Assignment #10: due Tuesday, January 8, 2008

Theoretical Astrophysics

Winter 2007/2008
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1. **Virial Theorem in Stellar Dynamics and Negative Heat Capacity** 25 pt

   Consider a system of N particles in which the potential between particles i and j, with $i, j \in N$, has the form

   $$\Phi(\vec{r}_i, \vec{r}_j) = \frac{C}{|\vec{r}_i - \vec{r}_j|^\alpha} \,, \tag{1}$$

   with constant C and exponent α.

   (a) Show that the scalar virial theorem has the form

   $$2K + \alpha W = 0 \,, \tag{2}$$

   where K is the kinetic energy and W is the potential energy.

   (b) Which values of α lead to a negative heat capacity?

   (c) Speculate about the long-term evolution of systems with negative heat capacity. What is the difference between stars and stellar clusters? What is the role of close binary systems in stellar clusters?

2. **Stability of Circular Orbits** 15pt

   Show that circular orbits in a given potential are unstable if the angular momentum per unit mass on a circular orbit decreases outward.

   Hint: Use cylindrical coordinates and look for stable trajectories with L_z = constant first.