected late-type stars with $\alpha = 2$ and $B_{\rm o}/B_{\rm eq} = 0.85$.

log g	$T_{\rm eff}$	$F_{\rm T}$
5.0	3000	2.1×10^5
5.0	5000	2.6×10^7
5.0	7000	4.0×10^8
5.0	9000	7.7×10^8
4.0	3000	7.5×10^5
4.0	5000	6.4×10^7
4.0	7000	1.2×10^9
4.0	9000	5.5×10^4
3.0	3000	2.1×10^6
3.0	5000	1.4×10^8
3.0	7000	2.7×10^9

radiative losses from the outer layers of the solar atmosphere. Analytical methods have been used by Choudhuri et al. (1993a,b) to investigate the efficiency of excitation of transverse tube waves by rapid foot point motions of magnetic flux tubes. They argue that occasional rapid motions can account for the entire energy flux needed to heat the quiet regions of the solar corona. Their main finding is that pulses are more efficient in supplying the energy to the solar corona than continuously excited tube waves.

Numerical results obtained by Huang et al. (1995) have shown that the efficiency of generation of *nonlinear transverse waves* at the top of the solar convection zone is high and that the amount of energy carried by these waves (approximately 10^9 erg/cm^2 s) is comparable to that estimated by Muller et al. (1994) based on their observational data.

In Paper I we found that the *transverse wave energy flux for the Sun* is approximately 10^8 erg/cm^2 s, which is one order of magnitude lower than numerically evaluated fluxes. The main reason for this difference is that our analytical approach does not allow for large amplitude motions observed on the Sun at the photospheric level (Muller 1989; Nesis et al. 1992; Muller et al. 1994) and also seen in time-dependent numerical simulations of solar convection (e.g., Nordlund & Dravins 1990; Nordlund & Stein 1991; Cattaneo et al. 1991; Steffen 1993; see also Nordlund et al. 1997). In addition, the analytical approach includes the correlation effects, which occur when the tube is shaken over a significant portion of its length, and are likely to reduce the efficiency of the generation of transverse tube waves.

Since there are no other calculations of transverse wave energy fluxes for late-type stars, we now compare the computed fluxes to previously calculated stellar wave energy fluxes carried by *longitudinal tube waves*. We begin with a comparison with the results obtained by Musielak et al. (2000), who used a similar analytical method to calculate *(linear) longitudinal wave energy spectra and fluxes* for stars with the same physical parameters as those considered in this paper. The comparison is given in Fig. 9. It shows that the energy carried by transverse tube waves is more than one order of magnitude larger than that carried by longitudinal tube waves. This conclusion is valid independently of $T_{\rm eff}$, with the tendency that the difference between the two fluxes is larger for cool stars. The obtained results are consistent with those given in Paper I for the Sun.

Finally, we compare our results with the *numerical longitudinal wave energy fluxes* of Ulmschneider et al. (2001b), who used an approach developed by Ulmschneider & Musielak (1998). These fluxes were obtained for late-type stars with $T_{\rm eff}$ ranging from 3500 K to 7000 K and gravities in the range log g = 3–5 (see Fig. 9). In their approach, the pressure fluctuations produced by the external turbulent motions are responsible for the generation of longitudinal tube waves. The fluctuations are described by a superposition of many partial waves with amplitudes determined by the turbulent flow and with random phases. Occasionally, this superposition produces large amplitudes fluctuations which excite *nonlinear longitudinal wave pulses.*

Clearly, the fluxes carried by nonlinear longitudinal tube waves are not only higher than those carried by linear longitudinal tube waves but they are also higher than the transverse wave energy fluxes calculated in this paper (see Fig. 9). The main reason for these differences is the fact that the analytical methods are restricted to linear waves and, in addition, they take into account the correlation effects, which significantly reduce the computed fluxes. As a result, the analytically calculated wave energy fluxes must be regarded as only lower bounds for the realistic amount of the wave energy generated in stellar convection zones.

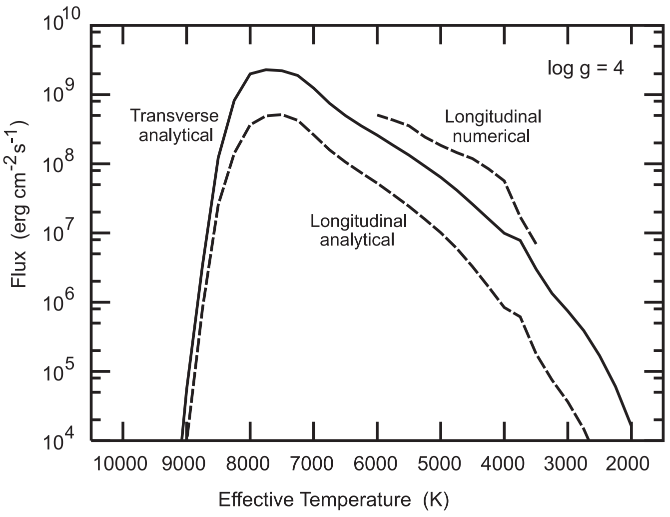

Fig. 9. Comparison between transverse and longitudinal wave energy fluxes computed for stars of different spectral types and $\log g = 4$. The fluxes were obtained for the mixing-length parameter $\alpha = 2$ and for the tube magnetic field $B_{\rm o}/{\rm B}_{\rm eq} = 0.85$ at $\tau_{5000} = 1$.

4. Conclusions

From our analytical studies of the generation of transverse tube waves propagating along a single magnetic flux tube embedded in atmospheres of different late-type stars, the following conclusions can be drawn.

1. The shapes of the computed wave energy spectra are similar for stars of different effective temperatures and gravities.

2. The spectra are not very sensitive to the strength of stellar magnetic fields but they do depend on the mixing-length parameter α. The maxima of the spectra for $\alpha = 1$ are near the cutoff frequency for transverse tube waves, while for $\alpha = 2$ they are a factor of two higher.

3. For higher values of the mixing-length parameter α, the total wave energy fluxes significantly increase and this effect is more prominent for hot stars than for cool ones. Increasing the magnetic field strength decreases these fluxes.

4. For stars with efficient convection, that is later than early F-stars, the total wave energy fluxes increase with higher effective temperature and with lower gravity. The maximum generated wave energies are similar for all gravities, and they occur in the range of the early F-stars.

5. Due to our analytical approach the obtained linear transverse wave energy fluxes are likely to represent only lower bounds for the realistic transverse wave energy fluxes generated in stellar atmospheres.

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