Stellar Astronomy and Astrophysics (SS08):

Exercise 5 (for June 19, 2008)

1. Period-Luminosity relation for Cepheids:

In the lecture we derived

1. the period-density relation $P \propto \rho^{-1/2}$,
2. A Mass-Luminosity relation $L \propto M^{\alpha}$,
3. and have shown that the instability strip for pulsation is restricted to a narrow strip of almost constant $T_{\rm eff}$ in the HRD.

a) If we assume that for Cepheids $\alpha = 4$ (a somewhat higher value than for main-sequence stars), show that a period-luminosity relation ($\log L$ vs. $\log P$ exists. Hint: Use Stefan-Boltzmann's law.

b) Assume that the instability strip is not at $T_{\rm eff} =$ const. but more accurately given by $\log L = \beta \log T_{\rm eff} + \delta$. How does the period-luminosity function look like in this case?

c) Is this relation sufficient to determine the distance of a Cepheid from measuring its period and an apparent magnitude?

2. Future of the Earth:

The sun continuously looses mass Δm since it is converting hydrogen into helium.

a) The present-day luminosity of the sun is $L_{\odot} = 3.846 \cdot 10^{26}\,\mathrm{W} = 3.846 \cdot 10^{33}\,\mathrm{erg\ s^{-1}}$. Assume that the sun had the same luminosity since it was formed 4.6 billion years ago. What is the total mass-loss by the conversion of mass into energy that the sun has experienced since its birth? By what amount did the semi-major axis of the earth orbit change due to this effect?

b) The sun also loosed mass by the solar wind. At the distance of the earth the mean velocity of the solar wind is $v = 500\,\mathrm{km/s}$ and the mean density amounts to $\rho = 5 \cdot 10^{6}$ protons per m^3. The mass of a proton is $1.672 \cdot 10^{-24}\,\mathrm{g}$. Assume that the solar wind is expelled isotropically and had the same value since 4.6 billion years (caution: actually the mass-loss was much higher during the T Tauri phase at the beginning of the life of the sun). What is the total mass-loss due to the solar wind since the birth of the sun and by what amount did the earth radius change due to this effect.

c) During the red-giant phase, particularly at the end of the AGB phase the sun will loose at significant part of its mass. Before the sun arrives at its largest radius (about 180 current solar radii of $6.96 \cdot 10^{10}\,\mathrm{cm}$) it looses about one third of its current mass. By what amount did the earth radius change due to this effect and does this help the earth to "survive"? (1 AU$= 1.496 \cdot 10^{13}\,\mathrm{cm}$).

3. Relative Abundances for CNO in equilibrium:

Assume that the CNO cycle is in equilibrium and the temperature is about $T = 2 \cdot 10^{7}\,\mathrm{K}$. In this case the lifetimes against proton capture is $\tau(^{15}\mathrm{N}) = 30$ years, $\tau(^{13}\mathrm{C}) = 1600$ years, $\tau(^{12}\mathrm{C}) = 6600$ years, $\tau(^{14}\mathrm{N}) = 6 \cdot 10^{5}$ years. Oxygen decays in $\tau(^{15}\mathrm{O}) = 1$ minute. What are the abundances of these CNO isotopes in equilibrium?