Nützliche Beziehungen der Vektoranalysis

Folgende Differentialoperationen wurden eingeführt:

Nabla-Operator:	$\boldsymbol{\nabla} \equiv \boldsymbol{\partial}$	$\equiv \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$		
Gradient:	$\operatorname{grad} \Phi$	$\equiv \boldsymbol{\nabla}\, \Phi$		
Divergenz:	$\operatorname{div} \boldsymbol{A}$	$\equiv \boldsymbol{\nabla} \cdot \boldsymbol{A}$		
Rotation:	$\operatorname{rot} \boldsymbol{A}$	$\equiv \boldsymbol{\nabla} \times \boldsymbol{A}$		
Laplace-Operator:	$\operatorname{div} \operatorname{grad} \Phi$	$\equiv \boldsymbol{\nabla} \cdot \boldsymbol{\nabla}\, \Phi$	$\equiv \Delta\, \Phi$	
	$\operatorname{div} \operatorname{rot} \boldsymbol{A}$	$= \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \boldsymbol{A})$	$= 0$	falls $\boldsymbol{A}$ 2×stetig db.
	$\operatorname{rot} \operatorname{grad} \Phi$	$= \boldsymbol{\nabla} \times \boldsymbol{\nabla}\, \Phi$	$= 0$	falls Φ 2×stetig db.
	$\operatorname{div} \boldsymbol{A}$	$= \boldsymbol{\nabla} \cdot \boldsymbol{A}$	$= 0$	$\Leftrightarrow$ $\boldsymbol{A}$ ist *quellenfrei.*
	$\operatorname{rot} \boldsymbol{A}$	$= \boldsymbol{\nabla} \times \boldsymbol{A}$	$= 0$	$\Leftrightarrow$ $\boldsymbol{A}$ ist *wirbelfrei.*

Weitere nützliche Beziehungen sind:

$$\begin{aligned}
\boldsymbol{\nabla} \cdot (\Phi \boldsymbol{F}) &= (\boldsymbol{\nabla}\Phi) \cdot \boldsymbol{F} + \Phi\, \boldsymbol{\nabla} \cdot \boldsymbol{F} \\
\boldsymbol{\nabla} \times (\Phi \boldsymbol{F}) &= (\boldsymbol{\nabla}\Phi) \times \boldsymbol{F} + \Phi\, \boldsymbol{\nabla} \times \boldsymbol{F} \\
\boldsymbol{\nabla} \cdot (\boldsymbol{F} \times \boldsymbol{G}) &= \boldsymbol{G} \cdot (\boldsymbol{\nabla} \times \boldsymbol{F}) - \boldsymbol{F} \cdot (\boldsymbol{\nabla} \times \boldsymbol{G}) \\
\boldsymbol{\nabla} \times (\boldsymbol{F} \times \boldsymbol{G}) &= \boldsymbol{F}\, (\boldsymbol{\nabla} \cdot \boldsymbol{G}) - \boldsymbol{G}\, (\boldsymbol{\nabla} \cdot \boldsymbol{F}) + (\boldsymbol{G} \cdot \boldsymbol{\nabla})\, \boldsymbol{F} - (\boldsymbol{F} \cdot \boldsymbol{\nabla})\, \boldsymbol{G} \\
\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{G}) &= \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{G}) - \Delta \boldsymbol{G}
\end{aligned}$$

Dabei ist Φ ein Skalarfeld und $\boldsymbol{A}$, $\boldsymbol{F}$, $\boldsymbol{G}$ sind Vektorfelder. Ausserdem gilt:

$$(\boldsymbol{F} \cdot \boldsymbol{\nabla}) = F_1 \frac{\partial}{\partial x_1} + F_2 \frac{\partial}{\partial x_2} + F_3 \frac{\partial}{\partial x_3} \,.$$