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Excitation of transverse magnetic tube waves in stellar convection zones

I. Analytical approach

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Abstract. Analytical treatment of the excitation of transverse magnetic tube waves in stellar convection zones is presented. The waves are produced by the interaction between thin and vertically oriented magnetic flux tubes embedded in stellar convection zones and the external turbulent motions. A general theory describing this interaction is developed and used to compute the wave energy spectra and fluxes for the Sun.

Key words. analytical methods – stars: chromosphere – stars: corona – stars: magnetic fields – MHD – linear waves

1. Introduction

The most prominent sources of the non-radiative energy required to heat stellar chromospheres and coronae, and to accelerate stellar winds, are obviously stellar convection zones. The fact that the wave energy is generated by the turbulent convection has been known for many years (e.g., Musielak 1990, with references to work going back some 50 years). In non-magnetic regions of stellar convection zones, the non-radiative energy is carried mainly by acoustic waves. The efficiency of excitation of these waves can be calculated by using the Lighthill-Stein theory of sound generation (Lighthill 1952; Stein 1967). In magnetic regions, which are typically associated with stellar magnetic flux tubes, three different types of tube waves can be generated: longitudinal, transverse and torsional (e.g., Spruit 1982). Physical properties of free longitudinal, transverse and torsional oscillations of magnetic flux tubes have been extensively studied in the literature (e.g., Roberts & Ulmschneider 1997). An analytical treatment based on the original Lighthill approach has been developed by Musielak et al. (1989, 1995) to describe the efficiency of generation of longitudinal tube waves in stellar convection zones. The main aim of this paper is to present an analytical treatment describing the generation of transverse tube waves. In principle, a similar approach can be developed to compute the exicitation of torsional tube waves, however, this is beyond the scope of the present paper.

Our recent work has shown that the description of turbulence given in the Lighthill-Stein theory is physically unjustified (Musielak et al. 1994). As a result, previously calculated stellar acoustic wave energy fluxes (e.g., Renzini et al. 1977; Bohn 1981, 1984) are *incorrect*. We have corrected the Lighthill-Stein theory by incorporating an improved description of the spatial and temporal spectrum of the turbulent convection and used this corrected theory to calculate new acoustic wave energy spectra and fluxes for late-type stars (Ulmschneider et al. 1996, 1999). These fluxes have been used by us to predict theoretically the level of the observed "basal flux" (Buchholz et al. 1998; Cuntz et al. 1999). This is an important result because it is the first time that the "basal flux" for late-type dwarfs has been predicted purely theoretically. The result also shows that non-magnetic regions of stellar atmospheres are heated by acoustic waves generated in stellar convection zones.

We have also incorporated the isolated small-scale magnetic (flux tube) structures (Stenflo 1978; Solanki 1993) into the theory of wave generation. The first calculations of this sort by Musielak et al. (1989) evaluated the energy flux carried by longitudinal tube waves from the solar convective zone, where the interaction between flux tubes and turbulent motions takes place, up into the overlying atmosphere, where the dissipation of the wave energy occurs. From that computation it became clear that longitudinal tube waves may play an important role

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in the heating of magnetically active regions of stellar atmospheres. Since then, we have significantly improved our analytical treatment of the generation of longitudinal tube waves (Musielak et al. 1995) and recently used it to compute the wave energy spectra and fluxes for a large number of late-type dwarfs and subgiants (Musielak et al. 2000). The analytical approach is restricted to linear waves but takes into account the cancellation and amplification (correlation) effects, which occur when magnetic flux tubes are excited at many points along their length.

The analytical approach has been supplemented by a numerical treatment that allows considering nonlinear waves but cannot incorporate the correlation effects (Ulmschneider & Musielak 1998). The latter approach has also been used by Ulmschneider et al. (2000) to calculate the amount of energy carried by longitudinal tube waves in atmospheres of non-solar late-type dwarfs and subgiants. Because of the difference in these two approaches, the analytically and numerically calculated fluxes can be seen as lower and upper bounds for the realistic stellar wave energy fluxes. Cuntz et al. (1998, 1999) have used such theoretical fluxes as the initial input to calculate the resulting heating in magnetic regions of their stellar chromospheric models.

As already discussed by Musielak et al. (1989), the amount of energy carried by longitudinal tube waves is much smaller than that carried by transverse tube waves. The fact that it is much easier to generate transverse tube waves than longitudinal ones has already been recognized by Spruit & Roberts (1983). There have been several attempts to estimate the amount of energy carried by these waves in the solar atmosphere. Various authors have based their estimates on observational data (Muller et al. 1989, 1994), others have used analytical (Choudhuri et al. 1993a,b) or numerical (Huang et al. 1995) methods. The obtained results (Ulmschneider & Musielak 1998) show that transverse tube wave are generated 30 or even 40 times more efficiently than longitudinal tube waves. Clearly, there is a lot of extra wave energy that is not accounted for in our current theoretical models. An important point is that this extra energy is carried by transverse waves, which are more difficult to damp than longitudinal tube waves (Narain & Ulmschneider 1991, 1996, also Ulmschneider et al. 1991) and, therefore, they can transfer the energy to much higher layers of stellar atmospheres. Obviously, some of the energy carried by these waves will leak to the external medium (e.g., Huang et al. 1999) where it will enhance the acoustic heating. Thus, it is essential to know the amount of energy carried by transverse tube waves in stellar atmospheres and include this energy in our theoretical models.

In this paper, we focus on the problem of the excitation of transverse tube waves by external convective turbulence. We assume that magnetic flux tubes are embedded in a magnetic field-free, turbulent and compressible medium, and that the waves are generated only by the external turbulent motions. In order to solve the problem analytically, we assume that the tubes are thin and vertically oriented. This assumption allows us to formally separate the generation of compressible tube waves considered by Musielak et al. (1989, 1995) from the incompressible tube waves discussed here. For non-vertical flux tubes, the generated waves would have both compressible and incompressible components and the coupling and energy transfer between these components would have to be accounted for. In this paper, we develop a general theory of the generation of purely transverse tube waves, which means that non-vertical flux tubes are not considered. The developed theory is then used to compute the resulting wave energy spectra and fluxes for the Sun. Nonsolar wave energy spectra and fluxes will be presented in a succeeding paper.

In Sect. 2 the derivation and solution of the inhomogeneous wave equation are outlined while Sect. 3 presents the calculated wave energy spectra and fluxes, and provides a discussion of the obtained results. Final remarks and conclusions are given in Sect. 4.

2. Inhomogeneous wave equation and its solution

2.1. Tube model

We consider an isolated magnetic flux tube embedded in a magnetic field-free, turbulent, compressible and isothermal medium. The tube is assumed to be thin, untwisted, and vertically oriented, with circular cross-section, and in temperature equilibrium with its surroundings. In addition, any distortions of the shape of the tube cross-section caused by the external force are neglected here by averaging physical quantities inside the tube over its cross-section (e.g., Spruit 1981). We also assume that transverse tube waves are excited by the external turbulence alone and that there are no other motions outside or inside the tube. Our approach is steady-state and restricted to the case of subsonic turbulence, for which $M (= u_{\rm t}/c_{\rm S}) < 1$, where M is the Mach number, and $u_{\rm t}$ and $c_{\rm S}$ are the rms turbulent velocity and sound speed, respectively.

To describe the transverse oscillations of the tube, we introduce a Cartesian coordinate system and assume that the axis of a non-oscillating tube is oriented along the z-axis. The gravitational acceleration assumed to be uniform and given by $\boldsymbol{g} = -g\hat{\boldsymbol{z}}$. We also consider a local cylindrical coordinate system $(r, \phi, \boldsymbol{l})$ within the tube, with $\boldsymbol{l}$ the vector along the tube (e.g., Spruit 1981). The tube magnetic field, $\boldsymbol{B}_o$, can then be expressed as $\boldsymbol{B}_o = B_o (r, \phi, \boldsymbol{l})\hat{\boldsymbol{l}}$, where $\hat{\boldsymbol{l}}$ is the unit vector along the tube; note that for non-oscillating tubes, $\hat{\boldsymbol{l}} = \hat{\boldsymbol{z}}$. In order to distinguish physical parameters inside and outside the tube, we introduce subscripts o and e to denote the internal and external parameters, respectively.

Because the tube is vertically oriented, the generation of longitudinal and transverse tube waves can be separated. In this paper, we consider the latter case (see Musielak et al. 1989 and 1995, for the former case), in which the waves are fully described by the perturbations of the tube velocity, $\boldsymbol{v}(z,t) = v_x(z,t)\hat{\boldsymbol{x}}$, and of the magnetic field, $\boldsymbol{b}(z,t) = b_x(z,t)\hat{\boldsymbol{x}}$; this restriction to oscillations in the x-direction is made without any loss of generality since there is no physical distinction between the x and y directions. Thus, the total magnetic field in the Cartesian coordinate system is given by $\boldsymbol{B}_o = B_{oz}\hat{\boldsymbol{z}} + b_x\hat{\boldsymbol{x}}$, with $b_x/B_{oz} = l_x$. For purely transverse tube waves, there is no variation in the tube cross section, and therefore the density (and pressure) perturbations can be neglected in the set of ideal MHD equations; the density ρ (and pressure p) within the tube can thus be replaced by their equilibrium values ρ_o (and p_o). In addition, the MHD equations can be further simplified by the two conditions $\nabla \cdot \boldsymbol{v} = 0$ and $(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = 0$; the first condition describes the incompressibility assumption and the second one is valid because we consider small-amplitude (linear) waves.

2.2. Equation of motion

The momentum equation for a tube oscillating with the velocity $\boldsymbol{v}$ can be written in the following form:

$$\rho_o \frac{\partial \boldsymbol{v}}{\partial t} = -\nabla \left(p_o + \frac{B_o^2}{8\pi} \right) + \frac{1}{4\pi}(\boldsymbol{B}_o \cdot \nabla)\boldsymbol{B}_o + \rho_o \boldsymbol{g}, \tag{1}$$

with the gas pressure, p_o, density, ρ_o, and magnetic field, $\boldsymbol{B}_o$, determined only within the tube.

Since there are turbulent motions outside the tube and their velocity is $\boldsymbol{u}$, the corresponding momentum equation is

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} = -\nabla p + \rho \boldsymbol{g}, \tag{2}$$

where ρ is the total external density, defined here as $\rho = \rho_e + \delta\rho_{\rm t} + \delta\rho_{\rm w}$, with ρ_e being the external gas density, and $\delta\rho_{\rm t}$ and $\delta\rho_{\rm w}$ represent density fluctuations caused by the turbulence and by the external acoustic waves, respectively. Similarly, for the total external pressure we have $p = p_e + \delta p_{\rm t} + \delta p_{\rm w}$. In general, both $\delta\rho_{\rm t}$ and $\delta p_{\rm t}$ may not be necessarily small when compared to ρ_e and p_e. Our approach, however, is restricted to subsonic turbulence, therefore, our analysis is constrained by the inequalities $< \delta\rho_{\rm t}\delta\rho_{\rm t} >_{\rm time}^{1/2} << \rho_e$ and $< \delta p_{\rm t}\delta p_{\rm t} >_{\rm time}^{1/2} << p_e$.

The external acoustic waves described by the terms with $\delta\rho_{\rm w}$ and $\delta p_{\rm w}$ can be excited by either the stresses exerted by the horizontal oscillations of the tube on its immediate surroundings (e.g., Huang 1996) or the external turbulent motions (Lighthill 1952; Stein 1967; Musielak et al. 1994); these waves may interact with the tube and lead to the excitation of transverse tube waves. According to Spruit (1982), the former process is of a very low efficiency for thin magnetic flux tubes and, therefore, it can be neglected (see also Huang et al. 1999). The problem of the acoustic wave generation in a convectively unstable and non-magnetized fluid has been solved only in the limit of weak wave motions that do not produce any backreaction on the turbulent flow. This means that the computed acoustic waves would only have minor effects on the tube especially when compared to the effects caused by the turbulent motions. As a result, in this paper, we will focus only on the interaction of flux tubes and turbulence, and neglect the influence of external acoustic waves on the tube by taking $\delta\rho_{\rm w} << \delta\rho_{\rm t}$ and $\delta p_{\rm w} << \delta p_{\rm t}$.

Using the above approximations, we write

$$(\rho_e + \delta\rho_{\rm t})\frac{\partial \boldsymbol{u}}{\partial t} = -\nabla(p_e + \delta p_{\rm t}) + (\rho_e + \delta\rho_{\rm t})\boldsymbol{g}. \tag{3}$$

The momentum equation for the tube and for the external medium are related to each other by the horizontal pressure balance

$$p_o + \frac{B_o^2}{8\pi} = p_e + \delta p_{\rm t}. \tag{4}$$

Combining Eqs. (1), (3) and (4), we obtain

$$\rho_o \frac{\partial \boldsymbol{v}}{\partial t} = \frac{1}{4\pi}(\boldsymbol{B}_o \cdot \nabla)\boldsymbol{B}_o + (\rho_o - \rho_e)\boldsymbol{g} + \rho_e \frac{\partial \boldsymbol{u}}{\partial t} + \delta\rho_{\rm t}\left(\frac{\partial \boldsymbol{u}}{\partial t} - \boldsymbol{g}\right). \tag{5}$$

Since we are only interested in transverse motions of the tube, we consider the perpendicular component of Eq. (5) and calculate the total force that restores the tube's equilibrium. Following Spruit (1981), we obtain

$$\rho_o \left(\frac{\partial \boldsymbol{v}}{\partial t}\right)_\perp = \rho_e \left(\frac{\partial \boldsymbol{u}}{\partial t}\right)_\perp + \frac{1}{4\pi}[\hat{\boldsymbol{l}} \times (\boldsymbol{B}_o \cdot \nabla)\boldsymbol{B}_o] \times \hat{\boldsymbol{l}} + (\rho_o - \rho_e)(\hat{\boldsymbol{l}} \times \boldsymbol{g}) \times \hat{\boldsymbol{l}} + \delta\rho_{\rm t}\left(\frac{\partial \boldsymbol{u}}{\partial t}\right)_\perp - \delta\rho_{\rm t}(\hat{\boldsymbol{l}} \times \boldsymbol{g}) \times \hat{\boldsymbol{l}}. \tag{6}$$

Defining $(\boldsymbol{v})_\perp = v_x$ and $(\boldsymbol{u})_\perp = u_x$, we may write Eq. (6) as

$$\rho_o \frac{\partial v_x}{\partial t} = \frac{B_o^2}{4\pi} k_x + (\rho_o - \rho_e) g l_x l_z + \rho_e \frac{\partial u_x}{\partial t} + \delta\rho_{\rm t}\left(\frac{\partial u_x}{\partial t} - g l_x l_z\right), \tag{7}$$

where the horizontal and vertical components of $\boldsymbol{l}$ are given by

$$l_x = \frac{\partial \xi}{\partial z} = \left(\frac{\partial}{\partial t}\right)^{-1} \frac{\partial v_x}{\partial z}, \tag{8}$$

and $l_z = 1$. In addition, the horizontal and vertical components of the curvature vector $\boldsymbol{k} = \partial\hat{\boldsymbol{l}}/\partial l$ are given by

$$k_x = \frac{\partial^2 \xi}{\partial z^2} = \left(\frac{\partial}{\partial t}\right)^{-1} \frac{\partial^2 v_x}{\partial z^2}, \tag{9}$$

and $k_z = 0$.

In the derived equation of motion (Eq. (7)) two different velocities (v_x and u_x) appear. Clearly, we need to consider the continuity of displacement at the tube surface to find a relationship between these velocities. For harmonic disturbances of the boundary, this constraint is of the form $\boldsymbol{v}/(\omega - \boldsymbol{k}\cdot\boldsymbol{v}_\parallel) = \boldsymbol{u}/\omega$, where ω is the frequency of a perturbation of the boundary, k is the corresponding wave number, and $\boldsymbol{v}_\parallel$ is the velocity of the internal fluid parallel to the boundary. In our case, $\boldsymbol{v}_\parallel = 0$, so that the constraint really is $\boldsymbol{v} = \boldsymbol{u}$. However, it must be noted that the direction of $\boldsymbol{v}$ and $\boldsymbol{u}$ is different since the former is directed toward the tube axis when the tube is bent, but the latter is acting in the opposite direction leading to bending the tube. Therefore, the velocities normal to the interface within and outside the tube can be written as $(\boldsymbol{v})_\perp = -(\boldsymbol{u})_\perp$ or equivalently as $v_x = -u_x$, which also gives $l_x = l_{\rm ox} = -l_{\rm ex}$.

Thus, we write Eq. (7) as

$$(\rho_o + \rho_e)\frac{\partial^2 v_x}{\partial t^2} - \frac{B_o^2}{4\pi}\frac{\partial^2 v_x}{\partial z^2} + (\rho_o - \rho_e) g \frac{\partial v_x}{\partial z} = \left(\frac{\partial}{\partial t}\right) \delta\rho_{\rm t} \left(\frac{\partial}{\partial t}\right)^{-1} \left(\frac{\partial^2 u_x}{\partial t^2} + g\frac{\partial u_x}{\partial z}\right). \quad (10)$$

Note that in this equation we use v_x for the first-order (wave) quantities and u_x for the second order quantities describing the external turbulence; this well-known procedure of separating the first and second-order variables was first introduced by Lighthill (1952) to treat the sound generation by turbulence. It must be also pointed out that the term $\rho_e \partial v_x/\partial t$ describes the apparent increase of inertia of the tube resulting from the backreaction of the external fluid on the oscillating tube (Basset 1961; Spruit 1981). More recently, some authors (Choudhuri 1990; Cheng 1992; Fan et al. 1994; Moreno-Insertis et al. 1996; Osin et al. 1999) have criticized the treatment of this term by Spruit and suggested several modifications. The criticism and the resulting modifications have been extensively discussed by Osin et al., who also proposed their own modification of the equation of motion. To support their analytical results, these authors have performed numerical calculations and concluded that any modifications become important only when there is a strong flow $v_\parallel \neq 0$ along the oscillating tube. Since we assume $v_\parallel = 0$ along the tube in our approach considered here, we take into account only the term originally introduced by Spruit.

2.3. Wave equation

To transform Eq. (10) into an inhomogeneous wave equation, we introduce the characteristic velocity, c_k, for transverse tube waves

$$c_k = \frac{B_{oz}}{\sqrt{4\pi(\rho_e + \rho_o)}}, \quad (11)$$

which is essentially the Alfvén velocity for transverse tube waves. Noting that $g(\rho_e - \rho_o)/(\rho_e + \rho_o) = c_k^2/2H$, and neglecting the third-order term $b_x^2\, \partial^2 v_x/\partial z^2$, we obtain

$$\left[\frac{\partial^2}{\partial t^2} - c_k^2 \frac{\partial^2}{\partial z^2} + \frac{c_k^2}{2H}\frac{\partial}{\partial z}\right] v_x(z,t) = S_x(z,t), \quad (12)$$

where the source function $S_x(z,t)$ is defined as

$$S_x(z,t) = \left(\frac{\partial}{\partial t}\right) \frac{\delta\rho_{\rm t}}{\rho_e + \rho_o} \left(\frac{\partial}{\partial t}\right)^{-1} \left(\frac{\partial^2 u_x}{\partial t^2} + g\frac{\partial u_x}{\partial z}\right) \cdot \quad (13)$$

To determine this function, we follow the standard procedure developed by Lighthill (1952) and assume that the external flow is known. Note also that in the limit of no motions outside the tube, we have $S_x = 0$, so that the wave equation obtained by Spruit (1981) for freely propagating transverse tube waves is recovered.

To remove the first derivative from the inhomogeneous wave equation, we make the following transformation

$$v_x(z,t) = \rho_o^{-1/4} v_1(z,t) \quad (14)$$

which yields

$$\left[\frac{\partial^2}{\partial t^2} - c_k^2 \frac{\partial^2}{\partial z^2} + \Omega_k^2\right] v_1(z,t) = \rho^{-1/4} S_x(z,t). \quad (15)$$

This is inhomogeneous Klein-Gordon equation for transverse tube waves and

$$\Omega_k = \frac{c_k}{4H}, \quad (16)$$

is the cutoff frequency for these waves (Spruit 1981). The condition for propagation of transverse tube waves is determined by the cutoff frequency Ω_k, which restricts these waves to be either propagating if $\omega > \Omega_k$ or evanescent for $\omega \leq \Omega_k$. The obtained cutoff frequency is "global" (the same along the entire tube) because both c_k and H are constant along the tube. The fact that $c_k = $ const along the tube reflects the decreasing of both the gas density and the tube magnetic field with height. We further note that in our model, the external medium is assumed to be isothermal, stratified, and convectively unstable (the Brunt-Vaisala frequency is negative), and therefore the medium supports only acoustic waves, which propagate for all frequencies $\omega > \Omega_{\rm S}$, where the latter is the acoustic cutoff defined by $\Omega_{\rm S} = c_{\rm S}/2H$ (Lamb 1908; Moore & Spiegel 1964). Therefore, Ω_k and $\Omega_{\rm S}$ are the only cutoffs that arise in our model. In order to compare them, we express the tube cutoff in terms of the acoustic cutoff, and obtain

$$\Omega_k = \frac{\Omega_{\rm S}}{\sqrt{2\gamma(2\beta+1)}}, \quad (17)$$

where $\beta = 8\pi p_o/B_o^2$. This result shows that transverse tube waves can propagate with frequencies much lower than external acoustic waves. The same conclusion can be drawn from a comparison between the cutoff obtained for longitudinal (Defouw 1976) and transverse (Spruit 1981) tube waves; note that the former is almost identical to the acoustic cutoff obtained for a thin and low-β flux tube with $\gamma = 5/3$. The differences between these two cutoff frequencies can in part account for the significantly broader wave energy spectra obtained here for transverse tube waves (see Sect. 3).

2.4. Source function

After deriving the inhomogeneous Klein-Gordon equation for transverse tube waves and discussing the propagation conditions for these waves, we now proceed with the calculation and discussion of the source function $S_x(z,t)$. As shown by Eq. (13), the source function depends on the external density and velocity field, and is completely determined when $\delta\rho_{\rm t}$ and u_x are known. Formally, we may use the continuity equation for the turbulence to express $\delta\rho_{\rm t}$ in terms of the turbulent velocity. This gives

$$\delta\rho_{\rm t} = \rho_e \left(\frac{\partial}{\partial t}\right)^{-1} \left(\frac{u_z}{H} - \frac{\partial u_z}{\partial z}\right). \tag{18}$$

Defining

$$S_{\rm t}(z,t) \equiv \rho^{1/4} S_x(z,t), \tag{19}$$

we write

$$S_{\rm t}(z,t) = \rho_o^{1/4} \frac{\rho_e}{\rho_o + \rho_e} \left(\frac{u_z}{H} - \frac{\partial u_z}{\partial z}\right) \times \left(\frac{\partial}{\partial t}\right)^{-1} \left(\frac{\partial^2 u_x}{\partial t^2} + g\frac{\partial u_x}{\partial z}\right). \tag{20}$$

This final expression for the source function obtained for transverse tube waves driven only by the external turbulence shows that the source term depends on both the x and z-components of the turbulent velocity. In addition, the source function depends on the term that explicitly displays the gravity and this term accounts for the effects of the fluctuating buoyancy force (Goldreich & Kumar 1988) on the wave generation (see Sect. 2.9, for more details). We emphasize that this source function describes the net generation rate, since the interaction of the tube with the external medium is already taken into account by the apparent increase of the tube inertia (see Sect. 2.2). Note also that our approach is formally not restricted to the Boussinesq approximation but instead the turbulence is treated as fully compressible.

2.5. Solution of the wave equation

Because all the coefficients in the inhomogeneous wave Eq. (15) are constant, the solution can be obtained by performing the Fourier transform in time and space. The result is

$$v_1(k,\omega) = \frac{S_{\rm t}(k,\omega)}{k^2 c_k^2 - (\omega^2 - \Omega_k^2)}, \tag{21}$$

where

$$S_{\rm t}(k,\omega) = \frac{1}{(2\pi)^2} \int\int S_{\rm t}(z,t) {\rm e}^{-i(\omega t - kz)} {\rm d}z{\rm d}t, \tag{22}$$

with $S_{\rm t}(z,t)$ from Eq. (20). After integrating the terms involving the velocity derivatives $\partial u_x/\partial t$,$\partial u_x/\partial z$ and $\partial u_z/\ \partial z$ by parts and assuming a finite turbulent region with $u_x, u_z \neq 0$, the source function can be written as

$$S_{\rm t}(k,\omega) = -\frac{\rho_o^{1/4}}{(2\pi)^2} \frac{\rho_e}{\rho_e + \rho_o} \left[k\omega + \frac{gk}{\omega H} - i\left(\frac{\omega}{H} - \frac{gk^2}{\omega}\right)\right] \times \int\int u_x(z,t) u_z(z,t) {\rm e}^{-{\rm i}(\omega t - kz)} {\rm d}z{\rm d}t. \tag{23}$$

We shall use this solution to calculate the wave energy fluxes in the following section. Note that for $S_{\rm t}(k,\omega) = 0$, the obtained solution reduces to the dispersion relation for transverse tube waves with a linear wave amplitude $v(k,\omega)$.

2.6. Wave energy fluxes and spectra

The energy flux for transverse tube waves can be evaluated from the general expression for the MHD energy flux given by Musielak & Rosner (1987, Eq. (4.2)). This gives the energy flux (erg cm^{-2} s^{-1}) in the z-direction in the form:

$$F(z,t) = -\frac{B_{oz}}{4\pi}\ b_x(z,t)\ v_x(z,t). \tag{24}$$

The mean wave energy flux can be calculated from the above equation by taking the time average, substituting $b_x = B_o l_x$ and transforming v_x to v_1. Using Eqs. (8) and (14) one obtains

$$< F(z,t) >_{\rm time} = -\frac{B_{oz}^2}{4\pi\rho_o^{1/2}} \times < v_1(z,t) \left(\frac{\partial}{\partial t}\right)^{-1} \left(\frac{\partial}{\partial z} + \frac{1}{4H}\right) v_1^*(z,t) >_{\rm time}. \tag{25}$$

The next step in our derivation of the wave energy flux is to express v_1 and its complex conjugate v_1^* in terms of their Fourier transforms and then take the time average over the time scale t_o. We find

$$< F(z,t) >_{\rm time} = -\frac{B_{oz}^2}{4\pi\rho_o^{1/2}} \lim_{t_0\to\infty} \frac{1}{t_0} \times \int_{-t_0/2}^{+t_0/2} {\rm d}t \int\int\int\int {\rm d}k'{\rm d}k''{\rm d}\omega'{\rm d}\omega'' v_1(k',\omega') \times v_1^*(k'',\omega'') \frac{-1}{i\omega''} \left(ik'' + \frac{1}{4H}\right) {\rm e}^{i(\omega'-\omega'')t - i(k'-k'')z}. \tag{26}$$

We perform the time-integration with the δ-function

$$\int_{-\infty}^{+\infty} {\rm e}^{i(\omega'-\omega'')t}{\rm d}t = 2\pi\delta(\omega' - \omega''),$$

and then the ω''-integration. Using

$$< F(z) >_{\rm time} = \int_{-\infty}^{+\infty} {\rm d}\omega < F(z,\omega) >_{\rm time} \tag{27}$$

we obtain for the frequency dependent flux

$$< F(z,\omega) >_{t_o} = \lim_{t_o\to\infty} \frac{B_{oz}^2}{2t_o\omega\rho_o^{1/2} c_k^4}\ I_1(k_o,\omega)\ I_2(k_o,\omega), \tag{28}$$

where

$$I_1(k_o,\omega) = \int_{-\infty}^{+\infty} \frac{S_t(k',\omega)e^{-ik'z}}{k'^2 - k_o^2}\, dk', \tag{29}$$

and

$$I_2(k_o,\omega) = \int_{-\infty}^{+\infty} \frac{S_t(k'',\omega)\ (k''-i/4H)e^{ik''z}}{k''^2 - k_o^2}\, dk'', \tag{30}$$

with

$$k_o^2 = \frac{\omega^2 - \Omega_k^2}{c_k^2}, \tag{31}$$

where for the propagating waves k_o^2 is always positive. Note also that the term $(-i/4H)$ is dropped in our calculations to have the real wave energy flux.

Now, the integrals $I_1(k_o,\omega)$ and $I_2(k_o,\omega)$ represent the asymptotic Fourier transforms and they are evaluated analytically in Appendix A. Using these results, we get

$$< F(z,\omega) >_{t_o} = \lim_{t_o\to\infty} \frac{\pi^2}{8t_o} \frac{B_{oz}^2}{\rho_o^{1/2}} \frac{1}{\omega k_o c_k^4}\ |S_t(k_o,\omega)|^2. \tag{32}$$

We next calculate $|S_t(k_o,\omega)|^2$ by averaging position (z_o) and time (t_o). From Eq. (23) we have

$$|S_t(k_o,\omega)|^2 = \frac{\rho_0^{1/2}}{(2\pi)^4}\eta_e^2|\chi(k_0,\omega)|^2 \int\int\int\int dz dz' dt dt'$$

$$\times\ < u_x u_z u_x' u_z' >\ e^{i\omega(t-t')-ik_0(z-z')} \tag{33}$$

where

$$\eta_e = \frac{\rho_e}{\rho_e + \rho_o}, \tag{34}$$

and

$$|\chi(k_o,\omega)|^2 = \frac{\omega^2}{H^2}(1+k_o^2H^2)\left(1+\frac{\omega_g^4}{\omega^4}\right), \tag{35}$$

with $\omega_g = \sqrt{gk_o}$. We follow Stein (1967) to average position and time and define

$$z_0 = \frac{1}{2}(z+z'), \quad t_0 = \frac{1}{2}(t+t'), \tag{36}$$

and the relative quantities

$$r = z - z', \quad \tau = t - t'. \tag{37}$$

Integrating over z_0 and t_0 we obtain

$$|S_t(k_o,\omega)|^2 = \frac{\rho_0^{1/2}}{(2\pi)^4}\eta_e^2|\chi(k_0,\omega)|^2 z_0 t_0$$

$$\times \int\int dr d\tau < u_x u_z u_x' u_z' > e^{i(\omega\tau - k_0 r)}. \tag{38}$$

It is seen that this calculation involves fourth order turbulent velocity correlations, $< u_x u_z u_x' u_z' >$, which are reduced here to the transverse ($< u_x u_x' >$) and longitudinal ($< u_z u_z' >$) second order correlations by using the properties of isotropic and homogeneous turbulence (e.g., Hinze 1975). The Fourier transform of these second order correlations gives the convolution integral, $J_c(k_o,\omega)$, which is defined below. Thus, we may write

$$|S_t(k_o,\omega)|^2 = t_o z_o \frac{\rho_o^{1/2}}{(2\pi)^2}\ \eta_e^2\ |\chi(k_o,\omega)|^2\ J_c(k_o,\omega), \tag{39}$$

with

$$J_c(k_o,\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} dr \int_{-\infty}^{+\infty} d\tau\ R_{xx}(r,\tau)R_{zz}(r,\tau)$$

$$\times e^{i(\omega\tau - k_o r)}, \tag{40}$$

and explicit formulas for the correlation tensors

$$R_{xx}(r,\tau) = << u_x(z,t)\ u_x(z+r,t+\tau) >_z>_t, \tag{41}$$

and

$$R_{zz}(r,\tau) = << u_z(z,t)\ u_z(z+r,t+\tau) >_z>_t \tag{42}$$

are derived in Appendix B. To evaluate the correlation tensors and the convolution integral, the turbulent energy spectrum must be prescribed (see Sect. 2.7). The final formula for the convolution integral can be found in Appendix C.

It is important to note that the source function $|S_t(k_o,\ \omega)|^2$ (see Eq. (39)) is completely determined by the external turbulent velocity field but it also depends on k_o, which represents the propagation condition for transverse tube waves through (see Eq. (31)). This dependence on k_o may imply that the source function approaches zero when $k_o \to 0$. However, this is *not* the case. In the limit of $k_o \to 0$ also $\omega_g \to 0$ and $|\chi(k_o = 0,\omega)|^2 = \omega^2/H^2$ (see Eq. (35)). Based on the results given in Appendix C, we find that the convolution integral $J_c(k_o = 0,\omega) \neq 0$. Hence, $|S_t(k_o = 0,\omega)|^2 \neq 0$ and some energy is generated when $\omega = \Omega_k$. The latter means that this energy cannot be carried away by transverse tube waves because the waves with the cutoff frequency Ω_k are non-propagating. Since we consider only propagating waves in this paper, we must remove the non-propagating component (see Sect. 3.2, for details).

Finally, we combine Eqs. (32) and (39), and derive the mean wave energy generation rate [in units of erg cm^{-2} s^{-1} Hz^{-1}]

$$<< F(\omega) >_{t_o}>_{z_o} = \pi \int_o^{z_{\rm turb}} dz\ \rho_e\eta_e \frac{\omega}{k_o}\frac{\Omega_k^2}{c_k^4}\left(1+\frac{\omega_g^4}{\omega^4}\right)$$

$$\times(1+k_o^2H^2)\ J_c(k_o,\omega), \tag{43}$$

where $z_{\rm turb}$ is the thickness of the turbulent region in the stellar convection zone. In deriving this expression, we have taken into account the fact that only *half* of the total generated flux propagates upward. In Sect. 3, we use Eq. (43) to compute the wave energy spectra carried by transverse tube waves propagating upward along a given flux tube embedded in the solar atmosphere.

2.7. Description of turbulence

By assuming that the source term is determined by a given external flow, we are obliged to specify the physical properties of the turbulence. Unfortunately, the properties of realistic turbulence occurring in the solar and stellar convection zones are presently unknown and currently no first principle theory of turbulence exists. Therefore, the properties of the turbulence occurring on and below the solar surface are usually determined by specifying a turbulent energy spectrum. For the turbulent energy spectrum in the solar atmosphere many different shapes have been proposed (e.g., Stein 1967; Bohn 1984; Musielak et al. 1989; Goldreich & Kumar 1988, 1990). Musielak et al. (1994) have combined some theoretical arguments about the turbulence with the results of numerical simulations (e.g., Cattaneo et al. 1991) as well as observational results (e.g., Zahn 1987; Muller et al. 1989, 1994) and suggested that the spatial and temporal parts of the turbulent energy spectrum can be described by an extended Kolmogorov spectrum with a modified Gaussian frequency factor. Recently, Nordlund et al. (1997) have argued that the Kolmogorov scaling with a power spectrum slope of $-5/3$ may not apply in regions of highly non-isotropic motions found in convection zone simulations. They also argue that the turbulence is reduced in rising bulk flows and enhanced in downflows. This could actually increase our tube wave generation rates as the magnetic tubes are indeed situated in the downflow regions.

The turbulent energy spectrum, $E(k,\omega)$, can be formally factored into a spatial and temporal part (e.g., Stein 1967)

$$E(k,\omega) = E(k)\Delta\left(\frac{\omega}{ku_k}\right), \tag{44}$$

where the mean velocity of the eddy with wave number k is given by

$$u_k = \left[\int_k^{2k} E(k')\mathrm{d}k'\right]^{1/2}, \tag{45}$$

and where the form of $E(k)$ and $\Delta(\omega/ku_k)$ must be prescribed. As already discussed by Musielak et al. (1994), the Kolmogorov spectrum must be well-represented, particularly as the tube excitation takes place at large optical depths (at the point of the maximum of the convective velocity at $\tau \approx 10$ to 100), where radiation effects are still minor. For the range of this spectrum we feel that it would be hard to deviate greatly from the typical length scale in a gravitational atmosphere, the scale height H, and the subsequent inertial breakup cascade to sizes as small as $H/100$. Thus, we specify the turbulent energy spectra appropriate for the solar and stellar convection zones by taking an extended Kolmogorov spectrum $E(k)$ and a modified Gaussian frequency factor $G(\frac{\omega}{ku_k})$. The extended Kolmogorov spatial spectrum is given by

$$E(k) = \begin{cases} 0 & 0 < k < 0.2k_\mathrm{t} \\ a\frac{u_\mathrm{t}^2}{k_\mathrm{t}}\left(\frac{k}{k_\mathrm{t}}\right) & 0.2k_\mathrm{t} \le k < k_\mathrm{t} \\ a\frac{u_\mathrm{t}^2}{k_\mathrm{t}}\left(\frac{k}{k_\mathrm{t}}\right)^{-5/3} & k_\mathrm{t} \le k \le k_\mathrm{d} \end{cases}, \tag{46}$$

where the factor $a = 0.758$ is determined by the normalization condition

$$\int_0^\infty E(k)\mathrm{d}k = \frac{3}{2}u_\mathrm{t}^2 \ , \tag{47}$$

and u_t is the rms turbulent velocity defined as

$$u_\mathrm{t} = \sqrt{\overline{u_x(r,t)^2}} = \sqrt{\overline{u_z(r,t)^2}}. \tag{48}$$

Note that for homogeneous and isotropic turbulence, u_t is independent of space and time, and is the same in either the x or z-direction. In addition, $k_\mathrm{t} = 2\pi/H$ and k_d is the wave number at which the turbulent cascade ends. According to Theurer et al. (1997), $k_\mathrm{d} = 2\pi/L$ with $L \approx 2.9$ cm, however, for our calculations performed here it is sufficient to take $L = H/100$.

The modified Gaussian frequency factor is described by

$$G\left(\frac{\omega}{ku_k}\right) = \frac{4}{\sqrt{\pi}}\frac{\omega^2}{|ku_k|^3}\mathrm{e}^{-(\frac{\omega}{ku_k})^2}. \tag{49}$$

The results given by Eqs. (44), (46) and (49) are used in Appendix C to evaluate the convolution integral (see Sect. 2.6).

2.8. Wave luminosities

To compute the total wave luminosity, we integrate Eq. (43) over the wave frequency ω and take into account the number of flux tubes on the stellar surface, N_t. In addition, we separate the dimensional factors by using the rms turbulent velocity u_t and the turbulent length scale l_t. The total wave luminosity [erg s^{-1}] due to transverse tube waves can then be written as

$$L_\mathrm{t} = (2\pi)^4 N_\mathrm{t}\int_o^{z_\mathrm{turb}}\left(\frac{1}{2}\rho_e u_\mathrm{t}^2\right)\ \frac{u_\mathrm{t}}{l_\mathrm{t}}\ M_\mathrm{t}^3\ \eta_e\ \mathrm{d}\bar{z}$$

$$\times\int_o^\infty \frac{\bar{\omega}}{\bar{k}_o\bar{c}_k}\ \bar{\Omega}_k^2\ (1+\bar{k}_o^2\ \bar{H}^2)\left(1+\frac{\bar{\omega}_g^4}{\bar{\omega}^4}\right)\ \bar{J}_\mathrm{c}(\bar{k}_o,\bar{\omega}), \tag{50}$$

where $M_\mathrm{t}(\equiv u_\mathrm{t}/c_k)$ is a coupling Mach number. The wave and cutoff frequencies are dimensionless, and written in terms of a characteristic turbulent frequency $\omega_\mathrm{t} = 2\pi u_\mathrm{t}/l_\mathrm{t}$. Similarly, the convolution integral is dimensionless, and is given as $J_\mathrm{c} = u_\mathrm{t}^3 l_\mathrm{t}^2 \bar{J}_\mathrm{c}$.

2.9. Wave sources

As shown by Eq. (50), the total wave luminosity for transverse tube waves depends on the third power of the Mach number, which clearly indicates that this is a dipole type of emission with respect to Mach number. It is interesting to note that the same type of emission was found to dominate in the generation of longitudinal tube waves (Musielak et al. 1989, 1995). This allows concluding that the interaction between the forced turbulence and magnetic flux tubes leads only to a dipole nature of the wave source emission. The dipole character of the excitation of both longitudinal and transverse tube waves distinguishes our results from those obtained earlier by Stein (1967), who found that acoustic waves are generated in the solar convective zone predominantly by quadrupole type of emission if no magnetic field is present, and by Ulmschneider & Stein (1982) who argued that a monopole type of emission should dominate in the wave generation if the magnetic field is present. The latter suggestion has been confirmed by calculations performed by Musielak & Rosner (1987), who found that monopole emission is primary responsible for generation of compressible (slow) MHD waves in an isothermal stratified medium with plasma $\beta < 1$, and by Lee (1993), who considered the generation of longitudinal (acoustic) waves in sunspots. However, the results obtained here are similar to those given by Musielak & Rosner (1987), who also found dipole type of emission for the generation of purely incompressible MHD (Alfven) waves in a stratified medium with an embedded weak uniform magnetic field (see also Collins 1989,b).

Having obtained Eqs. (43) and (50), we may now look for frequency domains in which the fluctuating buoyancy force dominates over the turbulent pressure. We define the critical frequency, $\omega_{\rm c}$, at which the contribution from both source terms is identical. This requires

$$\omega_{\rm c}^2 = \frac{\omega_g^4}{\omega_{\rm c}^2} = \frac{g^2}{c_k^2}\left(\frac{\omega_{\rm c}^2 - \Omega_k^2}{\omega_{\rm c}^2}\right), \tag{51}$$

and because

$$\frac{g^2}{c_k^2} = 4(2\beta+1)^2\Omega_k^2, \tag{52}$$

we obtain

$$\omega_{\rm c}^2 = 2(2\beta+1)\left[(2\beta+1) \pm 2\sqrt{\beta(\beta+1)}\right]\Omega_k^2. \tag{53}$$

For all $\omega > \omega_{\rm c}$ the turbulent pressure dominates in the wave generation; however, in the opposite limit, the contribution by the fluctuating buoyancy force is more important. As shown by Eq. (51), the critical frequency $\omega_{\rm c}$ is a sensitive function of the plasma β. For very low β, we find that $\omega_{\rm c} \approx \sqrt{2}\ \Omega_k$, which restricts the dominance of the fluctuating buoyancy force to a very narrow frequency regime $\Omega_k < \omega < \sqrt{2}\ \Omega_k$. If $\beta = 1$, then the fluctuating buoyancy force dominates for all wave frequencies that satisfy the condition $\Omega_k < \omega < 6\ \Omega_k$. Finally, in the limit of a high-β plasma, $\omega_{\rm c} \approx 4\beta\Omega_k$ and the frequency domain where the fluctuating buoyancy force dominates shows further broadening. Under typical conditions considered in this paper, it is likely that β is of order of unity, and therefore the fluctuating buoyancy force significantly contributes to the generated wave energy spectra and fluxes.

3. Results and discussion

In this paper, we present the results only for one magnetic flux tube embedded in the solar convection zone, which means that the filling factor, representing the number of magnetic flux tubes on the solar surface, will not be discussed here. In the following, we denote the flux carried by transverse waves (and given by Eq. (43)) as $F_{\rm tran}$ and the wave energy flux carried by longitudinal tube waves, which are also discussed here, as $F_{\rm long}$.

3.1. Solar convection zone model

The solar convection zone model used here to perform our calculations of the generation of transverse tube wave is a modified version of stellar envelope computer code originally used by Bohn (1981, 1984) and later modified by Theurer (1993) and Ulmschneider et al. (1996). To run this code, it is required to specify the effective temperature $T_{\rm eff}$ and the surface gravity g. The code employs a mixing-length description of convection, which assumes that the energy-containing eddies have spatial scales comparable to the local pressure scale height H (Prandtl 1925); this means that the so-called mixing-length parameter $\alpha = l_{\rm mix}/H$, where $l_{\rm mix}$ is the mixing-length, must be of the order of unity. In the solar and stellar convection zones, the mixing-length parameter may vary from 1.0 to 2.0 (Böhm-Vitense 1958; Trampedach et al. 1997). The code takes also into account the formation of hydrogen molecules and uses gray radiation transport. In all calculations described in this paper, we used $T_{\rm eff} = 5770$ K, $g = 2.736\ 10^4$ cm s^{-2}, and α ranging from 1.0 to 2.0. In our computations, we identified the turbulence velocity scale $u_{\rm t}$ with the convective velocity of the solar model. A comparison of the results obtained from this code with other gray calculations has been recently given by Ulmschneider et al. (1999), who also discussed potential difficulties of this code.

The described solar convection zone model is calculated for three different values of the mixing-length parameter α = 1.0, 1.5 and 2.0, and used to represent the external background medium that surrounds the tube. Then, for each of these models, we compute the distribution of physical parameters with height inside the tube by specifying the strength of the tube magnetic field at optical depth $\tau_{5000} = 1.0$. Observations show that magnetic flux tubes at the solar surface typically have field strengths of the order of B_{oz} = 1500 G (Stenflo 1978; Solanki 1993). On the other hand, it is known that a tube, completely devoid of gas but in pressure equilibrium with the outside gas, would have an equipartition field strength $B_{eq}/8\pi = p_e$, where p_e is the gas pressure outside the

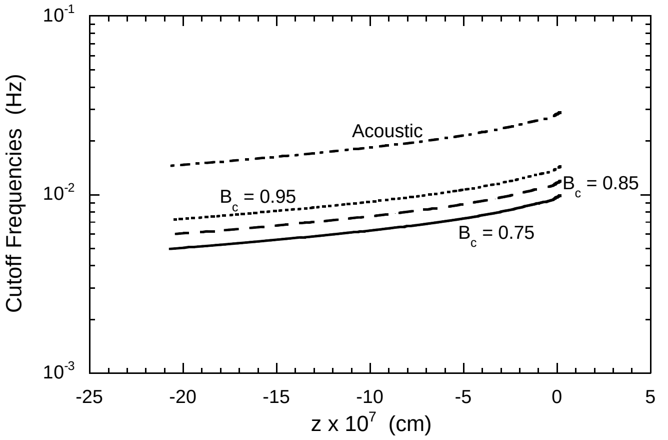

Fig. 1. The cutoff frequencies for acoustic and transverse tube waves are plotted as a function of depth, z, in the solar photosphere and convection zone. The effect of the magnetic field on the cutoff for transverse waves is shown

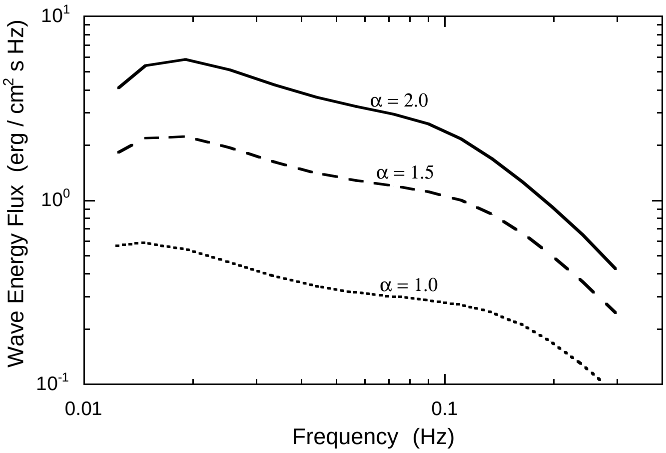

Fig. 2. Transverse tube wave energy spectra computed for three different solar convection zone models obtained with the mixing-length parameter $\alpha = 1.0$, 1.5 and 2.0. In all models, the tube magnetic field at $\tau_{5000} = 1.0$ was assumed to be $B_{\rm c} = 0.85$

tube. Taking $p_e = 1.17\ 10^5$ dyn cm^{-2} from model C of Vernazza et al. (1981) at the height $z = 0$, one gets an equipartition field strength of $B_{eq} = 1715\ G$. Thus, we find a ratio $B_{\rm c} \equiv B_{oz}/B_{eq} = 0.875$ and consider it as typical for the solar magnetic flux tubes. Since this ratio is likely to vary (e.g., Solanki 1993), we decided to consider three different dimensionless surface field strengths, namely, $B_{\rm c} = 0.75, 0.85$ and 0.95. This choice of the tube magnetic field strength has two main advantages: first, after the correct field strength is known, the appropriate energy fluxes carried by transverse waves can simply be interpolated and second, the dependence of these fluxes on the field strength can also be investigated.

3.2. Propagating wave fluxes

Transverse tube waves considered in this paper are propagating waves only if their frequencies are higher than the cutoff frequency, Ω_k, (see Eq. (16)). The obtained expression for Ω_k holds for an isothermal medium in which both the scale height H and the characteristic wave velocity c_k are constant. Our models of the solar convection zone are not isothermal and Ω_k changes with depth. To deal with this problem, we formally divide the model into equal size layers and assume that each of these layers is isothermal, so that Ω_k can be treated as a locally constant quantity. As shown in Fig. 1, the changes of Ω_k and the acoustic cutoff frequency, $\Omega_{\rm S}$, are small throughout the region of the wave generation in the convection zone. However, it is seen that Ω_k calculated for the magnetic field $B_{\rm c} = 0.85$ is almost 2.5 times lower than $\Omega_{\rm S}$; this simply means that the frequency interval for propagating transverse tube waves is much broader than that for acoustic waves. The results presented in Fig. 1 also demonstrate that the effects of the magnetic field on Ω_k are relatively small.

As discussed in Sect. 2.6, the source function given by Eq. (39) is non-zero at the cutoff frequency Ω_k, which means that some energy is generated in the form of non-propagating waves. The problem is common for this kind of calculations (see Ulmschneider et al. 2000, for extensive discussion of this problem) and the non-propagating component must be removed. To achieve this goal, we multiply the wave energy flux given by Eq. (43) by a factor $(1 - \Omega_k^2/\omega^2)^2$, which reduces the non-propagating component to zero as $\omega \to \Omega_k$; factors of this form typically appear in expressions for group velocities for different waves propagating in inhomogeneous media (e.g., Lighthill 1960). The effects caused by this factor on the wave energy spectra and fluxes are relatively small because it mainly affects only two frequency points near the cutoff (see Figs. 2 and 3) and reduces the total wave energy fluxes by 10% or less. All results presented in the next two subsections have been obtained by using this factor. This guarantees that the generated energy is always carried away by the propagating transverse tube waves.

3.3. Wave energy spectra

The dependence of the calculated wave energy spectra on the mixing-length parameter α is shown in Fig. 2. It is seen that the shape of the spectrum slightly changes when α increases. In addition, the maximum shifts toward higher frequencies when α is increased.

The wave energy spectra presented in Fig. 3 have been computed for $\alpha = 2.0$ and for three different values of the dimensionless magnetic field strength $B_{\rm c} = 0.75$, 0.85 and 0.95. The results show that the shape of the spectrum is practically independent from the magnetic field, except some small modifications at low frequencies. It is also seen that the maximum shifts toward higher frequencies when $B_{\rm c}$ is increased; however, this effect is likely to be caused by the decrease of the cutoff frequency with increasing magnetic field strength.

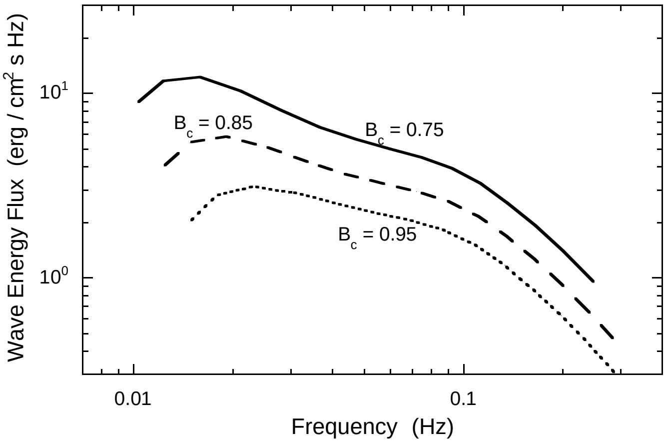

Fig. 3. Transverse tube wave energy spectra computed for three different values of the tube magnetic field: $B_c = 0.75$, 0.85 and 0.95. In all models, the tube magnetic field was specified at $\tau_{5000} = 1.0$ and the mixing-length parameter was taken $\alpha = 2.0$

Table 1. Total wave energy fluxes $F_{\rm tran}$ (erg/cm^2 s) generated as transverse waves in a magnetic flux tube embedded in the solar convection zone are compared to the fluxes for longitudinal tube waves, $F_{\rm long}$, computed by Musielak et al. (1995)

α	$B_c = B_{oz}/B_{eq}$	$F_{\rm tran}$	$F_{\rm long}$
1.0	0.85	2.3 10^7	6.4 10^6
1.5	0.85	5.8 10^7	1.4 10^7
2.0	0.85	1.2 10^8	2.4 10^7
2.0	0.75	1.8 10^8	5.6 10^7
2.0	0.95	7.6 10^7	6.0 10^6

3.4. Wave energy fluxes

The total wave energy fluxes for one magnetic flux tube embedded in the solar convection zone are obtained by integrating the wave energy spectra over the frequency range $\Omega_k \geq \omega \leq 25\Omega_k$. The results are presented in Table 1. It is seen that the total flux $F_{\rm tran}$ increases with α, which reflects the fact that the convective velocity depends on α. Based on these results, the dependence of $F_{\rm tran}$ on α can approximately be given as

$$F_{\rm tran} \approx 2.3 \quad 10^7 \ \alpha^{2.4} \ \text{ergs cm}^{-2}\ \text{s}^{-1} \ . \tag{54}$$

This α-dependence of the transverse wave generation rate is higher than the value of 1.8 found for longitudinal tube waves by Musielak et al. (1995) but it is much lower than the value of 3.8 obtained by Musielak et al. (1994) for the acoustic wave generation.

The results presented in Table 1 also show that the total wave energy flux $F_{\rm tran}$ decreases with increasing magnetic field strength B_c. The dependence of $F_{\rm tran}$ on B_c can approximately be fitted by the expression:

$$F_{\rm tran} \approx 6.4 \quad 10^7 \ B_c^{-3.6} \ \text{ergs cm}^{-2}\ \text{s}^{-1} \ . \tag{55}$$

It is interesting to compare this dependence with that of longitudinal tube waves. For linear longitudinal waves energy fluxes are also given in Table 1. We find that $F_{\rm long} \approx 4.2 \ 10^6 B_c^{-9.4}$ (see Musielak et al. 1995, 2000; also Ulmschneider & Musielak 1998). The comparison of these results clearly indicates that the process of shaking of magnetic flux tubes, which is responsible for the generation of transverse tube waves, does not depend as strongly on the magnetic field as the process of squeezing these tubes, which leads to the excitation of longitudinal tube waves.

The difference can be explained by the role played by the "stiffness" of magnetic flux tubes in the wave generation process. The "stiffness" is determined by the strength of the magnetic field, B_c, and is greater for $B_c = 0.95$ than for $B_c = 0.75$. Greater "stiffness" means that there is less gas inside the tubes and thereby less material to support the propagation of longitudinal tube waves. In longitudinal waves the potential energy density is due to the compression of the gas. For transverse waves the gas density is not as important because the potential energy density is due the tension of the magnetic field resulting from the curvature. From Table 1, we find that $F_{\rm tran}$ decreases by a factor of 2 and $F_{\rm long}$ decreases by one order of magnitude when the "stiffness" is increased from $B_c = 0.75$ to $B_c = 0.95$. This gives $F_{\rm tran}/F_{\rm long} \approx 3$ for $B_c = 0.75$ and $F_{\rm tran}/F_{\rm long} \approx 13$ for $B_c = 0.95$ and $F_{\rm tran}/F_{\rm long} \approx 5$ for $B_c = 0.85$. Thus, the amount of gas inside the tubes is essential for the existence of longitudinal tube waves but its effect on transverse tube waves is much reduced.

Now, by comparing directly $F_{\rm tran}$ and $F_{\rm long}$, we see that the transverse wave energy fluxes are always much higher than the corresponding longitudinal fluxes. For typical solar values, $\alpha = 2$ and $B_c = 0.85$, we find that $F_{\rm tran}/F_{\rm long} = 5$. This simply means that it is much easier for the external turbulent motions to shake magnetic flux tubes than to squeeze them (e.g., Spruit & Roberts 1983).

3.5. Comparison to previous results

The results presented here must now be compared to those obtained in previous papers. The efficiency of the generation of transverse tube waves in Sun has been calculated by Huang et al. (1995), who investigated the nonlinear response to purely transverse shaking of a thin and vertically oriented flux tube embedded in the solar atmosphere. In this approach, the shaking velocities are represented by a spectrum of partial waves with random phases and their amplitudes determined from a description of the convective turbulence. The shaking is imposed on the tube at only one specific height, which means that the correlation effects (see Sect. 1) are not accounted for. Typical wave energy fluxes obtained by this method are of the order of 10^9 erg/cm^2 s. Hence, they are one order of magnitude higher than the fluxes computed here by using our analytical approach. Clearly, the fluxes calculated with the nonlinear effects included must be regarded as upper bounds for the realistic wave energy fluxes carried by transverse tube waves in the solar atmosphere. However, the analytically computed fluxes represent only lower bounds for the realistic fluxes.

The fact that nonlinear effects and particularly shocks can be important in the excitation of transverse tube waves has been demonstrated by Choudhuri et al. (1993,b) and Zhugzhda et al. (1994), who used analytical methods, and by Muller et al. (1994), who based their estimates on observations. The results obtained by Muller et al. seem to indicate that the actual wave energy fluxes carried by transverse tube waves can be as high as 10^{10} erg/cm^2 s. However, the latter has to be taken with caution because of a number of assumptions underlying this estimate.

In Sect. 3.4, we have already compared the obtained transverse wave energy fluxes with the fluxes calculated analytically for longitudinal tube waves by Musielak et al. (1995). The comparison shows that the former are 5 times (or more) higher than the latter ones. Finally, we need to compare our results to those obtained by Ulmschneider & Musielak (1998), who have used similar method as Huang et al. (1995) to compute the efficiency of the nonlinear generation of longitudinal tube waves. Without taking the correlation effects into account, they obtained for the Sun the flux of the order of 2 10^8 erg/cm^2 s, which is very similar to the flux found here for transverse tube waves. Clearly, the contribution to the chromospheric (and likely coronal) heating by transverse tube waves must be included in theoretical chromospheric models recently constructed by Cuntz et al. (1998, 1999). The wave energy spectra and fluxes computed in this paper for transverse tube waves can be used as the initial input to these theoretical models.

4. Conclusions

From our analytical studies of the generation of linear transverse waves in a magnetic flux tube embedded in the solar convection zone, the following general conclusions can be drawn.

1. As a result of the interaction between the magnetic flux tube and the external turbulent motions transverse tube waves are generated by the process of dipole emission;
2. The shapes of the wave energy spectra calculated for different values of the mixing length parameter, α, and the tube magnetic field strength, $B_c = B_{oz}/B_{eq}$ (where $B_{oz} = 1500$ G is the observed and B_{eq} the equipartition field strength on the solar surface), are not very sensitive to changes of these parameters;
3. Typical fluxes carried by linear transverse tube waves along one solar magnetic flux tube are of the order of 10^8 erg/cm^2 s. They are one order of magnitude lower than the fluxes carried by nonlinear transverse waves. The linear transverse fluxes are between 3 to 13 times higher than the fluxes carried by linear longitudinal waves along the same tube;
4. The total transverse wave energy fluxes do not depend as strongly on the strength of the tube magnetic field as the longitudinal fluxes. The difference can be explained by the different roles played by the "stiffness" of the magnetic tube in the wave generation process;
5. Our results are valid only for linear excitation, which means that the obtained wave energy fluxes must be regarded as only lower bounds for realistic energy fluxes carried by these waves.

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Appendix A: The asymptotic Fourier transforms

To evaluate the integrals $I_1(k_o, \omega)$ and $I_2(k_o, \omega)$, we follow Musielak et al. (1995) and write them in the following general form:

$$I_{1,2}(k_o, \omega) = \oint_C \frac{F_{1,2}(\eta, \omega)}{(\eta - k_o)(\eta + k_o)} \, d\eta, \tag{A.1}$$

where $F_{1,2}$ represent the numerators of the integrands of Eqs. (29) and (30), η is a complex number, and C is a closed contour to be specified (see below). Since the integrand has two simple poles at $\eta = \pm k_o$, with k_o being always real for the propagating waves, *Cauchy's principal value* must be used to evaluate these integrals. It is also required that the integrands in Eq. (A1) go to zero faster than $1/\eta$. Using this criterion, we find that the integrands of I_1 and I_2 are convergent only in the lower (η^*) and upper (η) half of the complex plane, respectively.

Thus, we choose the contour C in the lower half of the complex plane for the integral I_1 and perform the integration in the counterclockwise direction along contours surrounding the poles $\eta^* = \pm k_o$. By applying the residue theorem, we obtain the Cauchy's principal value of the integral I_1 to be given by

$$P\, I_1(k_o, \omega) = -\frac{i\pi}{2k_o} \left[F_1(k_o, \omega) - F_1(-k_o, \omega) \right]. \tag{A.2}$$

Now, for the integral I_2, we take the contour C to be in the upper part of the complex plane and perform the integration in the clockwise direction along contours surrounding the poles $\eta = \pm k_o$. This gives

$$P\, I_2(k_o, \omega) = \frac{i\pi}{2k_o} \left[F_2(k_o, \omega) - F_2(-k_o, \omega) \right]. \tag{A.3}$$

According to the so-called radiation condition (e.g., Sommerfeld 1909; also Lighthill 1960), only outward propagating waves are present in our approach because there are no other wave sources which could generate inward propagating waves. Therefore, to satisfy this condition, we must assume that $F_1(-k_o, \omega) = 0$ and $F_2(-k_o, \omega) = 0$. To obtain Eq. (32) in the main text, we take $F_1(k_o, \omega) = |S_o(k_o, \omega)| \exp(ik_o z)$ and $F_2(k_o, \omega) = k_o |S_o(k_o, \omega)| \exp(-i\, k_o z)$, and omit P as the obtained results are convergent for all values of k_o and ω.

Appendix B: The correlation tensors

In general, the correlation tensor R_{ij} is defined as

$$R_{ij}(\boldsymbol{r},\tau) \equiv << u_i(\boldsymbol{x},t)\ u_j(\boldsymbol{x}+\boldsymbol{r},t+\tau) >_x>_t, \quad \text{(B.1)}$$

where the average is taken over all points $\boldsymbol{x}$ of a large volume, which can be considered infinite, and over all times t, which are long compared to all other time scales, and thus can also be considered infinite. A situation in which R_{ij} is independent of $\boldsymbol{x}$ and t is called *time-independent homogenous turbulence*. We can introduce the spectral tensor, $\Phi_{ij}(\boldsymbol{k},\omega)$, and Fourier transform $R_{ij}(\boldsymbol{r},\tau)$. This gives

$$\Phi_{ij}(\boldsymbol{k},\omega) = \frac{1}{(2\pi)^4}\int \mathrm{d}^3r \int_{-\infty}^{+\infty} \mathrm{d}\tau\ R_{ij}(\boldsymbol{r},\tau)\ \mathrm{e}^{i(\omega\tau-\boldsymbol{k}\cdot\boldsymbol{r})} \quad \text{(B.2)}$$

and its inverse transform

$$R_{ij}(\boldsymbol{r},\tau) = \int \mathrm{d}^3k \int_{-\infty}^{+\infty} \mathrm{d}\omega\ \Phi_{ij}(\boldsymbol{k},\omega)\ \mathrm{e}^{-i(\omega\tau-\boldsymbol{k}\cdot\boldsymbol{r})}. \quad \text{(B.3)}$$

For homogeneous and isotropic turbulence, $R_{ij}(\boldsymbol{r},\tau) = R_{ij}(r,\tau)$ and $\Phi_{ij}(\boldsymbol{k},\omega) = \Phi_{ij}(k,\omega)$, where $r = |\boldsymbol{r}|$ and $k = |\boldsymbol{k}|$. Clearly, the correlation tensors can be evaluated when the corresponding spectral tensors are known. It was first shown by Batchelor (1960) that the spectral tensors for the homogeneous and isotropic turbulence can be expressed in terms of a 3-D turbulent energy spectrum, $E(k,\omega)$, and given by

$$\Phi_{ij}(k,\omega) = \frac{E(k,\omega)}{8\pi k^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right). \quad \text{(B.4)}$$

Taking $\mathrm{d}^3k = k^2 \sin\theta\ \mathrm{d}k\ \mathrm{d}\theta\ \mathrm{d}\phi$ and performing analytically the angle integration over ϕ and θ (see Musielak et al. 1995, for details), we obtain

$$R_{xx}(r,\tau) = \int_0^\infty \mathrm{d}\omega \cos\omega\tau \int_0^\infty \mathrm{d}k\ E(k,\omega) \times \left(\frac{\sin kr}{kr} + \frac{\cos kr}{k^2r^2} - \frac{\sin kr}{k^3r^3}\right), \quad \text{(B.5)}$$

and

$$R_{zz}(r,\tau) = 2\int_0^\infty \mathrm{d}\omega \cos\omega\tau \int_0^\infty \mathrm{d}k\ E(k,\omega) \times \left(\frac{\sin kr}{k^3r^3} - \frac{\cos kr}{k^2r^2}\right). \quad \text{(B.6)}$$

According to Eq. (44) in the main text, the turbulent energy spectrum can be separated in a spatial and temporal factor, namely, $E(k,\omega) = E(k)\Delta(\omega/ku_k)$, where $E(k)$ and $\Delta(\omega/ku_k)$ are represented by the extended Kolmogorov energy spectrum (see Eq. (46)) and by the modified Gaussian frequency factor (see Eq. (49)), respectively. Formally, the ω-integration in Eqs. (B5) and (B6) can be performed, and the result is

$$\int_0^\infty \mathrm{d}\omega\ \cos\omega\tau\ E(k,\omega) = E(k)\int_0^\infty \mathrm{d}\omega\ \Delta\left(\frac{\omega}{ku_k}\right)\ \cos\omega\tau = E(k)\left(1-\frac{\alpha^2\tau^2}{2}\right)\ \mathrm{e}^{-\alpha^2\tau^2/4} \quad \text{(B.7)}$$

where $\alpha = ku_k$.

Appendix C: The convolution integral

We want to evaluate the convolution integral (see Eq. (40)) given in the following form:

$$J_\mathrm{c}(k_o,\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\mathrm{d}r\int_{-\infty}^{+\infty}\mathrm{d}\tau\ R_{xx}(r,\tau)R_{zz}(r,\tau) \times \mathrm{e}^{i(\omega\tau-k_o r)}, \quad \text{(C.1)}$$

where the correlation tensors $R_{xx}(r,\tau)$ and $R_{zz}(r,\tau)$ are given by Eqs. (B5) and (B6). Using these results and also taking into account Eq. (B6), we obtain

$$J_\mathrm{c}(k_o,\omega) = \frac{1}{2\pi^2}\int_{-\infty}^{+\infty}\mathrm{d}r\ \mathrm{e}^{-ik_o r}\int_o^\infty \mathrm{d}k_1\int_o^\infty \mathrm{d}k_2 \times \left[\left(\frac{\sin k_1r}{k_1r}+\frac{\cos k_1r}{k_1^2r^2}-\frac{\sin k_1r}{k_1^3r^3}\right)\left(\frac{\sin k_2r}{k_2^3r^3}-\frac{\cos k_2r}{k_2^2r^2}\right)\right] \times \int_{-\infty}^{+\infty}\mathrm{d}\tau \mathrm{e}^{i\omega\tau}\left(1-\frac{\alpha^2(k_1)\tau^2}{2}\right)\left(1-\frac{\alpha^2(k_2)\tau^2}{2}\right) \times E(k_1)\ E(k_2)\ \mathrm{e}^{-\alpha^2(k_1)\tau^2/4}\ \mathrm{e}^{-\alpha^2(k_2)\tau^2/4}. \quad \text{(C.2)}$$

Now, the τ-integration can be performed analytically (see Musielak et al. 1995) and the result is

$$J_\mathrm{c}(k_o,\omega) = \frac{1}{4\pi}\int_{-\infty}^{+\infty}\mathrm{d}r\ \mathrm{e}^{(-ik_o r)}\int_o^\infty \mathrm{d}k_1\int_o^\infty \mathrm{d}k_2 \times \left[\left(\frac{\sin k_1r}{k_1r}+\frac{\cos k_1r}{k_1^2r^2}-\frac{\sin k_1r}{k_1^3r^3}\right)\left(\frac{\sin k_2r}{k_2^3r^3}-\frac{\cos k_2r}{k_2^2r^2}\right)\right] \times E(k_1)\ E(k_2)\ g(k_1,k_2,\omega), \quad \text{(C.3)}$$

where

$$g(k_1,k_2,\omega) \equiv \frac{4}{\sqrt{\pi}}\ \frac{1}{(\alpha_1^2+\alpha_2^2)^{1/2}}\left[\frac{3\alpha_1^2\alpha_2^2}{(\alpha_1^2+\alpha_2^2)^2} + 2\omega^2\frac{\alpha_1^4+\alpha_2^4-4\alpha_1^2\alpha_2^2}{(\alpha_1^2+\alpha_2^2)^3} + 4\omega^4\frac{\alpha_1^2\alpha_2^2}{(\alpha_1^2+\alpha_2^2)^4}\right] \times\ \exp[-\omega^2/(\alpha_1^2+\alpha_2^2)]. \quad \text{(C.4)}$$

where $\alpha_1 = \alpha(k_1) = k_1u_{k1}$ and $\alpha_2 = \alpha(k_2) = k_2u_{k2}$.

The integral over r can also be evaluated analytically (Musielak et al. 1995). We define

$$I(k_o,k_1,k_2) \equiv \int_{-\infty}^{+\infty}\mathrm{d}r\ \left[\left(\frac{\sin k_1r}{k_1r}+\frac{\cos k_1r}{k_1^2r^2}-\frac{\sin k_1r}{k_1^3r^3}\right) \times \left(\frac{\sin k_2r}{k_2^3r^3}-\frac{\cos k_2r}{k_2^2r^2}\right)\right]\ \mathrm{e}^{-ik_or}, \quad \text{(C.5)}$$

and rewrite it as

$$I(k_o,k_1,k_2) = \frac{1}{4}\int_{-\infty}^{+\infty}\mathrm{d}r\ \left[\frac{\sin k_1r}{k_1r} - \frac{1}{k_1^2}\frac{\mathrm{d}^2}{\mathrm{d}r^2}\left(\frac{\sin k_1r}{k_1r}\right)\right] \times \left[\frac{\sin k_2r}{k_2r} + \frac{1}{k_2^2}\frac{\mathrm{d}^2}{\mathrm{d}r^2}\left(\frac{\sin k_2r}{k_2r}\right)\right]\ \mathrm{e}^{-ik_or}. \quad \text{(C.6)}$$

With

$$\frac{\sin kr}{kr} \pm \frac{1}{k^2}\frac{\mathrm{d}^2}{\mathrm{d}r^2}\left(\frac{\sin kr}{kr}\right) =$$

$$\frac{1}{2}\int_{-1}^{+1} \mathrm{d}u \mathrm{e}^{ikru} \mp \frac{1}{2}\int_{-1}^{+1} \mathrm{d}u\ u^2 \mathrm{e}^{ikru},$$

$I(k_o, k_1, k_2)$ can also be expressed as

$$I(k_o, k_1, k_2) = \frac{1}{16}\int_{-1}^{+1} \mathrm{d}u \int_{-1}^{+1} \mathrm{d}v\ [(1+u^2)(1-v^2)] \times \int_{-\infty}^{+\infty} \mathrm{e}^{-ir(k_o - k_1 u + k_2 v)}\ \mathrm{d}r. \quad \text{(C.7)}$$

Introducing $x = k_1 u$ and $y = k_2 v$, we have

$$I(k_o, k_1, k_2) = \frac{\pi}{8k_1k_2}\int_{-k_1}^{+k_1} \mathrm{d}x \int_{-k_2}^{+k_2} \mathrm{d}y\ \left[1 + \frac{x^2}{k_1^2} - \frac{y^2}{k_2^2} - \frac{x^2y^2}{k_1^2k_2^2}\right]\ \delta(k_o - x - y). \quad \text{(C.8)}$$

To perform the integration over y, we define the distribution Π such that $\Pi(y) = 1$ for $|y| \leq 1/2$ and $\Pi(y) = 0$ else, and write

$$\int_{-k_2}^{+k_2} \mathrm{d}y\ f(y) = \int_{-\infty}^{+\infty} \mathrm{d}y\ f(y)\Pi\left(\frac{y}{2k_2}\right). \quad \text{(C.9)}$$

With this because of the peak of the δ-function at $y = k_o - x$ we get

$$I(k_o, k_1, k_2) = \frac{\pi}{8k_1^3k_2^3}\left[-\frac{1}{5}x^5 + \frac{k_o}{2}x^4 + \frac{1}{3}(k_2^2 - k_1^2 - k_o^2)x^3 + k_o k_1^2 x^2 + (k_2^2 - k_o^2)k_1^2 x\right]_A^B \Pi\left(\frac{k_o - x}{2k_2}\right)_A^B \quad \text{(C.10)}$$

where the limits A and B have to be selected by taking into account the distribution $\Pi(y)$ and the integration limits of the integral $I(k_o, k_1, k_2)$. There are 6 different cases. *Case 1*: Π to the right of I, no overlap, integral vanishes, $I(k_o, k_1, k_2) = 0$. *Case 2*: Left corner of Π overlaps right corner of I, then $A = k_o - k_2$ and $B = k_1$, and

$$I(k_o, k_1, k_2) = \frac{\pi}{24k_1k_2}\left[2(k_2+2k_1) - 3k_o - k_o^3\left(\frac{1}{k_1^2} - \frac{1}{k_2^2}\right) + 2k_o^2\left(\frac{k_2}{k_1^2} - 2\frac{k_1}{k_2^2}\right) - \frac{3}{2}k_o\left(\frac{k_2^2}{k_1^2} - 3\frac{k_1^2}{k_2^2}\right) + \frac{2}{5}\left(\frac{k_2^3}{k_1^2} - 4\frac{k_1^3}{k_2^2}\right) + \frac{1}{10}\frac{k_o^5}{k_1^2k_2^2}\right]. \quad \text{(C.11)}$$

Case 3: I is completely inside Π, then $A = -k_1$ and $B = k_1$, and

$$I(k_o, k_1, k_2) = \frac{\pi}{3k_2}\left(1 - \frac{k_o^2}{k_2^2} - \frac{2}{5}\frac{k_1^2}{k_2^2}\right). \quad \text{(C.12)}$$

Case 4: Π is completely inside I, then $A = k_o - k_2$ and $B = k_o + k_2$, and

$$I(k_o, k_1, k_2) = \frac{\pi}{6k_1}\left(1 + \frac{k_o^2}{k_1^2} + \frac{1}{5}\frac{k_2^2}{k_1^2}\right). \quad \text{(C.13)}$$

Case 5: Right corner of Π overlaps left corner of I, then $A = -k_1$ and $B = k_o + k_2$, and

$$I(k_o, k_1, k_2) = \frac{\pi}{24k_1k_2}\left[2(k_2+2k_1) + 3k_o + k_o^3\left(\frac{1}{k_1^2} - \frac{1}{k_2^2}\right) + 2k_o^2\left(\frac{k_2}{k_1^2} - 2\frac{k_1}{k_2^2}\right) + \frac{3}{2}k_o\left(\frac{k_2^2}{k_1^2} - 3\frac{k_1^2}{k_2^2}\right) + \frac{2}{5}\left(\frac{k_2^3}{k_1^2} - 4\frac{k_1^3}{k_2^2}\right) - \frac{1}{10}\frac{k_o^5}{k_1^2k_2^2}\right]. \quad \text{(C.14)}$$

Case 6: Π to the left of I, no overlap, integral vanishes, $I(k_o, k_1, k_2) = 0$.

For propagating waves, $k_o > 0$, and we have: *Case 3* if $k_2 > k_1 + k_o$, *Case 4* if $k_2 < k_1 - k_o$, *Case 6* if either $k_2 + k_1 \leq k_0$ or $k_1 = 0$ or $k_2 = 0$, and *Case 2* in all other situations.

For non-propagating waves, $k_o < 0$, and we have: *Case 3* if $k_2 > k_1 + |k_o|$, *Case 4* if $k_2 < k_1 - |k_o|$, *Case 6* if either $k_2 + k_1 \leq |k_0|$ or $k_1 = 0$ or $k_2 = 0$, and *Case 5* with k_o to be replaced by $|k_o|$ in all other situations.

Finally, the convolution integral is evaluated from the following formula:

$$J_c(k_o, \omega) = \frac{1}{4\pi}\int_o^\infty \mathrm{d}k_1 \int_o^\infty \mathrm{d}k_2\ E(k_1)\ E(k_2)\ g(k_1, k_2, \omega) \times I(k_0, k_1, k_2), \quad \text{(C.15)}$$

where the function g is given by Eq. (C4) and the function I by Eqs. (C11)–(C14), depending on the considered case.

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