Assignment #2: due Tuesday, Oct. 30

Theoretical Astrophysics

Winter 2006/2007
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1. **Boltzmann equation with external potential** 30 pt

Given a gas at constant temperature T in an external gravitational potential $\Phi(\vec{x})$. Assume the distribution function can be separated in form $f = g(\vec{x})\, f_0(\vec{w})$, where

$$f_0(\vec{w}) = \left(\frac{m}{2\pi\, k\, T}\right)^{3/2} \exp\left(-\frac{\mathrm{m}\,\vec{w}^2}{2k\,T}\right) \tag{1}$$

is the Maxwell distribution function. Determine $g(\vec{x})$ from the Boltzmann transport equation.

2. **Equilibrium Distribution Function for Boltzmann's Equation** 40 pt

Assume the following distribution function (see also script equation 1.18),

$$f(\vec{x}, \vec{w}, t) = \exp\left[\alpha(\vec{x}, t) + \frac{m}{2}\beta(\vec{x}, t)\left(2\vec{v}(\vec{x}, t)\cdot\vec{w} - \vec{w}^2\right)\right] \,, \tag{2}$$

where $\vec{v}(\vec{x}, t) = \langle\vec{w}\rangle$ is the local mean (bulk) velocity and $\alpha(\vec{x}, t)$ and $\beta(\vec{x}, t)$ are scalar functions that depend on space and time only.

(a) Show that for the distribution function $f(\vec{x}, \vec{w}, t)$ the collision term in the Boltzmann equation vanishes when assuming fully elastic collisions (i.e. for ideal gases).

(b) Show furthermore that in general $f(\vec{x}, \vec{w}, t)$ is *not* a solution of Boltzmann's equation and find constraints on $\alpha(\vec{x}, t)$, $\beta(\vec{x}, t)$, and $\vec{v}(\vec{x}, t)$ such that Boltzmann's equation is satisfied.

(c) Compute the particle density $n(\vec{x}, t)$ and use the result to find the bulk velocity $\vec{v}(\vec{x}, t) = \langle\vec{w}\rangle = \int d^3w\, \vec{w} f \,/\, n(\vec{x}, t)$.

(d) Compute the mean energy per particle $\epsilon(\vec{x}, t)$ which is defined via $n(\vec{x}, t)\, \epsilon(\vec{x}, t) = \int d^3w\, m\vec{w}^2/2\, f$.