Assignment #11: due Tuesday, Jan. 15

Theoretical Astrophysics

Winter 2007/2008

lecturer: Ralf Klessen, ZAH/ITA, Albert-Ueberle-Str. 2, 69120 Heidelberg

1. **Stellar dynamics around a black hole** 40 pt

   A black hole of mass M is imbedded in the center of an infinite, homogeneous, three-dimensional sea of mass-less test particles. Far from the hole, the test particles follow a Maxwell Boltzmann velocity distribution,

   $$f_0(\vec{v}) = \frac{n_0}{(2\pi\,\sigma^2)^{3/2}} \exp\left(-\frac{v^2}{2\sigma^2}\right) , \tag{1}$$

   where n_0 is the density normalization and σ the velocity dispersion (which we assume to be fixed, i.e. we use an isothermal approximation).

   (a) Show that the density distribution of the test particles that are *not bound* to the hole is

   $$n(r) = n_0 \left\{ 2\sqrt{\frac{r_H}{\pi\, r}} + e^{r_H/r} \left[1 - \mathrm{erf}\left(\sqrt{\frac{\mathrm{r_H}}{\mathrm{r}}}\right)\right]\right\} , \tag{2}$$

   where $r_H = GM/\sigma^2$ is the "sphere of influence" of the black hole, and the error function is defined as

   $$\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x \mathrm{d}t\, e^{-t^2} . \tag{3}$$

   Hint: Assume the age of the system is larger than the relaxation time. Hence, there are always stars in the high-velocity tail of the Maxwell Boltzmann distribution that exceed the local escape velocity. Use this to calculate the fraction of unbound stars at each radius. Assume furthermore the system is in thermal equilibrium, then the density distribution should be that of an isothermal sphere, i.e. $n(r) \propto \exp[-\Phi(r)/\sigma] \propto \exp[r_{\rm bh}/r]$. The final density profile follows from the combination of both constraints.

   (b) Show that close to the black hole, i.e. for $r \ll r_H$, $n(r) \propto r^{-1/2}$. Thus there is a weak density cusp around the hole.

2. **Mean free path of gas particles and photons** 20 pt

   (a) Calculate the mean free path of the nitrogen molecules you are currently breathing in and out. Take the radius of N_2 to be 1 Å and assume a (room) temperature of 20 °C. What is the average time between collisions?

(b) Calculate how far you could see if the atmosphere here in Heidelberg had the opacity of the solar photosphere. Use the simple value for electron scattering, $\kappa \approx 0.2\,\mathrm{cm^2\,g^{-1}}$.

(c) What is the average mean free path of a photon in Sun if we assume a mean density of $1.4\,\mathrm{g\,cm^{-3}}$ and again take only electron scattering into account?

(d) How long does it take a photon released at the center of the Sun to escape through the surface? Assume the photons "diffuse" outwards, so that the succession of collisions (actually, absorption and reemission events) can be described as a random walk.