

## Assignment #6: due Tuesday, Nov. 27

### Theoretical Astrophysics

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#### 1. Accretion Disk

60 pt

Consider a thin (non-selfgravitating), axisymmetric, disk in cylindrical coordinates,  $(R, \theta, z)$  whose evolution can be described by the continuity and Navier-Stokes equation

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = \vec{g} - \frac{1}{\rho} \nabla P + \vec{\eta} \quad (1)$$

(here given in Cartesian coordinates), where  $\vec{\eta}$  is the viscous friction and  $\vec{g}$  is the gravitational acceleration due to the central star with mass  $M$ . Assume that the velocity does not vary with disk height  $z$  and  $v_z = 0$ .

- (a) Use the surface density  $\Sigma = \int \rho dz$  to rewrite the continuity equation in the form

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R} \frac{\partial}{\partial R} (R \Sigma v_R) = 0 \quad (2)$$

and derive the equation which determines the evolution for the angular momentum

$$\frac{\partial}{\partial t} (\Sigma R^2 \Omega) + \frac{1}{R} \frac{\partial}{\partial r} (\Sigma R^3 \Omega v_R) = T \quad (3)$$

where  $\Omega = v_\theta/R$  is the angular velocity and the function  $T$  involves the viscous terms.

- (b) Assume that the viscous force per unit area  $f_{\text{visc}}$  is proportional to the rotational shear  $A = R d\Omega/dR$ , i.e.

$$f_{\text{visc}} = \nu \rho A \quad (4)$$

where  $\nu$  is the kinetic viscosity, derive the viscous term  $T$ .

- (c) Now assume that the disk matter is in Keplerian orbits around the central star, i.e.  $\Omega = \sqrt{GM/R^3}$ , show that the disk evolution is then governed by the equation

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left( R^{1/2} \frac{\partial}{\partial R} (\nu \Sigma R^{1/2}) \right). \quad (5)$$

- (d) Consider a steady state, thin accretion disk, i.e.  $|\nabla P/\rho| \ll GM/r^2$ ,  $\partial\vec{v}/\partial t = 0$ . Show that the mass infall rate  $\dot{m} = -2\pi R \Sigma v_R$  is given by

$$\dot{m} = 3\pi\nu\Sigma \left[ 1 - \left( \frac{R_*}{R} \right)^{1/2} \right]^{-1} \quad (6)$$

where  $R_*$  are the radii with vanishing shear (say the stellar surface or the inner disk radius).

- (e) Given that the energy dissipation per unit area of the disk surface is given by

$$\dot{E} = -\nu\Sigma R^2 \left( \frac{d\Omega}{dR} \right)^2, \quad (7)$$

show that the total luminosity emitted from the accretion disk is then

$$L = \frac{GM\dot{m}}{2R_*}. \quad (8)$$

Compare this to the total potential energy released and discuss possible discrepancies.